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The Bohr Model of the Atom: A Critical Evaluation of its Impact and Limitations in Modern Physics

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Abstract

This study investigated the properties and behavior of orbiting leptons, with a focus on their speed, wavelength, frequency, and energy. The research revealed the dependence of lepton speed on the size of the orbit and the magnitude of the charge of the nucleus. The speeds of electrons and muons were found to be similar, but muons being closer to the nucleus exhibited a greater sensitivity to nuclear charge. The analysis also demonstrated that the wavelengths of leptons increase linearly with the principal quantum number and the charge of the nucleus. Moreover, the comparison between Bohr and de Broglie wavelengths highlighted the influence of mass and speed on the wavelength differences. The research provided valuable insights into the relationship between speed, wavelength, frequency, and energy of orbiting leptons in atomic systems. The observations underscored the impact of relativistic motion on these parameters and highlighted the need to account for these effects in theoretical calculations. By examining computed values of lepton speed and analyzing their implications for different energy states and atoms, researchers gained a deeper understanding of lepton properties and the nature of orbital motion. The study incorporated the relativistic motion of leptons, leading to higher energies than those calculated by Niels Bohr. While the differences were small, they underscored the importance of considering relativistic corrections in energy level calculations. The findings emphasized the significance of incorporating these corrections to achieve accurate results, particularly in simple energy level calculations. The impact of the Bohr model extends beyond atomic theory and finds applications in various fields, including atomic physics, chemistry, and materials science. Therefore, this study contributed to our understanding of orbiting leptons, their properties, and their behavior within atomic systems. It highlighted the importance of incorporating relativistic corrections in energy level calculations and provided insights into the relationship between speed, wavelength, frequency, and energy. The investigation also emphasized the significance of the Bohr model in explaining atomic behavior, while acknowledging the advancements and limitations of modern theoretical frameworks like quantum mechanics.

1. Introduction

The Bohr Model of the Atom, proposed by Niels Bohr in 1913, revolutionized our understanding of atomic structure by introducing the concept of quantized energy levels and specific electron orbits, bridging the gap between classical physics and atomic phenomena. Bohr's postulation that electrons undergo discrete energy transitions by absorbing or emitting quanta of energy successfully accounted for the observed spectral lines in atomic emission and absorption spectra [1,2]. The model also played a pivotal role in the development of modern physics by providing the foundation for quantum mechanics [3,4]. However, as scientific knowledge expanded, the Bohr Model encountered limitations, such as the explanation of electron spin, atomic orbital shapes, and the wave-particle duality of matter. Its reliance on classical mechanics and fixed electron orbits, also presented challenges for complex atomic and molecular systems [5,6]. While the Bohr model successfully accounted for the hydrogen spectrum, it is now considered outdated, although the concept of orbital angular momentum quantization remains foundational. The incorporation of de Broglie's standing waves into the Bohr Model fell short in describing electron trajectories classically, leading to the development of quantum mechanics by Schrödinger and Heisenberg. A comprehensive study is essential to address unresolved issues in research and investigate overlooked aspects of lepton motion and characteristics within the context of the wavefunction's descriptive capability [7]. The Bohr Model's impact on atomic theory is acknowledged and its limitations and the significance of studying simple and muonic atoms contribute to our understanding of modern physics. Simple atoms have minimal changes in lepton energy states due to nuclear size effects, fluctuating vacuum fields and other nuclear structure factors. Conversely, highly charged muonic atoms, like the hydrogen-like muonic atom, offer unique opportunities to study and manipulate nuclear structure effects with precision [8-11]. These atoms play a crucial role in verifying quantum electrodynamics, studying bound state theory, and accurately determining fundamental physical constants [12]. Despite advances in Quantum Electrodynamics, wave mechanics remains unquestioned, while the original Bohr model has been abandoned [13]. Therefore, this study focuses on investigating the limitations of the Bohr Model in explaining lepton relativistic mass and emphasizes the significance of considering this aspect when studying kinematic properties. The study assesses the strengths and weaknesses of the model and explores subsequent advancements in atomic theory, with a particular emphasis on quantum mechanics. It critically evaluates the impact of the Bohr Model on modern physics by examining its historical context and fundamental principles. The study addresses concepts like speed, frequency, de Broglie wavelength, Bohr-de Broglie wavelength, relativistic mass and relativistic energy levels.

