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A Review on de Broglie Stationary Wave Hypothesis from Classical Mechanics

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Abstract

This study examines the relationship between wavelengths and properties of orbiting leptons in atomic systems, with a focus on comparing the Bohr and de Broglie hypotheses. The analysis reveals that the orbitals are not equally spaced, contradicting initial assumptions. The de Broglie equations support this finding by demonstrating a non-linear relationship between wavelength and the principal quantum number, n. The number of wavelengths, denoted as η, is shown to be proportional to n, indicating that each orbital has a consistent number of wavelengths. The study argues that inner leptons moving at higher speeds exhibit shorter wavelengths, while outer leptons moving at slower speeds have longer wavelengths. Furthermore, it is observed that energy is higher in s state orbitals or perfectly circling orbits, which could offer insights into unresolved nuclear structure effects like the Lamb shift. The analysis points out limitations in both Bohr's consideration of lepton speed and the de Broglie hypothesis's failure to compare wavelength and lepton speed. By deepening our understanding of wavelength relationships, lepton speed, and orbital properties, this research paves the way for advancing atomic system theories and addressing unresolved phenomena. It underscores the importance of developing a comprehensive framework that reconciles the differing perspectives of the Bohr and de Broglie hypotheses to fully comprehend the behavior of orbiting leptons.

1. Introduction

The study of atomic physics and the behavior of subatomic particles has led to numerous breakthroughs in our understanding of the universe and the fundamental laws of nature. Atomic physics has numerous practical applications, including the development of new materials, medicine, and technology. The continuous evolution of atomic models and scientific inquiry demonstrates the importance of advancing our knowledge of the world around us. Atomic nuclei are quantum bound states of positively charged proton and the uncharged neutron, giving a total number of *A* nucleons [1]. The discovery of the structure of the atomic nucleus, after the discovery of different kinds of radioactive decays, begins with the fundamental paper by Rutherford (1911), in which he explained the large-angle alpha particle scattering from gold that had been discovered earlier by Geiger and Marsden. Back in the early 1900s, Greiger and Marsden carried out scattering experiments with metallic foil targets such as aluminum, copper, silver, and gold (of charge *Ze*) using alpha particles as the probe particles. They observed that most of the alpha particles were not deflected at all; but that of the particles that had suffered deflection, some were deflected back to the same side of the foil from where they were initially generated. In fact, about one in 20,000 alpha particles were scattered through an angle greater than 90° by a gold film 0.4 microns thick. Rutherford then showed mathematically the probability that α–particle will be scattered through a solid angle. His mathematical relation agrees very well with experimental data. Rutherford's analysis of the Greiger-Marsden experiment indicated that the chief portion of an atom is empty space, but that there exists somewhere inside the atom, a very massive positively charged region that makes the entire atom overall electrically neutral. Rutherford suggested that an atom contains at its center a charge whose magnitude is equal to an integral multiple of the electronic charge and is surrounded by a sphere with a homogeneous distribution of electrons [2-4]. Rutherford theory gave birth to the idea of the nucleus and the hypothesis of the proton, a positively charged particle within the nucleus. It was showed that the nuclear charge number *Z* equaled the atomic number. Using the first mass separators, Soddy in 1913 was able to show that one chemical element could contain atomic nuclei with different masses, forming different isotopes. This model, like many others of the time, consisted of protons and electrons. All of these structure suggestions occurred before James Chadwick discovered the neutron in 1932, which not only explained certain difficulties of previous models (e.g., the problems of the confinement of the electron or the spins of light nuclei), but opened the way to a very rapid expansion of our knowledge of the structure of the nucleus. Shortly after the discovery of the neutron, Heisenberg in 1932, proposed that the proton and neutron are two states of the nucleon classified by a new spin quantum number, the isospin.

An electron was discovered by the investigation of cathode rays [5], and Rutherford scattering showed that the positive charge and almost the entire mass of an atom is concentrated in its center in form of an atomic nucleus [6]. Also, the spectral density of black-body radiation explained by Planck using the quantum hypothesis, motivated the Bohr model of the atom [7], according to which the electron can revolve around the nucleus only on certain quantized orbits. The Bohr atomic model is one of the starting points of the descriptions of energy spectrum of the hydrogen-like atoms [8]. In 1913, he developed the postulates of a new quantum theory, that the angular momentum of an electron in the hydrogen atom was quantized and calculates the allowed energy of the electron in one of its allowed orbit. The measurement and theory of radiative transition in Bohr model of hydrogen atom provides information on the Rydberg constant, the proton, deuteron charge radii and the relative atomic mass of the electron [9]. Based on Bohr theory, Sommerfeld, (1916) developed a formula that takes into account the magnetic interaction between the spin of the electron on its own axis and its orbital motion [10]. This gives rise to the fine structure of energy levels of hydrogen-like atoms. However, despite the success of describing the quantized energies, the Bohr-Sommerfeld model has difficulties with the generalization to many-electron systems [11,12].

