

Global Solar Radiation Analysis Using Response Surface Methodology in Bauchi, Nigeria

Edema, O.G., Abdullahi, M., Akinbolusere, A. O., and Isa A. I.

Department of Physics/Electronics, Federal Polytechnic, Auchi, Nigeria.

Corresponding Author: edemagregori@gmail.com

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Abstract

In this paper, meteorological data was used to estimate the global solar radiation in Bauchi, Bauchi state, Nigeria. The study examines the impact of cloud cover, relative humidity and the daily values of global solar radiation in Bauchi, Nigeria, using different proposed empirical models for a period of 10 years (1985-1994). The Angstrom model was used as the base, while other regression equations were developed by modifying Angstrom equation. The accuracy of the developed model was tested using statistical indicator that is, Response Surface Methodology (RSM). The equation with high value of correlation coefficient is considered as the best equation and is given as,

$$\frac{H}{H_0} = a_0 + a_1 \left(\frac{\%}{s_0} \right) + a_2 (T) + a_3 (C_c) + a_4 \left(\frac{\%}{s_0} T \right) + a_5 \left(\frac{\%}{s_0} C_c \right) + a_6 (C_c T) + a_7 \left(\frac{\%}{s_0} C_c T \right)$$

It was observed that the new model can be used for estimating daily values of global solar radiation with a higher accuracy and has good adaptability to highly variable weather condition. The equation could be employed in estimating global solar radiation of location that has the same geographical location information as Bauchi.

1.0.Introduction

The quantity of solar radiation reaching the Earth's surface varies dramatically as a function of changing atmospheric conditions as well as the changing position of the Sun through the day. Accurate data of global solar radiation is necessary at various steps of the design, simulation, and performance evaluation of any research work involving solar energy [1, 2, 3]. The solar radiation modeling has shown significant progress in recent decades, reaching at present integration in geographic information systems that allow quantification at its spatial distribution, also provide detailed estimates or forecast climate changes which have improved significantly in recent years. Several models have been proposed for generation of global radiation. Global solar radiation is of economic importance as a renewable energy alternative, recently global solar radiation has been studied due to its importance in providing energy for Earth's climate system. The solar radiation reaching the Earth's surface depends upon climatic conditions of a location, which is essential to the prediction, and design of a solar energy system. The random nature of global solar radiation is included in all proposals, but the way of implementing this in a model varies significantly [4, 5, 6]. Meteorological parameters such as

sunshine duration, cloud cover, relative humidity, and temperature play an important role in the solar energy budget of the Earth and in the transfer of energy between the surface and the atmosphere. Meteorological parameters are frequently used as predictors of atmospheric conditions (examples are humidity, precipitation, temperature, pressure, cloudiness, solar radiation and wind), since acquiring detailed atmospheric conditions require advance measurement. Meteorological parameters have been used to estimate atmospheric transmittance coefficient in parametric model also called meteorological model [7, 8, 9]. Response-surface methodology comprises a body of methods for exploring for optimum operating conditions through experimental methods. It consists of a group of mathematical and statistical techniques used in the development of an adequate functional relationship between a response of interest. A number of correlations involving global solar radiation and sunshine hours for different locations in Nigeria have been studied by different researchers. Among these are, [10] presented a quantitative review and classification of empirical models for predicting global solar radiation in West Africa. [11] developed models for estimating global solar radiation using common meteorological data in Akure, Nigeria. [12] developed a model for estimation of global solar radiation in Bauchi with regression coefficients $a = 0.24$ and $b = 0.46$. [13] developed models for empirical correlation of global solar radiation with Meteorological data for Onne, Nigeria. Also, [14] developed empirical models for evaluation of various global solar radiation models for Nigeria. [15] estimated the global and diffuse solar radiations for selected cities in Nigeria and [16] studied the relationship between air temperature oscillations and solar variability on short and medium time scales. They presented the responses of regional climates to solar variations on shorter time scales using two datasets: one for the air temperature in Nanjing and the Greenwich sunspot number, and the other for the air temperature in Shijiazhuang and the United States sunspot number. [17] developed empirical models for the correlation of Global Solar Radiation with meteorological data for Northern Nigeria. [18] presented methods and strategy for modeling daily global solar radiation with measured meteorological data – A case study in Nanchang station, China. The aim of this research is the development of equations which correlate the global solar radiation with meteorological parameters for Bauchi (latitude of 10.283° N, longitude of 9.817° E and an altitude of 610 m) in northern Nigeria.