2. Methodology

The starting point for a theory of lepton eigenenergies and eigenstates should be a well-known, non-relativistic Schrodinger equation,

$$\hat{H}_0\psi_{nlm} = E_n\psi_{nlm}$$

where ψ_{nlm} is the unperturbed eigenstates and

$$\hat{H}_0 = -\frac{\hbar^2}{2m_l}\nabla^2 - \frac{Zke^2}{r}$$

is the Hamiltonian represents the total energy for a lepton interacting with a point-charge nucleus. In this context, the mass and charge of the lepton are denoted as m_l and *-e*, respectively, while Z represents the number of protons inside the nucleus of charge $+Ze$. The Schrödinger equation provides the solution for the lepton's unperturbed eigenenergies, which describe its energy levels in the absence of external influences [14,15]:

$$E_n^l = -\frac{m_l c^2}{2}\left(\frac{Z\alpha}{n}\right)^2 \quad (1)$$

where m_l is the rest mass of the electron (which is approximately $9.11 \times 10^{-31}\,kg$, $1.88577 \times 10^{-27}\,kg$ for muon), c is the speed of light and n is the principal quantum number which takes values of 1, 2, 3, …, ∞. [16]. The fine-structure constant α is given by

$$\alpha = \frac{ke^2}{\hbar c} = \frac{\gamma}{\hbar c} \tag{2}$$

where $\hbar = h/2\pi$ is the Planck's constant, $\gamma = ke^2$ and e^- is the charge of lepton. This fundamental constant plays an important role throughout atomic physics. The concept of relativistic mass refers to the mass of an object as it appears to an observer in motion relative to the object. In the case of the electron, its relativistic mass increases as its speed approaches the speed of light, due to the effects of time dilation and length contraction predicted by special relativity [15]. In 1905, Einstein stated the assumption of a constant mass m in Newton's second law as:

$$F = \frac{d(mv)}{dt}$$

where v is the velocity and that m has to be corrected as relativistic mass of an orbiting lepton as:

$$m_l^r(n) = m_l\gamma = m_l\left(1 - \frac{v_n^2}{c^2}\right)^{-1/2} = m_l\left(1 - \frac{1}{2}\frac{v_n^2}{c^2} + \cdots\right) \tag{3}$$

where v_n is the velocity of lepton and m_l is lepton rest mass [6]. Thus the modified energy levels of lepton can be determined by taken into account the relativistic motion of leptons as

$$E_n^l(\gamma) \approx -\frac{m_l c^2}{2}\left(\frac{Z\alpha}{n}\right)^2\left[1 - \left(\frac{v_n}{c}\right)^2\right] \tag{4}$$

Therefore, the kinetic energy of the orbiting lepton is

$$E(\gamma) = (\gamma - 1)m_l c^2 \simeq -\frac{1}{2}\frac{v_n^2}{c^2}m_l c^2 \tag{5}$$

It was found that the ratio of the speed in the orbit to the speed of light can be obtain by comparing (1) and (5) as

$$\frac{v_n}{c} = \frac{Z\alpha}{n}$$

Thus, relativistically, an electron bounds to nucleus of charge +*Ze* in the Bohr orbit has speed

$$v_n = \frac{Z\alpha c}{n} = \frac{\gamma}{\hbar}\left(\frac{Z}{n}\right) \tag{6}$$

The speed according to (5) is charge dependence (not mass). The well-known Bohr's quantization of orbital angular momentum says [7],

$$m_l^r v_n = \frac{n\hbar}{r_n} \tag{7}$$

And the lepton relativistic mass can be obtained from the Planck's quantum of action by rearranging de Broglie relation as [17,18],

$$\lambda_{\text{de Broglie}} = \frac{h}{m_l^r v_n} \tag{8}$$

By putting (7) into (8), we find

$$\lambda_{\text{Bohr-de Broglie}} = 2\pi a_l \frac{n}{Z} \tag{9}$$

where the use of $r_n = a_l n^2/Z$ has been made. This gives the lepton frequency,

$$f = \begin{cases} \dfrac{v_n}{\lambda_{\text{de Broglie}}} \\ \dfrac{v_n}{2\pi a_l n} \end{cases} \tag{10}$$

where v_n is defined from equation 5. The relations (4), (6), (8), (9) and (10) which represent some properties of orbiting lepton are computed and the results are shown in Table 1 to Table 12 (see Appendix).

3. Results and Discussion

In Figure 1, the computed values of lepton speed for different atoms and energy states (n) were presented. This analysis aimed to investigate whether different orbitals could have the same wavelength. The results indicated a clear dependence of lepton speed on the size of the orbit and the charge of the nucleus. The study conducted in this research focused on investigating the relationship between the speed, wavelength, and frequency of orbiting leptons in atomic systems. The results confirmed the expected dependence of lepton speed on both the size of the orbit and the magnitude of the charge of the nucleus, as demonstrated in Figure 1. Specifically, it was observed that the inner lepton (n = 1) of heavier atoms, such as Francium, moves at a speed close to the speed of light. Interestingly, both electrons and muons were found to move at the same speed around the nucleus, despite their different positions relative to the nucleus. The muon, being approximately 207 times closer to the nucleus than the electron, is more sensitive to the nuclear charge. It was observed that the inner lepton (n = 1) of heavier atoms, such as Francium, exhibited a speed close to the speed of light ($3.00 \times 10^8\ m/s$), specifically measuring approximately $1.18 \times 10^8\ m/s$. This finding suggested that lepton speed decreases as the principal quantum number (n) increases, implying that lepton movement is faster for smaller values of n. Notably, both electrons and muons were found to move at the same speed around the nucleus, as depicted in Figure 1. Despite their identical speeds, it is important to acknowledge that muons are approximately 207 times closer to the nucleus than electrons. This close proximity may render muons more sensitive to the nuclear charge. The results obtained from this analysis are valuable in understanding the behavior of leptons in atomic systems.