2. Methodology

Starting with the well-known Bohr's quantization of orbital angular momentum [13],

$$m_l v_n r_n = m_l \omega r_n^2 = n\hbar \qquad (1)$$

where $\hbar$ is the Planck's constant, m_l is the lepton mass, e^- is the charge of lepton r_n is the distance of lepton from the center of nucleus, v_n is the tangential velocity and *n* is the principal quantum number which takes values of 1, 2, 3, …, ∞. The allowed radii for lepton in circular orbits can obtain using (1) as

$$r_n = \frac{n\hbar}{m_l v_n} \tag{2}$$

The centripetal force F_{centr} required keeping the lepton in a circular orbit is

$$F_{\text{centr}} = \frac{m_l v_n^2}{r_n} \tag{3}$$

and the electric force of attraction between the nucleus and lepton are according to Coulomb's law given by

$$F_C = -\frac{Z\gamma}{r_n^2} \tag{4}$$

where $\gamma = ke^2$ is the coulomb constant. When the forces (4) and (3) balanced, then,

$$r_n = \frac{Z\gamma}{m_f v_n^2} \tag{5}$$

Comparing (2) and (5) gives the allowed radii for lepton in circular orbits,

$$r_n = \frac{n^2\hbar^2}{Z\gamma m_l} = \frac{a_l}{Z} n^2 \tag{6}$$

where $a_l = \hbar^2/\gamma m_l$ is called the Bohr radius. For an electron $a_0 = \hbar^2/\gamma m_e$ and for muon $a_\mu = \hbar^2/\gamma m_\mu$. The speed of orbiting lepton can be deduced by rearranging (2) and the substitution of (6) as follows:

$$v_n = \frac{n\hbar}{m_l}\left(\frac{Z}{n^2 a_l}\right) = \frac{\gamma}{\hbar}\left(\frac{Z}{n}\right) \tag{7}$$

However, the wavelength of orbiting leptons can be determined from the Bohr's quantization (1) as

$$\lambda = \frac{2\pi m_l v_n}{\hbar} \frac{r_n^2}{n} \tag{8}$$

where the use of $v_n = \lambda f$ has been made [14]. Using equation (7) and (6) the lepton wavelength (8) becomes,

$$\lambda_{\text{Bohr}}^l = \frac{2\pi m_f a_l^2}{\hbar} v_n \left(\frac{n^3}{Z^2}\right) = \begin{cases} 2\pi a_0 \left(\frac{n^2}{Z}\right), & \text{for an electron} \\ 2\pi a_\mu \left(\frac{n^2}{Z}\right), & \text{for an muon} \end{cases} \tag{9}$$

This gives the classical wavelength of an orbiting lepton. The question arises on how to prove the Bohr hypothesis of angular momentum quantization. This motivated the de Broglie by coming up with the idea that the orbiting lepton can behaves as a wave.

2.1 de Broglie Hypothesis

Based on Einstein theory of special relativity, the energy and momentum of a photon are related by [15], $E^2 = (pc)^2 + (mc^2)^2$ or

$$E = \frac{p}{c} \tag{10}$$

for photon. The Plank quantization of light [16], $E = nhv$, Einstein 1905, relates the energy of light with its frequency v over Planck's constant h [17]:

$$E = \frac{nhc}{\lambda} \tag{11}$$

de Broglie unify (10) and (11), come up with the idea that leptons possess wave-like properties [18,19],

$$\lambda_{\text{de Broglie}} = \frac{h}{p} = \frac{h}{m_l v_n} \tag{12}$$

In 1923 the French physicist Louis de Broglie suggested that a material particle having nonzero mass also possesses wave properties uniquely related to its mass and energy This hypothesis gives an interesting physical insight into Bohr's quantization rule (1). By considering an orbiting lepton as a wave, it orbit round the nucleus having the de Broglie wavelength of

$$\lambda^{l}_{\text{de Broglie}} = \frac{h}{m_f}\frac{a_l m_l}{\hbar}\left(\frac{n}{Z}\right) = \begin{cases} 2\pi a_0 \left(\frac{n}{Z}\right), & \text{for an electron} \\ 2\pi a_\mu \left(\frac{n}{Z}\right), & \text{for an muon} \end{cases} \tag{13}$$

Equation (13) gives the possible wavelength of orbiting lepton when according to de Broglie exhibit wave behavior. However, the two equations (10) and (13) did not match as they differed by the multiple of *n*.