2.0 Materials and Method

The raw data collected from Nigerian Meteorological Agency, Oshodi, Lagos was processed into required formats with information on year, month, day, time and measured parameters. The monthly means of the daily values of the meteorological parameters were evaluated by taking the average of all the days in every month for each month for each year of the data. The daily average temperature was evaluated by taking mean value of the maximum and minimum temperature for any day and its monthly means evaluated alongside the other parameter.

The data obtained from Nigerian Meteorological Agency, Oshodi, Lagos includes; global solar radiation, sunshine duration, relative humidity, cloud cover and ambient temperature for Bauchi. The collected data cover the period of ten years (1985-1994). Bauchi is on latitude 10.283° N, longitude 9.817° E and altitude 610.0m above the sea level. These values were obtained by the use of GPS (Global Positioning System) equipment. The most convenient and widely used correlation for predicting solar radiation was developed by Angstrom. To develop the model, the global solar radiation measured using Gun-Bellini distillate was converted to useful form ($MJ / m^2 / day$) using a conversion factor of 1.1364 proposed by [17]. The first correlation proposed for estimating the monthly mean global solar radiation on the horizontal

surface H ($MJ / m^2 / day$) using the sunshine duration data was done by [19]. [20] has put the Angstrom correlation in more convenient form as:

$$\frac{H}{H_0} = a + b \left(\frac{S}{S_0} \right) \quad 1$$

Where H is the measured monthly mean of the daily global solar radiation, H_0 is the monthly mean extraterrestrial solar radiation on horizontal surface, S is the monthly mean daily brightness sunshine hours. S_0 is the maximum possible monthly mean daily sunshine hour or the day length, $\frac{H}{H_0}$ is the clearness index, $\frac{S}{S_0}$ is the fraction of sunshine hours ‘a’ and ‘b’ are regression constants.

Therefore, Clearness index $K_T = \frac{H}{H_0}$ is defined as the ratio of the measured monthly mean of the daily global solar radiation (H), to the monthly mean extraterrestrial solar radiation on horizontal surface (H_0).

The above equation has been found to be very convenient to a large number of locations and most widely used correlation.

The extraterrestrial solar radiation on horizontal surface is given by [1],

$$H_0 = \frac{24}{\pi} I_{SC} \left(1 + 0.0033 \cos \frac{360}{365} n \right) \left(\cos L \cos \delta \sin \omega_s + \frac{2\pi}{360} \omega_s \sin L \sin \delta \right) \quad 2$$

where I_{SC} is the solar constant, n is the monthly mean value of observed sunshine hour, δ = solar declination, ω_s = hour angle and L is the latitude angle for the location in degrees.

$$\omega_s = \cos^{-1} (-\tan L \tan \delta)$$

$$\delta = 23.45 \sin \left(\frac{360}{365} \{284 + n\} \right)$$

$$S_0 = \frac{2}{15} \cos^{-1} (-\tan L \tan \delta) \quad 3$$

Latitude = 10.283° N

Longitude = 9.817° E

Altitude = 610.0m above the sea level

All the parameters in equation 1, help in the determination of H_0 by computations. The value of the measured monthly average of the daily solar radiation on the horizontal surface and the extraterrestrial solar radiations H/H_0 are used with some parameters for the solar radiation analysis as used in the models in this work.

2.1. Data Collection

Table 1: Mean monthly daily global solar radiation (H), sunshine duration (S), mean relative humidity (RH), mean cloud cover (C_C), monthly ambient temperature (T) and monthly mean extraterrestrial solar radiation (H_0).

Month	H ($MJ / m^2 / day$)	S (h)	RH (%)	C_C (octas)	T (°C)	H_0 ($MJ / m^2 / day$)
JAN	14.12	7.1	26.2	6.09	29.26	31.84
FEB	15.54	8.02	32.1	5.53	32.74	34.52
MAR	16.98	7.71	37.3	5.14	35.37	36.84

APR	17.28	7.71	47.4	5.7	39.7	37.93
MAY	16.5	7.27	64.2	6.54	34.42	37.63
JUN	15.08	7.1	90.3	6.58	32.05	37.07
JUL	13.29	6.38	92.8	7.17	29.69	37.15
AUG	13.29	5.22	96.4	7.45	29.34	37.56
SEP	14.55	6.81	94.9	6.07	30.43	37.04
OCT	16.08	7.87	78.1	6.36	31.82	35.01
NOV	16.24	8.34	40.4	5.34	31.34	32.36
DEC	14.77	7.66	33.2	5.72	30.61	30.93
TOTAL	183.72	87.19	733.3	73.69	386.77	425.88

$$Y = \frac{2(\max - x)}{\max - \min} - 1 \quad 4$$

max is the maximum value recorded in the year and min is the minimum value recorded in the year and x is the value recorded for a particular month in the year.