Figure 2 displayed the relationship between wavelength and the principal quantum number (n) and the charge of the nucleus (Z). The wavelengths were found to increase linearly with both n and Z. In the de Broglie model, the wavelength for electrons was in the order of nanometers ($10^9\ m$) and Ångströms ($Å$) in the Bohr-de Broglie model. For muons, which have a significantly higher mass than electrons, the wavelengths were shorter and in the order of $10^{-12}\ m$. It is noteworthy that these values differed from those calculated using the rest mass of the leptons, indicating the influence of relativistic motion on the observed wavelengths.

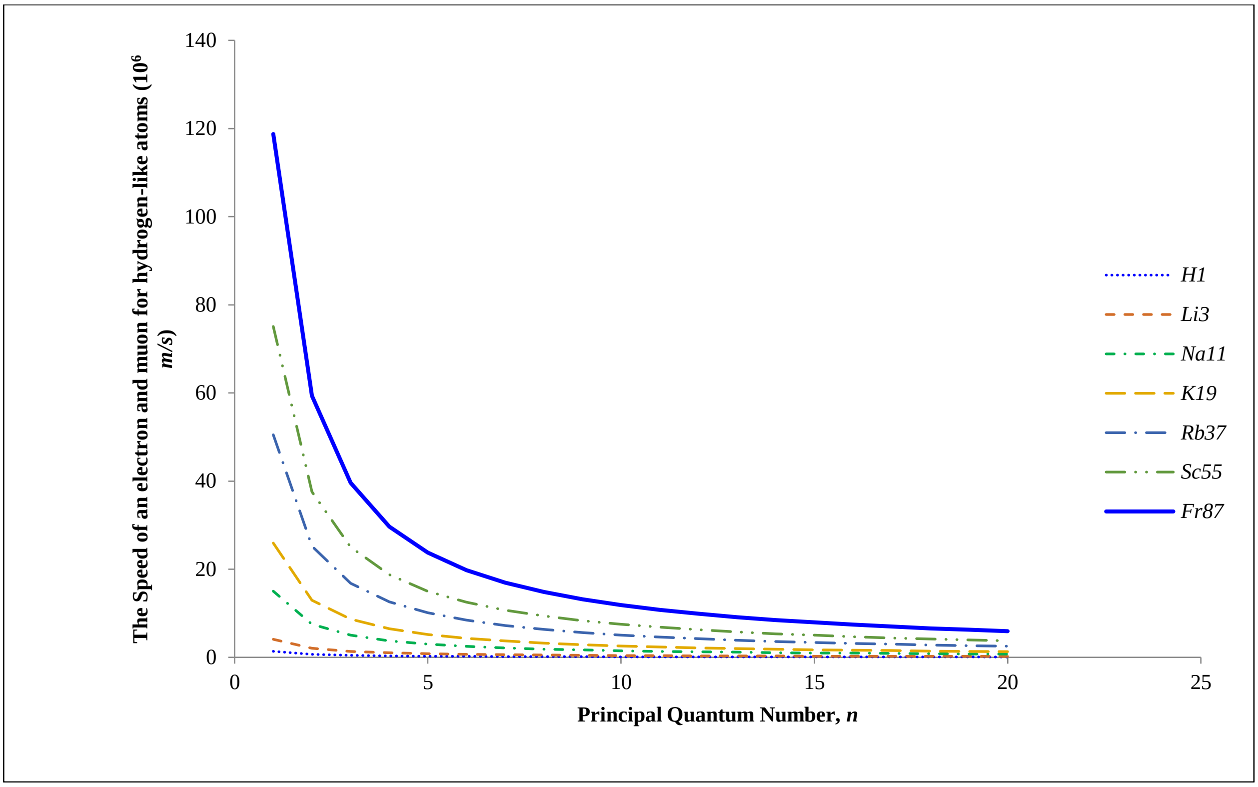


Figure 1: The speed of an orbiting lepton (both electron and muon) round some hydrogen-like atoms