2.2 The de Broglie Circular Orbit

Here the Bohr's first postulate (1) will be used to determine the length of an orbit as follows:

$$pr_n = n\hbar$$

where *p* is the linear momentum of lepton in an allowed orbit of radius *r*. using de Broglie's wavelength (12), Bohr's quantization rule can be written as,

$$\frac{h}{\lambda_{\text{de Broglie}}} r_n = n\hbar$$

Assuming that the lepton's orbit within an atom encompasses, *n* de Broglie wavelengths, we can derive the following expressions for the circumference of a circular orbit and the de Broglie wavelengths of the lepton:

$$2\pi r_n = n\lambda_{\text{de Broglie}} \tag{14}$$

Equation 14 demonstrates that the permissible orbits are those where the circumference of the orbit can accommodate a whole number of de Broglie wavelengths. This stated that the lepton wavelength takes only integer value of the wavelengths. This statement asserts that the lepton wavelength assumes only integer values. This relationship can be justified by considering the particle nature of leptons using the Bohr model and the wave-particle duality of leptons using the de Broglie model. Here, the parameter or symbol η denotes the number of waves.

2.3 The Parameter for Number of Waves

Here the two wavelengths (9) and (13) will be tested using the de Broglie hypothesis (14) and determine the possible value of constant, η as follows:

$$\eta = \frac{2\pi a_l}{\lambda^l_{\text{Bohr}}}\left(\frac{n^2}{Z}\right)$$

Therefore, the parameter,

$$\eta^e = \frac{2\pi a_0}{\lambda^e_{\text{Bohr}}}\left(\frac{n^2}{Z}\right) = \frac{\lambda^e_{\text{de Broglie}}}{\lambda^e_{\text{Bohr}}} \quad (15)$$

for an electron and

$$\eta^\mu = \frac{2\pi a_\mu}{\lambda^\mu_{\text{Bohr}}}\left(\frac{n^2}{Z}\right) = \frac{\lambda^\mu_{\text{de Broglie}}}{\lambda^\mu_{\text{Bohr}}} \quad (16)$$

for muon. The relations (8), (9), (13), (15) and (16) will be examining to determine the behavior of orbiting lepton.

3. Results and Discussion

The relations (8), (10), (13), (15), and (16), which represent some properties of the orbiting lepton, were computed using Microsoft Excel. The results of these calculations are presented in Table 1 to Table 4. It is observed that the de Broglie and Bohr wavelengths are the same for hydrogen atoms and when n = 1 for both electron and muonic atoms. However, as the orbit size increases, the difference between the wavelengths starts to increase. According to equation (15) - (16), the wavelength will decrease with an increase in the nuclear charge, Z. Another interesting observation is that both leptons have the same wavelength value in the n = 1 orbit.

Table 1: The comparison of orbital length using Bohr and de Broglie wavelength for Z = 1