Table 2: Extraterrestrial solar radiations (H/H_0), sunshine hour (S/S_0), mean temperature (T) and cloud cover (C_c) obtained from table 1 using equation 4.

H/H_0	S/S_0	T ($^{\circ}\text{C}$)	C_c (octas)
0.21	0.73	-1	-0.18
0.3	0.81	-0.33	-0.66
0.45	0.3	0.17	-1
0.37	-0.16	1	-0.52
0.14	-0.01	-0.01	0.21
-0.28	-0.1	-0.47	0.25
-0.95	-0.62	-0.92	0.76
-1	-1	-0.98	1
-0.47	-0.15	-0.78	-0.19
0.42	0.5	-0.51	0.06
1	0.9	-0.6	-0.83
0.67	1	-0.74	-0.5

2.2. Models and Data Analysis

The regression analysis computation obtained in the software MINITAB, using three meteorological parameters; monthly mean average temperature (T), Sunshine hours (S/S_0) and Cloud cover (C_c).

Model Equations with one variable

Model 1

$$\frac{H}{H_0} = a_0 + a_1 \left(\frac{S}{S_0} \right) \quad 5$$

The coefficient of correlation of 0.8758 exists between the clearness index and relative sunshine duration also coefficient of determination of 0.767 implies 76.7% of clearness index can be accounted using relative sunshine duration.

Model 2

$$\frac{H}{H_0} = a_0 + a_1(T) \tag{6}$$

The correlation of coefficient of 0.3899 exists between the clearness index and monthly mean daily temperature also coefficient of determination of 0.152 implies 15.2% of clearness index can be accounted using monthly mean daily temperature.

Model 3

$$\frac{H}{H_0} = a_0 + a_1(C_c) \tag{7}$$

The coefficient of correlation of 0.8468 exists between the clearness index and cloud cover also coefficient of determination of 0.7171 implies 71.7% of clearness index can be accounted using monthly mean cloud cover.

2.3. Model Equations with no Interaction

The equations (8) - (14) can be modified by incorporating extra parameters to the set of correlation equations for two and three variables.

Model 4

$$\frac{H}{H_0} = a_0 + a_1\left(\frac{s}{s_0}\right) + a_2(T) \tag{8}$$

The coefficient of correlation of 0.9566 exists between the clearness index, relative sunshine duration and monthly mean daily temperature, the coefficient of determination of 0.9150 implies 91.5% of clearness index can be accounted by using relative sunshine duration and monthly mean daily temperature.

Model 5

$$\frac{H}{H_0} = a_0 + a_1\left(\frac{s}{s_0}\right) + a_2(C_c) \tag{9}$$

The correlation of coefficient of 0.9165 exists between the clearness index, relative sunshine duration and monthly mean cloud cover, the coefficient of determination of 0.842 implies 84.2% of clearness index can be accounted using relative sunshine duration and monthly mean cloud cover.

Model 6

$$\frac{H}{H_0} = a_0 + a_1(T) + a_2(C_c) \tag{10}$$

The correlation of coefficient of 0.8468 exists between the clearness index, monthly average daily temperature, and monthly mean cloud cover, the coefficient of determination of 0.7170 implies 71.7% of clearness index can be accounted using monthly mean daily temperature and monthly mean cloud cover.

Model 7

$$\frac{H}{H_0} = a_0 + a_1\left(\frac{s}{s_0}\right) + a_2(T) + a_3(C_c) \tag{11}$$

The coefficient of correlation of 0.9566 exists between the clearness index, relative sunshine duration, monthly mean daily temperature and monthly mean cloud cover, the coefficient of determination of 0.9150 implies 91.5% of clearness index can be accounted using relative sunshine duration, monthly average daily temperature and monthly mean cloud cover.

2.4. Model Equations with Interaction

Model 8

$$\frac{H}{H_0} = a_0 + a_1 \left(\frac{s}{s_0}\right) + a_2 (T) + a_3 \left(\frac{s}{s_0} T\right) \quad 12$$

The coefficient of correlation of 0.9566 exists between the clearness index, relative sunshine duration, monthly mean daily temperature and product of relative sunshine duration and monthly mean daily temperature, the coefficient of determination of 0.9150 implies 91.5% of clearness index can be accounted using relative sunshine duration, monthly average daily temperature and product of relative sunshine duration and monthly mean daily temperature.

Model 9

$$\frac{H}{H_0} = a_0 + a_1 \left(\frac{s}{s_0}\right) + a_2 (C_c) + a_3 \left(\frac{s}{s_0} C_c\right) \quad 13$$

The coefficient of correlation of 0.9187 exists between the clearness index, relative sunshine duration, monthly mean cloud cover and product of relative sunshine duration and monthly mean cloud cover, the coefficient of determination of 0.8440 implies 84.4% of clearness index can be accounted using relative sunshine duration, monthly mean cloud cover and product of relative sunshine duration and monthly mean cloud cover.