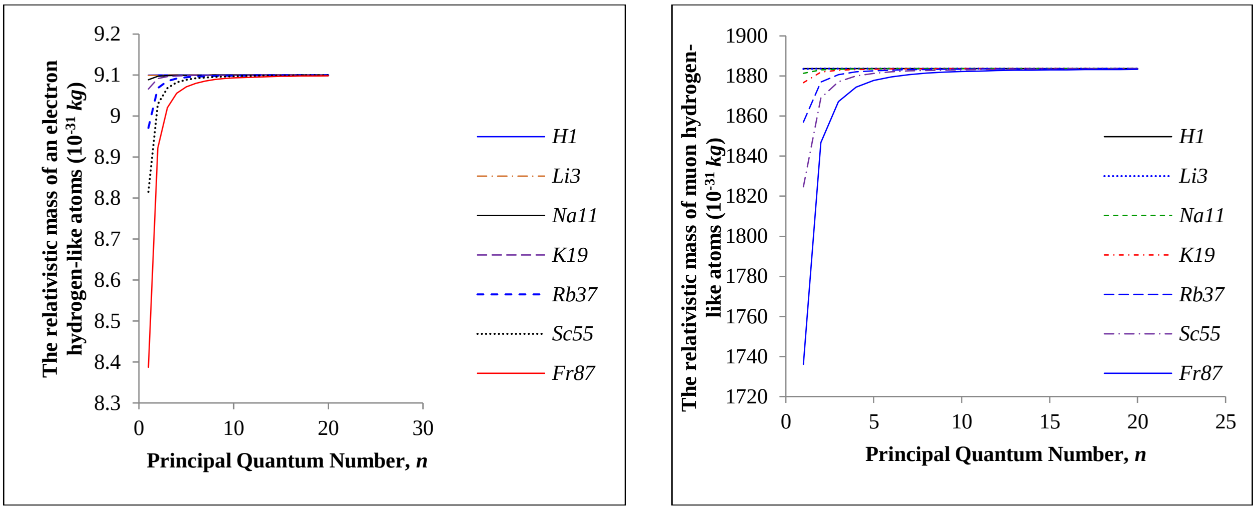
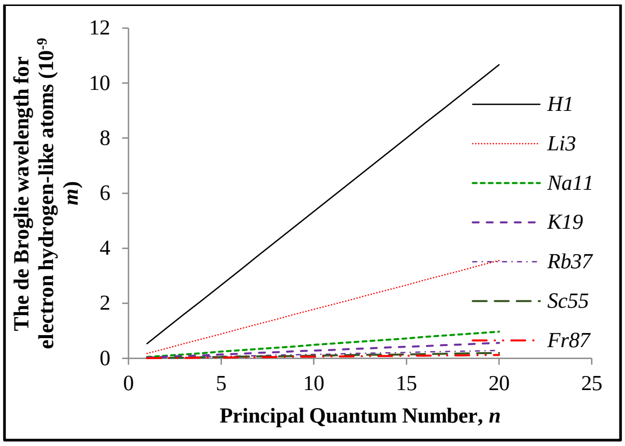


Figure 2 (*a*): The mass of orbiting electron for some hydrogen-like atoms **(*b*):** The mass of orbiting muon for some hydrogen-like atoms



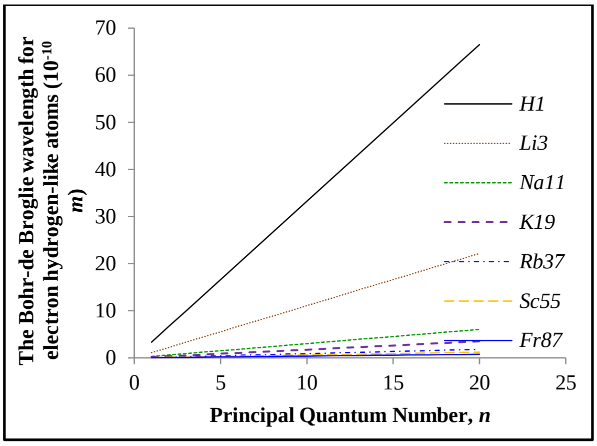
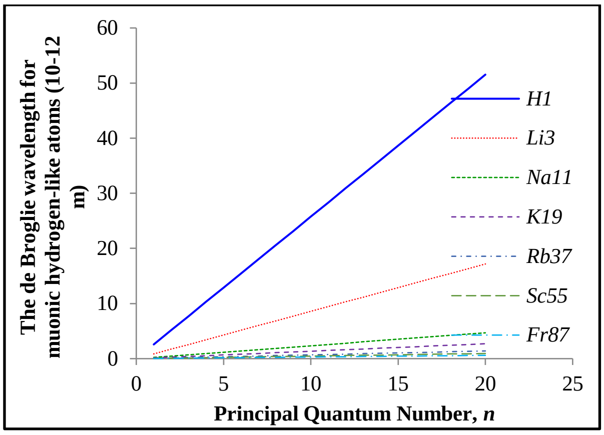


Figure 3 (*a*): The de Broglie wavelength of electron hydrogen-like atoms **(*b*):** The Bohr-de Broglie wavelength of electron hydrogen-like atoms