Quantum No.	Bohr Hypothesis		de Broglie Hypothesis		Relative Wavelength Shifts	
n	λ_e (10^{-8} m)	λ_μ (10^{-10} m)	λ_e (10^{-9} m)	λ_μ (10^{-11} m)	η^e	η^μ
1	0.03320000	0.01600000	0.33230000	0.16050000	10.0090361	10.0312500
2	0.13290000	0.06420000	0.66470000	0.32110000	5.00150489	5.00155763
3	0.29910000	0.14450000	0.99700000	0.48170000	3.33333333	3.33356401
4	0.53170000	0.25690000	1.32940000	0.64220000	2.50028211	2.49980537
5	0.83080000	0.40140000	1.66180000	0.80280000	2.00024073	2.00000000
6	1.19640000	0.57800000	1.99410000	0.96340000	1.66675025	1.66678201
7	1.62850000	0.78670000	2.32650000	1.12400000	1.42861529	1.42875302
8	2.12700000	1.02760000	2.65880000	1.28460000	1.25002351	1.25009731
9	2.69200000	1.30060000	2.99120000	1.44510000	1.11114413	1.11110257
10	3.32350000	1.60570000	3.32350000	1.60570000	1.00000000	1.00000000
11	4.02140000	1.94290000	3.65590000	1.76630000	0.90911125	0.90910495
12	4.78580000	2.31220000	3.98820000	1.92680000	0.83334030	0.83331892
13	5.61670000	2.71360000	4.32060000	2.08740000	0.76924173	0.76923644
14	6.51400000	3.14710000	4.65290000	2.24800000	0.71429229	0.71430841
15	7.47780000	3.61280000	4.98530000	2.40860000	0.66668004	0.66668512
16	8.50810000	4.11050000	5.31760000	2.56910000	0.62500441	0.62500912
17	9.60490000	4.64040000	5.65000000	2.72970000	0.58824142	0.58824670
18	10.76800000	5.20240000	5.98230000	2.89030000	0.55556278	0.55557051
19	11.99700000	5.79650000	6.31470000	3.05080000	0.52635659	0.52631761
20	13.29400000	6.42280000	6.64700000	3.21140000	0.50000000	0.50000000

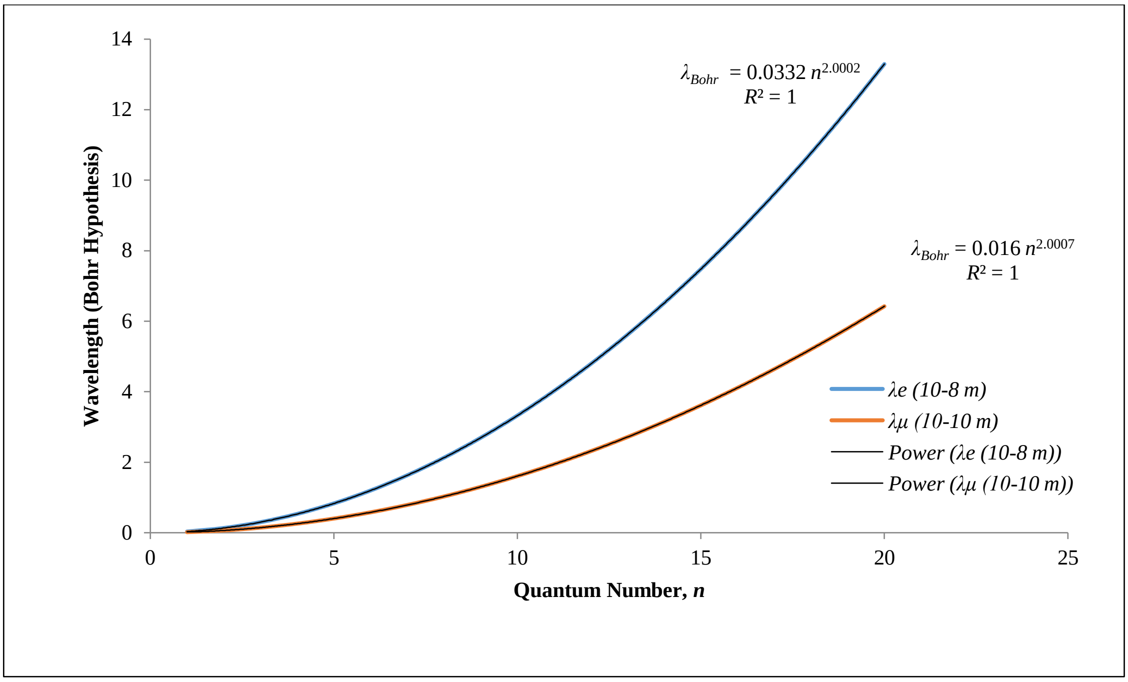

Figure 1: The Bohr wavelength of leptons as function of principal quantum number, *n*

The non-linearity of the variation of the Bohr wavelength with the principal quantum number, *n*, can be observed from Figure 2. The value of the wavelength increases quadratically with the quantum number, *n*, as shown by equations (19) and (20):

$$\lambda^{e^-}_{\text{Bohr}}(n) = 0.0332\, n^2 \qquad (19)$$

and

$$\lambda^{\mu^-}_{\text{Bohr}}(n) = 0.016\, n^2 \qquad (20)$$

These equations indicate that the lepton orbitals are not equally spaced. We have determined the wavelengths as follows:

$$\lambda^{\mu}(n) = \begin{cases} 3.320\, nm & \text{for } e^- \\ 0.0016\, nm & \text{for } \mu^- \end{cases} \qquad (21)$$

However, when considering the circumference, C^n, we have two possibilities based on the hypotheses:

$$C^n = \begin{cases} \lambda n^2, & \text{for Bohr Hypothesis} \\ \lambda n^2, & \text{for de Broglie Hypothesis} \end{cases} \qquad (22)$$

This explains the energy quantization since there cannot be equal spacing between lepton orbits. This is due to the fact that the lepton moves slower at higher orbitals, despite the constant number of wavelengths.