Model 10

$$\frac{H}{H_0} = a_0 + a_1 \left(\frac{s}{s_0}\right) + a_2 (T) + a_3 (C_c) + a_4 \left(\frac{s}{s_0} T\right) + a_5 \left(\frac{s}{s_0} C_c\right) + a_6 (C_c T) + a_7 \left(\frac{s}{s_0} C_c T\right) \quad 14$$

The coefficient of correlation of 0.9701 exists between the clearness index, relative sunshine duration, monthly mean daily temperature, monthly mean cloud cover and with their interactions as shown in equation 14, the coefficient of determination of 0.9410 implies 94.1% of clearness index can be accounted using relative sunshine duration, monthly mean daily temperature, monthly mean cloud cover and with their interactions.

The Response surface methodology (RSM) used in this work is a collection of mathematical and statistical techniques useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response. The linear and quadratic models' equations provided the best estimate for the monthly values. The monthly data processed in preparation for the correlation of mean average temperature, cloud cover (clearness index), sunshine duration for Bauchi are shown in the Tables 3 and 4. Comparing the results, we can see that all the regression equations gave very good results.

3.0. Results and Discussion

When the measured and predicted values of solar radiation obtained from model equations were plotted, they showed almost the same curve. This revealed that the model equations can be used to predict the solar radiation of any part of the world that possesses similar climatic factors like that of Bauchi town. The following plotted graphs (Figure1 – Figure10) illustrated these points better. The vertical axis is the measure of the global solar radiation ($MJ / m^2 / day$)

, and the horizontal axis is the total month in the year from January to December for all the graphs.

It is also observed that the calculated values from the models used are in good agreement with the measured data. Almost all the figures above show variation of the parameters throughout the years. This helps in analyzing the global solar radiation, which can be applied to other regions in the globe that have closer climatic factors.

A number of multi-linear regression equations were developed to predict the relationship between global solar radiations with one or more combinations of the following meteorological parameters: fraction of sunshine or sunshine duration, mean average temperature, and cloud cover (clearness index) for Bauchi for a period of ten years (1985 – 1994). Equation 10 gives a good solar radiation analysis for Bauchi town. The model indicates good agreement between the measured and estimated values for Bauchi town, with high value of R and R² adjusted. The regression equation with the highest values of R and R² was developed using MINITAB software which is given as,

$$\frac{H}{H_0} = 0.109 + 1.02\left(\frac{S}{S_0}\right) + 0.531(T) - 0.075(C_c) + 0.33\left(\frac{S}{S_0}T\right) + 0.65\left(\frac{S}{S_0}C_c\right) + 0.322(C_cT) + 1.15\left(\frac{S}{S_0}C_cT\right) \quad 15$$

where H/H_0 is the clearness index, S/S_0 is the fraction of sunshine hour, C_c is the cloud cover and T is the ambient temperature.

4.0. Conclusion

Using the Angstrom model as the base, other regression equations were developed by modifying the Angstrom equation. The values of the correlation coefficient (R) and coefficient of determination (R^2) were determined for each equation. The monthly mean daily global solar radiation, fraction of sunshine duration, maximum temperature, cloud cover and monthly mean ambient temperature have been employed in this study to develop several correlation equations. Three variables have been developed with different types of equations obtained. It was observed that equation (10) has the highest value of correlation coefficient and coefficient of determination, which gives good results. The equation could be employed in the estimation of global solar radiation in Bauchi and other similar locations. The recommended model equation will in no small measure help in predicting the solar radiation in Bauchi town.

Table3: Monthly mean of the hour angle (ω), Solar declination (Δ), Extraterrestrial solar radiation (H), Sunshine duration (S), Relative humidity (RH), Cloud cover (C_c) and Temperature (T).