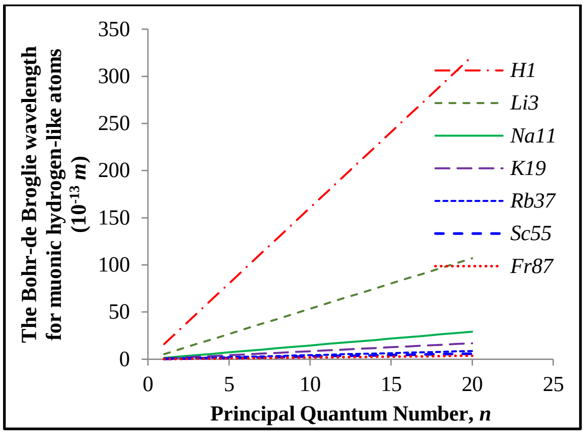
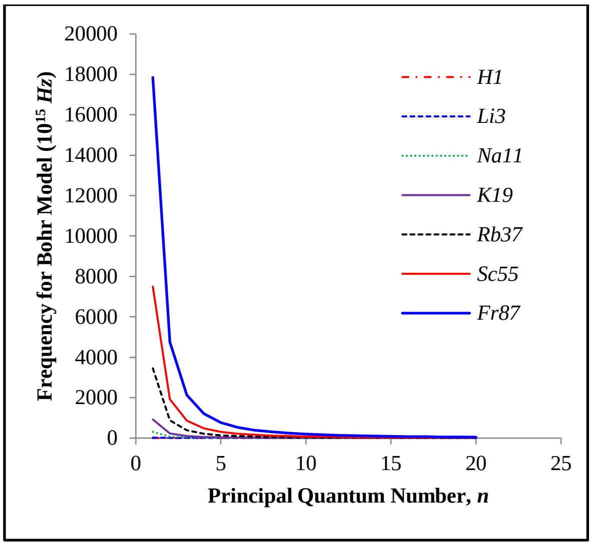


Figure 4(*a*): The de Broglie wavelength of muonic hydrogen-like atoms **(*b*):** The Bohr-de Broglie wavelength of muonic hydrogen-like atoms



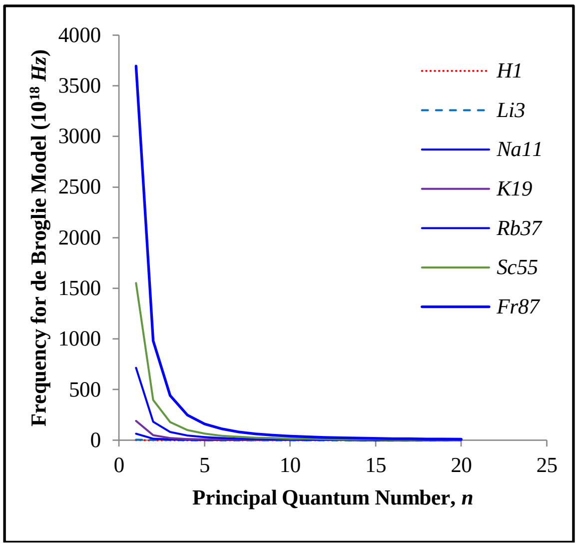


Figure 5(*a*): The frequency of electron hydrogen-like atoms from Bohr Model **(*b*):** The frequency of muon hydrogen-like atoms using de Broglie Model

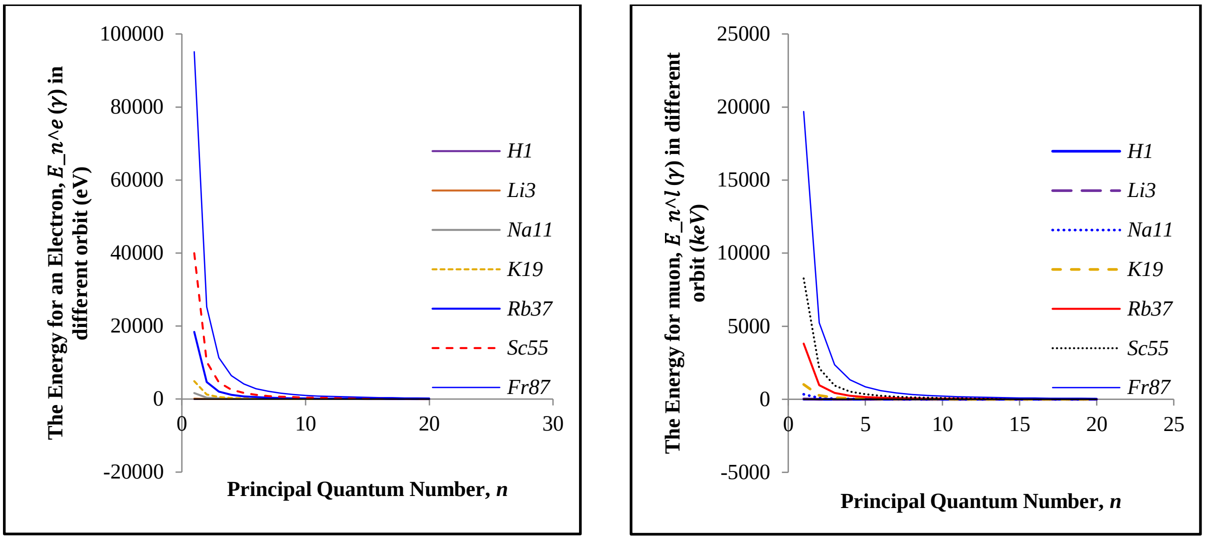


Figure 6: (*a*) The Energy for electron, $E_n^e(\gamma)$ in different orbit (*keV*), (*b*) The Energy for muon, $E_n^{\mu}(\gamma)$ in different orbit (*keV*)