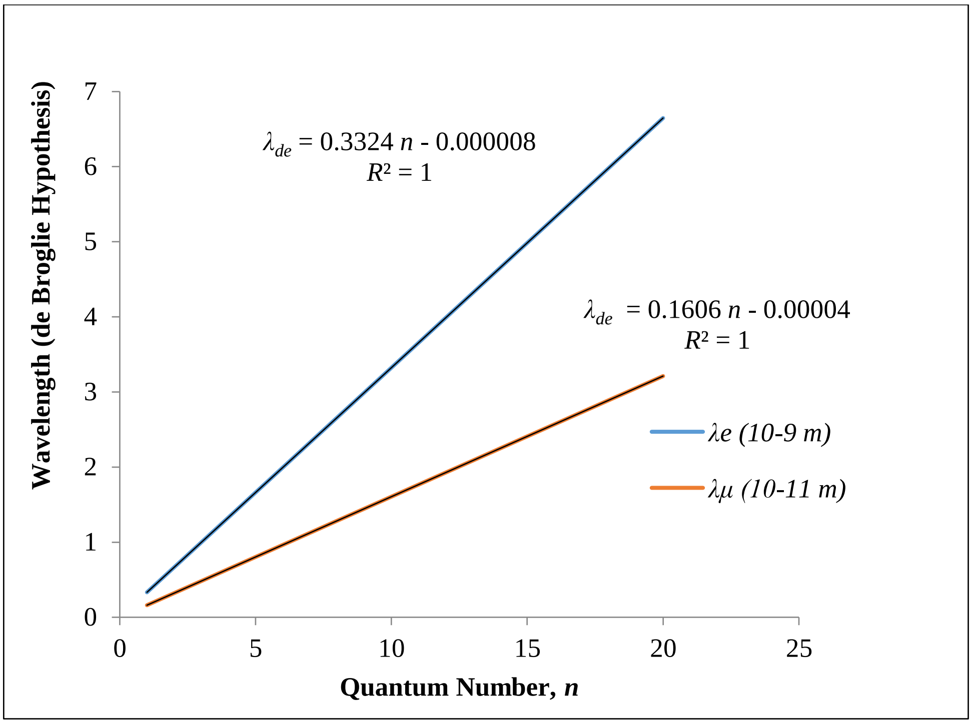

Figure 2: The de Broglie wavelength of leptons as function of principal quantum number, *n*

Contrary to the initial statement, upon observing Figure 2, it becomes evident that the variation of the Bohr wavelength with the principal quantum number, *n*, is not linear. This indicates that the orbitals are not equally spaced for both electrons and muons. The following de Broglie wavelength equations describe the relationship with the quantum number, *n*:

$$\lambda_{\text{de Broglie}}^{e^-}(n) = 0.3324\, n \tag{23}$$

and

$$\lambda_{\text{de Broglie}}^{\mu^-}(n) = 0.1606\, n \tag{24}$$

However, this seems to contradict the dependence of energy on the n^2 interval between quantum states. The number of wavelengths, denoted as η, is proportional to the quantum number, *n*. It is observed that the number of wavelengths, η, varies with the quantum number, *n*, and that the spacing between orbitals increases linearly with η. The slopes of the lines in Figure 2 demonstrate that the changes in the number of wavelengths are consistent for both the de Broglie and Bohr hypotheses.

The determined wavelengths are as follows:

$$\lambda^{\mu}(n) = \begin{cases} 0.3324\, nm & \text{for } e^- \\ 0.0160\ \text{Å} & \text{for } \mu^- \end{cases} \tag{25}$$

Considering the circumference, denoted as C^n, we have two possibilities based on the hypotheses:

$$C^n = \begin{cases} \lambda n, & \text{for Bohr Hypothesis} \\ \lambda n, & \text{for de Broglie Hypothesis} \end{cases} \tag{26}$$

To reconcile the Bohr and de Broglie hypotheses, the parameter η must take the form: $2\pi r_n = \eta\lambda$, where $\eta = \varrho n$, and the constant ϱ defines the number of waves leptons can have for a complete cycle around a nucleus. It cannot be assumed that each orbital has a different number of wavelengths. It is argued here that leptons could only have different wavelengths when subjected to different interactions, such as ($+Ze$), similar to their mass, distance, r_n, and frequency. To support this argument, it is necessary to revisit the properties of leptons and determine if it is possible for different orbitals to have the same wavelength.