Month	ω	Δ	H	S	RH	C_c	T
JAN	84.656	-20.202	31.84	7.1	26.2	6.09	29.26
FEB	87.478	-16.354	34.52	8.02	32.1	5.53	32.74
MAR	89.334	-12.624	36.84	7.71	37.3	5.14	35.37
APR	94.181	12.5	37.93	7.71	47.4	5.7	39.7
MAY	93.71	19.235	37.63	7.27	64.2	6.54	34.42
JUN	94.547	23.209	37.07	7.1	90.3	6.58	32.05

JUL	94.018	20.759	37.15	6.38	92.8	7.17	29.69
AUG	92.388	12.62	37.56	5.22	96.4	7.45	29.34
SEP	90.223	1.195	37.04	6.81	94.9	6.07	30.43
OCT	88.02	-10.713	35.01	7.87	78.1	6.36	31.82
NOV	86.302	-20.055	32.36	8.34	40.4	5.34	31.34
DEC	84.282	23.704	30.93	7.66	33.2	5.72	30.61
TOTAL	1079.139	33.274	425.88	87.19	733.3	73.69	386.77

Table 4: Regression constant of the model used

Equations	a ₀	a ₁	a ₂	a ₃	a ₄	a ₅	a ₆	a ₇	R ²
Model1	-0.0866	0.863	-	-	-	-	-	-	76.7%
Model2	0.251	0.417							15.2%
Model3	-0.042	-0.854							71.7%
Model4	0.0913	0.861	0.412						91.5%
Model5	-0.0850	0.541	-0.431						84.2%
Model6	-0.040	0.005	-0.852						71.7%
Model7	0.0879	0.849	0.404	-0.016					91.5%
Model 8	0.0914	0.856	0.411	-0.007					91.5%
Model 9	-0.061	0.546	-0.421	0.078					84.4%
Model 10	0.109	1.02	0.531	-0.075	0.33	0.65	0.322	1.15	94.1%

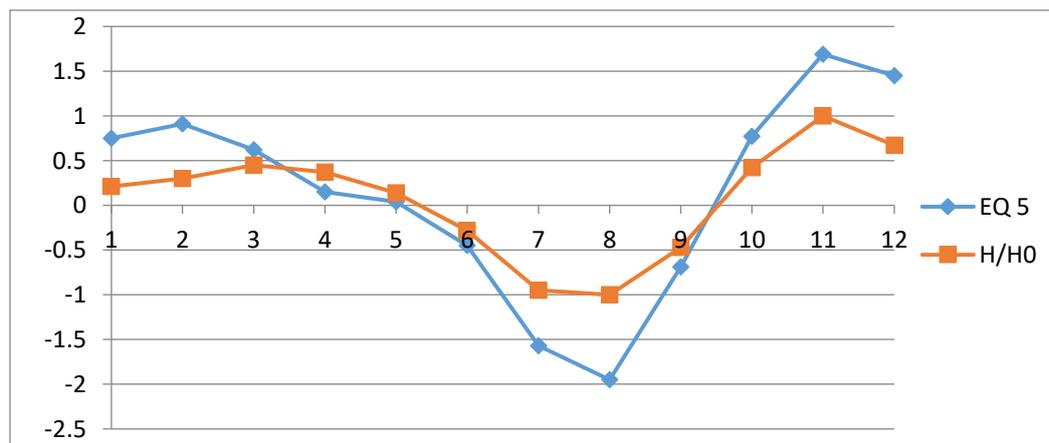


Figure: Graph of measured global solar radiation and equation 5 against months of the year

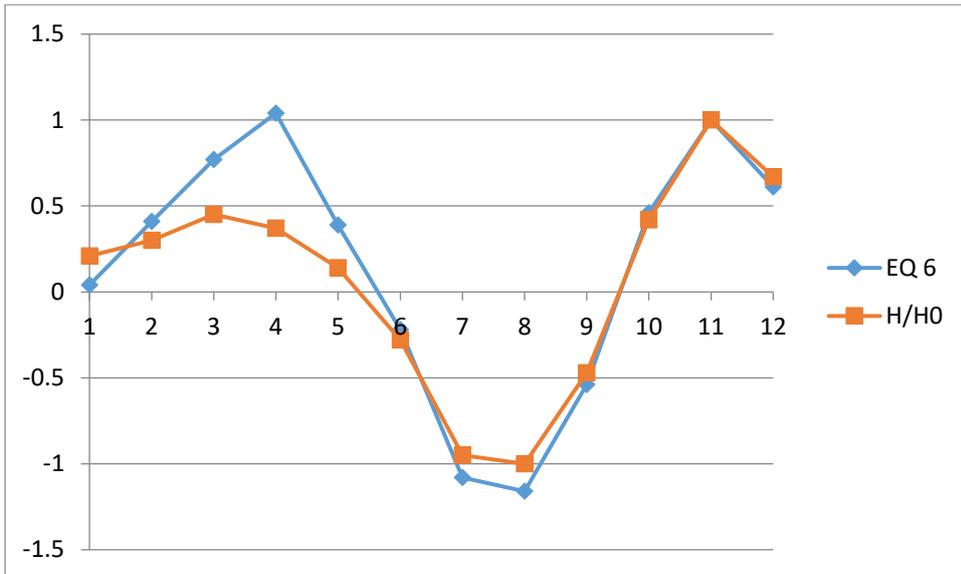


Figure 2: Graph of measured global solar radiation and equation 6 against months of the year