To further analyze the relationship between Bohr and de Broglie wavelengths, Figure 3 and 4 provided a comparison, revealing that the two wavelengths differ by a multiple of "*n*." These wavelengths were compared with the lepton's circular orbit, which was determined for each hypothesis. The variation of lepton frequency with the principal quantum number, *n*, was illustrated in Figure 5. It was observed that electrons could complete approximately 3.5×10^{14} orbits around the nucleus of Francium in just one second, while muons could complete about 7.4×10^{19} orbits in the same time frame. Despite the electrons and muons moving at the same speed, their frequencies differed. This disparity can be attributed to the fact that muons, being closer to the nucleus, have a chance of completing around 10^3 orbits before an electron completes one orbit. Additionally, it was noted that for light nuclei with smaller charges, the average number of orbits completed by electrons or muons was approximately 4×10^{12} times the charge of the nucleus and about 850 times the size of the nuclear charge for muonic atoms.

The speed of an electron can vary depending on its environment and specific conditions. In a vacuum, electrons can potentially reach the maximum speed of light, approximately 299,792,458 meters per second. However, in most practical scenarios, electrons do not travel anywhere near the speed of light. Within atoms, electrons are typically bound to the nucleus and move in quantized energy levels, where their speeds are usually a few percent of the speed of light. In electrical circuits, electrons exhibit drift velocities on the order of millimeters per second (*mm*/*s*) as they flow through conductors. In semiconductors or transistors, electrons can achieve speeds of a few thousand meters per second. In high-energy particle accelerators like the Large Hadron Collider, electrons can approach speeds close to the speed of light, reaching energies on the order of tera-electron-volts (*TeV*). At such high energies, relativistic effects on mass and velocity become significant, and the behavior of particles is described by relativistic quantum mechanics.

The concept of relativistic mass is valuable for understanding the behavior of high-energy particles, including electrons in particle accelerators or cosmic rays. As the velocity of an electron approaches the speed of light, the denominator of the equation describing relativistic mass approaches zero, resulting in an infinitely large relativistic mass. However, in practical situations, the maximum velocity of an electron is constrained by its energy and the potential difference across an electric field. It is important to note that the concept of relativistic mass has faced criticism due to its potential to cause confusion and its lack of invariance under Lorentz transformations, which are fundamental in special relativity. Alternative approaches, such as the relativistic energy-momentum relation, have gained popularity for their clearer and more consistent treatment of relativistic effects.

The Bohr model of the atom, proposed by Niels Bohr, introduced the concept of quantized energy levels and explained the emission and absorption of photons. This model has helped explain many properties of atoms and their spectra and served as a foundation for the development of quantum mechanics. The Bohr model, however, has limitations and inaccuracies, particularly in its inability to fully account for the behavior of electrons in complex atoms and under extreme conditions. Despite these limitations, the Bohr model remains a useful and intuitive model for understanding electron behavior in atoms. Its predictions for atomic spectra and energy levels have been largely confirmed by experimental evidence. Modern theoretical frameworks, such as quantum mechanics, provide a more accurate and comprehensive understanding of atomic behavior. Quantum mechanics incorporates wave-particle duality and treats electrons as probability distributions described by wavefunctions. This approach allows for a more detailed description of electron behavior, accounting for effects such as electron spin and electron-electron interactions. Quantum mechanics has significantly advanced our understanding of atoms and their interactions with electromagnetic radiation. The impact of the Bohr model extends beyond atomic theory and has practical applications in fields such as atomic physics, chemistry, and materials science. The model's insights into energy levels and electron configurations have contributed to our understanding of chemical bonding and the behavior of materials. Additionally, the Bohr model has influenced the development of technologies such as electron microscopy, which relies on the interaction of electrons with matter to generate images at the atomic scale.

This research also highlighted the impact of relativistic motion on the energy of orbiting leptons. The findings revealed that when considering the relativistic effects, the calculated energy of orbiting leptons was higher compared to the values calculated using Niels Bohr's model. Although the differences were relatively small, it is crucial to incorporate these corrections, particularly in simple energy level calculations. This indicates the importance of considering relativistic effects when studying atomic systems and further emphasizes the significance of accurate theoretical calculations. Thus, this study provided valuable insights into the relationship between the speed, wavelength, frequency, and energy of orbiting leptons in atomic systems. By considering the computed values of lepton speed at different energy states and for various atoms, researchers can deepen their understanding of the interplay between lepton properties and the nature of orbital motion. Moreover, the incorporation of relativistic effects in these calculations sheds light on the impact of relativistic motion on the observed parameters. Overall, this research contributes to the comprehension of wave-particle duality and the behavior of subatomic particles within the framework of quantum mechanics.