This study represents a significant exploration into the intricate interplay between the wavelengths characterizing orbiting leptons within atomic systems, with a pronounced focus on a comparative analysis of the enduring hypotheses formulated by Bohr and de Broglie. The research embarks on a meticulous examination of the fundamental assumption that orbitals are uniformly spaced, revealing a strikingly non-linear relationship between wavelength and the principal quantum number (n), substantiated by the equations devised by de Broglie. Intriguingly, the investigation uncovers an emerging pattern wherein the number of wavelengths, denoted as η, exhibits a consistent proportionality to n. This compelling observation suggests that each orbital possesses an inherently unique and consistent wavelength pattern, defying the conventional belief in uniformity [20,21].

Furthermore, the study unearths a captivating correlation between the velocity of leptons and their associated energy levels within these orbitals. In particular, it discerns that s-state orbitals or those trajectories resembling perfect circles manifest substantially higher energy levels, a revelation that could potentially cast light upon hitherto unresolved enigmas within nuclear structure effects, notably the Lamb shift [22,23]. Notably, this insightful analysis also accentuates the constraints inherent in Bohr's treatment of lepton speed and the de Broglie hypothesis's incapacity to forge a direct nexus between lepton speed and wavelength, thereby shedding light on the imperfections of these foundational models.

As a pivotal contribution to the burgeoning realm of atomic physics, this research assumes an essential role in propelling the frontiers of atomic system theories and enriching our comprehension of atomic behavior. It accentuates the pressing need for the development of a comprehensive theoretical framework that strives to harmonize the divergent perspectives and premises offered by the Bohr and de Broglie hypotheses [20,21]. Through its painstaking exploration of wavelength dynamics, lepton velocity intricacies, and orbital attributes, this study not only significantly contributes to existing academic discourse but also emerges as a beacon guiding future investigations into the intriguing world of atomic phenomena.

4. **Conclusion**

Based on our findings, we can conclude that the circumferences of orbitals are equal to the integer value of the constant ϱ, and different orbitals have a different number of wavelengths for the orbiting lepton. We argue that the number of wavelengths is the same for each orbital due to the following reasons: The inner leptons move faster, resulting in a higher frequency and shorter wavelength, while the outer leptons move slower, leading to a lower frequency and longer wavelength. Additionally, in cases where leptons have degrees of freedom, such as *nlm*, their speed (and thus wavelength) varies according to the shape of the orbitals. This energy is found to be greater in the s state orbitals or perfectly circling orbits. Understanding and solving nuclear structure effects, particularly the Lamb shift, where lepton motion is affected, could benefit from considering these factors. Bohr's approach considered the speed of the lepton but not the nature of the wavelength. On the other hand, the de Broglie hypothesis, although it only aimed to support the Bohr hypothesis and has not been extensively applied to other atomic system properties, did not compare the wavelength and the speed of the lepton. It appears that further work needs to be done to fully validate the de Broglie hypothesis. For future studies, it is recommended to determine the exact number of wavelengths for each orbital, which could provide valuable insights.

5. Recommendations

To further advance the study, the following recommendations are proposed:

a. Determine the exact number of wavelengths associated with each orbital to understand the relationship between wavelength and orbital properties.
b. Explore how variations in lepton motion and wavelength contribute to nuclear structure effects, such as the Lamb shift, and develop theoretical frameworks to explain these phenomena.
c. Develop an integrated framework that considers both lepton speed and the nature of the wavelength, reconciling the perspectives of the Bohr and de Broglie hypotheses.
d. Employ advanced measurement techniques, like electron scattering or muonic atoms, to experimentally validate the observed wavelength variations and enhance understanding of lepton behavior.
e. Extend the understanding of wavelength variations to predict additional atomic system properties, broadening the scope of research in atomic physics.

By following these recommendations, the one can advance our understanding of lepton behavior, improve measurement accuracy and provide valuable insights into atomic nuclei and their interactions.

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