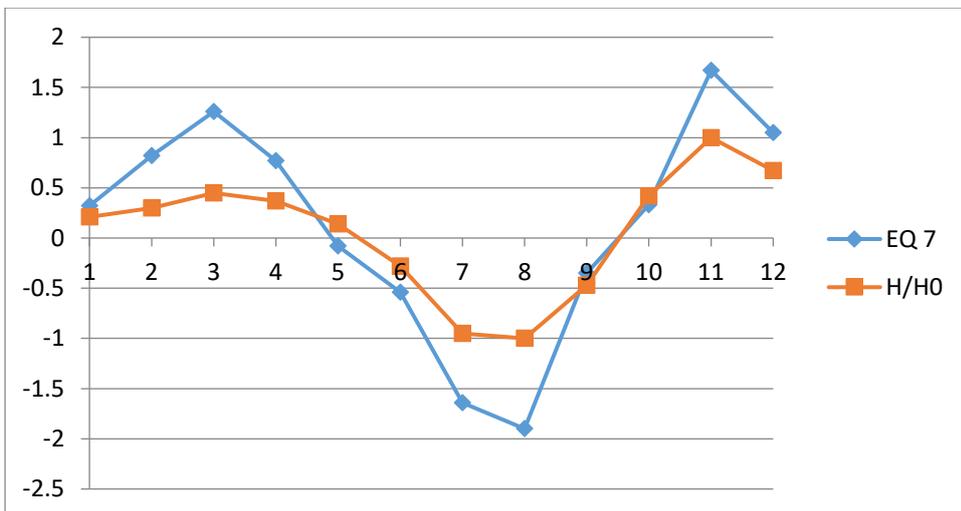


Figure 3: Graph of measured global solar radiation and equation 7 against months of the year

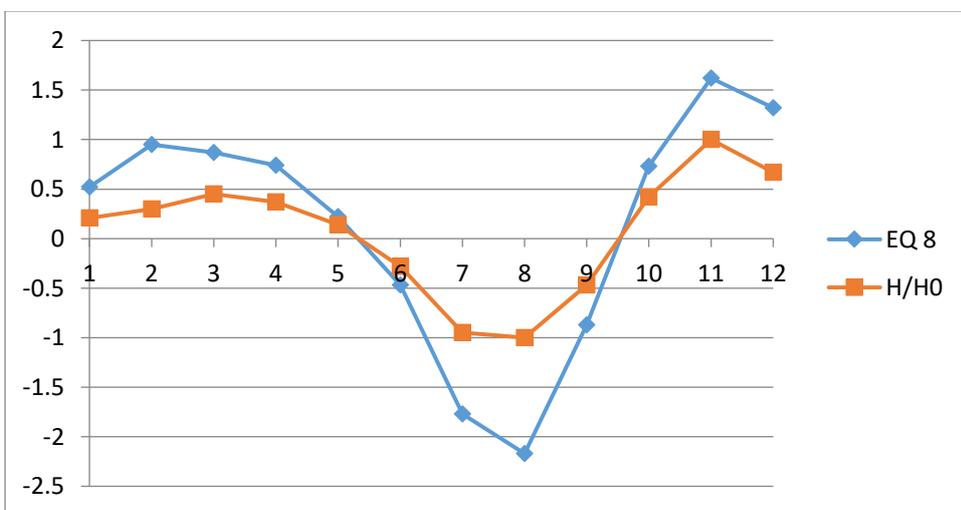


Figure 4: Graph of measured global solar radiation and equation 8 against months of the year