4. Conclusion

In conclusion, the understanding of electron speed and behavior is dependent on various factors and contexts. While electrons can potentially reach the speed of light in a vacuum, their speeds are typically much lower in practical scenarios, such as within atoms or electrical circuits. The concept of relativistic mass is valuable for high-energy particles but has limitations and alternative approaches due to its potential for confusion and lack of invariance under Lorentz transformations. The Bohr model, despite its limitations, introduced the concept of quantized energy levels and explained the emission and absorption of photons, providing valuable insights into atomic behavior. However, modern theoretical frameworks like quantum mechanics offer a more accurate and comprehensive understanding of electron behavior, incorporating wave-particle duality and accounting for various factors such as electron spin and interactions. The Bohr model's impact extends beyond atomic theory and finds applications in fields like atomic physics, chemistry, and materials science. It has contributed to our understanding of chemical bonding, material behavior, and technologies such as electron microscopy. Thus, while the Bohr model and the concept of relativistic mass have played significant roles in advancing our knowledge of electrons and atoms, they are part of a larger framework of theories and approaches that continue to refine our understanding of the quantum world.

Based on the study, the following recommendations are proposed for further research:

i. Examining the properties of leptons in excited states can provide valuable insights into their behavior under different energy conditions.

ii. To uncover patterns and variations in lepton properties and behavior, further exploration should encompass a broader range of elements and isotopes.

iii. Further investigation is needed to apply quantum mechanics to accurately describe the behavior of leptons in atomic systems.

iv. Conducting direct experiments to measure lepton properties, such as speed, wavelength, frequency, and energy, will provide concrete evidence and strengthen our understanding of orbiting leptons.

v. Exploring specific applications of the Bohr model and the insights gained from this research in fields such as atomic physics, chemistry, and materials science can advance these disciplines.

vi. Further research should involve the development of new theoretical frameworks or the refinement of existing ones. These frameworks should better incorporate relativistic effects to accurately describe the behavior of leptons in atomic systems.

By following these recommendations, researchers can enhance our knowledge of lepton properties, refine theoretical models, validate findings through experiments, identify practical applications, and advance scientific disciplines in atomic physics, chemistry, and materials science.

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Appendix

Table 1: The speed of an orbiting lepton (both electron and muon) round some hydrogen-like atoms

Quantum No. *n*	The Speed of an electron and muon for hydrogen-like atoms (10^6 *m/s*)						
	H_1	Li_3	Na_{11}	K_{19}	Rb_{37}	Sc_{55}	Fr_{87}
1	1.3650000	4.0950000	15.015000	25.935000	50.505000	75.075000	118.75500
2	0.6825000	2.0475000	7.5075000	12.967500	25.252500	37.537500	59.377500
3	0.4550000	1.3650000	5.0050000	8.6450000	16.835000	25.025000	39.585000
4	0.3412500	1.0237500	3.7537500	6.4837500	12.626250	18.768750	29.688750
5	0.2730000	0.8190000	3.0030000	5.1870000	10.101000	15.015000	23.751000
6	0.2275000	0.6825000	2.5025000	4.3225000	8.4175000	12.512500	19.792500
7	0.1950000	0.5850000	2.1450000	3.7050000	7.2150000	10.725000	16.965000
8	0.1706250	0.5118750	1.8768750	3.2418750	6.3131250	9.3843750	14.844375
9	0.1516667	0.4550000	1.6683333	2.8816667	5.6116667	8.3416667	13.195000
10	0.1365000	0.4095000	1.5015000	2.5935000	5.0505000	7.5075000	11.875500
11	0.1240909	0.3722727	1.3650000	2.3577273	4.5913636	6.8250000	10.795909
12	0.1137500	0.3412500	1.2512500	2.1612500	4.2087500	6.2562500	9.8962500
13	0.1050000	0.3150000	1.1550000	1.9950000	3.8850000	5.7750000	9.1350000
14	0.0975000	0.2925000	1.0725000	1.8525000	3.6075000	5.3625000	8.4825000
15	0.0910000	0.2730000	1.0010000	1.7290000	3.3670000	5.0050000	7.9170000
16	0.0853125	0.2559375	0.9384375	1.6209375	3.1565625	4.6921875	7.4221875
17	0.0802941	0.2408824	0.8832353	1.5255882	2.9708824	4.4161765	6.9855882
18	0.0758333	0.2275000	0.8341667	1.4408333	2.8058333	4.1708333	6.5975000
19	0.0718421	0.2155263	0.7902632	1.3650000	2.6581579	3.9513158	6.2502632
20	0.0682500	0.2047500	0.7507500	1.2967500	2.5252500	3.7537500	5.9377500