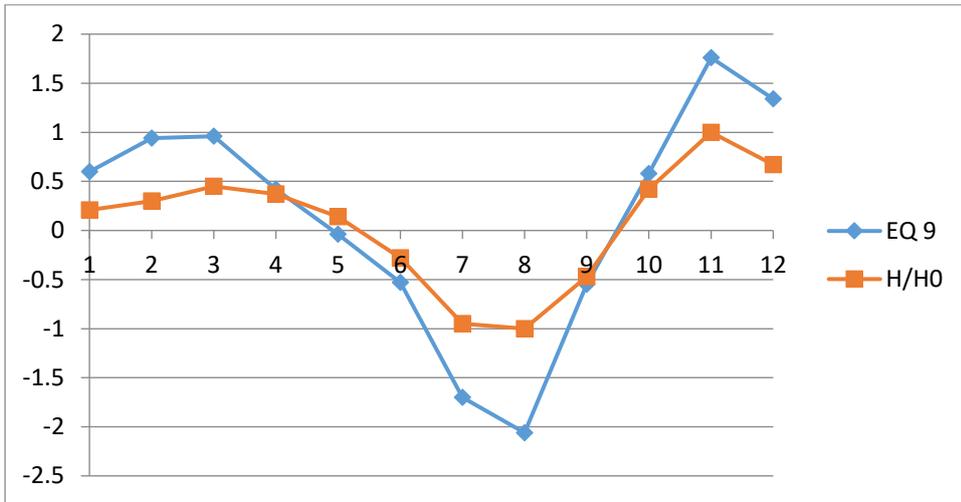


Figure 5:Graph of measured global solar radiation and equation 9 against months of the year

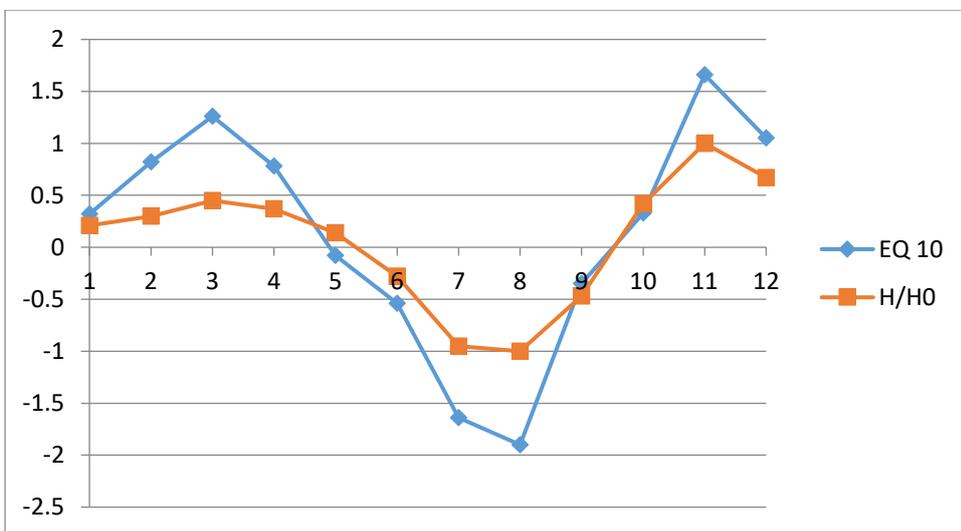


Figure 6 the graph of measured global solar radiation and equation10 against months of the year.

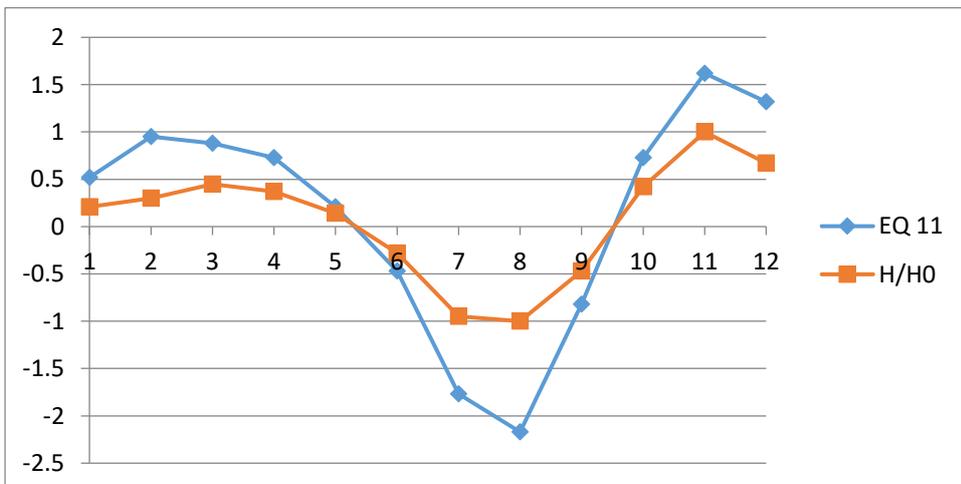


Figure 7 the graph of measured global solar radiation and equation11 against months of the year

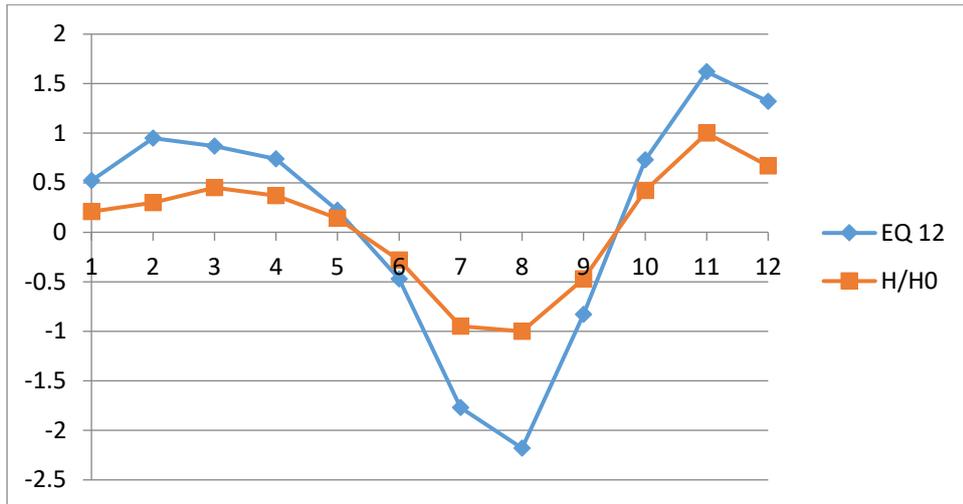


Figure 8: Graph of measured global solar radiation and equation12 against months of the year

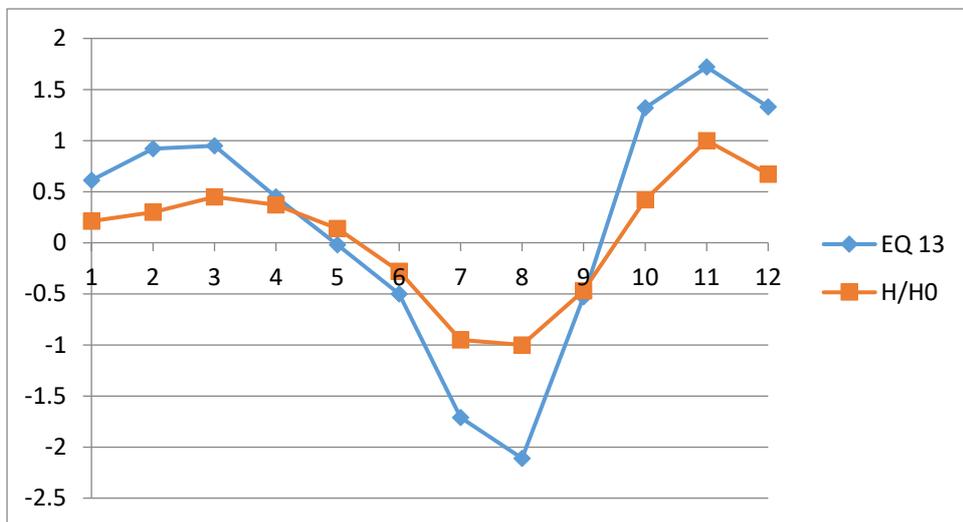


Figure 9: Graph of measured global solar radiation and equation13 against months of the year

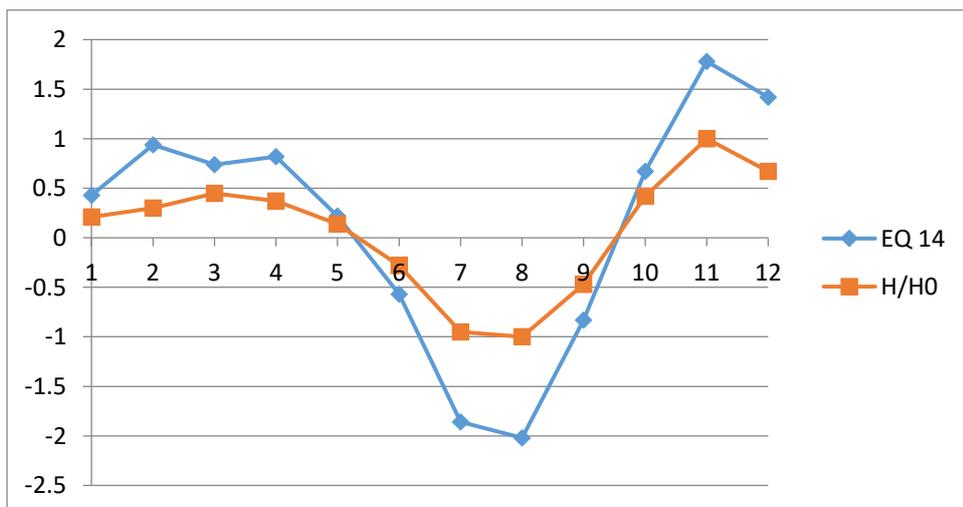


Figure10: Graph of measured global solar radiation and equation14 against months of the year

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