Table 2: The mass of orbiting electron for some hydrogen-like atoms

Quantum No. *n*	The relativistic mass of an electron hydrogen-like atoms (10^{-31} *kg*)						
	H_1	Li_3	Na_{11}	K_{19}	Rb_{37}	Sc_{55}	Fr_{87}
1	9.0999058041	9.0991522331	9.0886022391	9.0659951091	8.9710451631	8.8150559661	8.3870276380
2	9.0999764510	9.0997880583	9.0971505598	9.0914987773	9.0677612908	9.0287639915	8.9217569100
3	9.0999895341	9.0999058041	9.0987335821	9.0962216791	9.0856716851	9.0683395521	9.0207808490
4	9.0999941123	9.0999470143	9.0992876402	9.0978746941	9.0919403229	9.0821909979	9.0554392270
5	9.0999962326	9.0999660898	9.0995440900	9.0986398048	9.0948418070	9.0886022391	9.0714811060
6	9.0999973838	9.0999764510	9.0996833955	9.0990554200	9.0964179215	9.0920848883	9.0801952120
7	9.0999980781	9.0999826991	9.0997673931	9.0993060231	9.0973682691	9.0941848161	9.0854495440
8	9.0999985285	9.0999867540	9.0998219103	9.0994686738	9.0979850807	9.0955477495	9.0888598060
9	9.0999988370	9.0999895341	9.0998592867	9.0995801870	9.0984079650	9.0964821721	9.0911978720
10	9.0999990582	9.0999915224	9.0998860225	9.0996599512	9.0987104517	9.0971505598	9.0928702760
11	9.0999992220	9.0999929939	9.0999058041	9.0997189674	9.0989342571	9.0976450911	9.0941076660
12	9.0999993457	9.0999941123	9.0999208491	9.0997638550	9.0991044799	9.0980212223	9.0950488030
13	9.0999994431	9.0999949841	9.0999325581	9.0997987881	9.0992369541	9.0983139411	9.0957812290
14	9.0999995195	9.0999956748	9.0999418483	9.0998265058	9.0993420673	9.0985462040	9.0963623860
15	9.0999995814	9.0999962326	9.0999493430	9.0998488672	9.0994268674	9.0987335821	9.0968312340
16	9.0999996324	9.0999966885	9.0999554773	9.0998671682	9.0994962704	9.0988869371	9.0972149510
17	9.0999996742	9.0999970662	9.0999605615	9.0998823361	9.0995537897	9.0990140341	9.0975329670
18	9.0999997097	9.0999973838	9.0999648221	9.0998950470	9.0996019915	9.0991205432	9.0977994680
19	9.0999997388	9.0999976513	9.0999684276	9.0999058041	9.0996427841	9.0992106815	9.0980250070
20	9.0999997643	9.0999978806	9.0999715052	9.0999149878	9.0996776125	9.0992876402	9.0982175690

Table 3: The mass of orbiting muon for some hydrogen-like atoms

Quantum No. *n*	The relativistic mass of muon hydrogen-like atoms (10^{-31} *kg*)						
	H_1	Li_3	Na_{11}	K_{19}	Rb_{37}	Sc_{55}	Fr_{87}
1	1883.6805014	1883.5245123	1881.3406635	1876.6609876	1857.0063488	1824.7165850	1736.1147211
2	1883.6951254	1883.6561281	1883.1101659	1881.9402469	1877.0265872	1868.9541462	1846.8036804
3	1883.6978336	1883.6805014	1883.4378515	1882.9178876	1880.7340388	1877.1462873	1867.3016357
4	1883.6987812	1883.6890320	1883.5525415	1883.2600617	1882.0316468	1880.0135366	1874.4759200
5	1883.6992201	1883.6929806	1883.6056266	1883.4184396	1882.6322540	1881.3406635	1877.7965889
6	1883.6994584	1883.6951254	1883.6344629	1883.5044719	1882.9585098	1882.0615719	1879.6004089
7	1883.6996022	1883.6964187	1883.6518504	1883.5563468	1883.1552317	1882.4962569	1880.6880556
8	1883.6996954	1883.6972581	1883.6631354	1883.5900155	1883.2829117	1882.7783841	1881.3939798
9	1883.6997593	1883.6978336	1883.6708723	1883.6130987	1883.3704488	1882.9718096	1881.8779595
10	1883.6998050	1883.6982451	1883.6764067	1883.6296099	1883.4330635	1883.1101659	1882.2241471
11	1883.6998390	1883.6985497	1883.6805014	1883.6418263	1883.4793912	1883.2125339	1882.4802869
12	1883.6998646	1883.6987812	1883.6836158	1883.6511180	1883.5146273	1883.2903930	1882.6751022
13	1883.6998847	1883.6989617	1883.6860395	1883.6583491	1883.5420495	1883.3509858	1882.8267144
14	1883.6999005	1883.6991047	1883.6879626	1883.6640867	1883.5638079	1883.3990642	1882.9470139
15	1883.6999133	1883.6992201	1883.6895140	1883.6687155	1883.5813616	1883.4378515	1883.0440654
16	1883.6999239	1883.6993145	1883.6907838	1883.6725038	1883.5957280	1883.4695960	1883.1234949
17	1883.6999326	1883.6993927	1883.6918362	1883.6756436	1883.6076345	1883.4959051	1883.1893242
18	1883.6999399	1883.6994584	1883.6927182	1883.6782747	1883.6176122	1883.5179524	1883.2444899
19	1883.6999459	1883.6995138	1883.6934645	1883.6805014	1883.6260563	1883.5366111	1883.2911764
20	1883.6999512	1883.6995613	1883.6941016	1883.6824025	1883.6332658	1883.5525415	1883.3310368

Table 4: The de Broglie wavelength of electron hydrogen-like atoms

Quantum No. *n*	The de Broglie wavelength for electron hydrogen-like atoms (10^{-9} *m*)						
	H_1	Li_3	Na_{11}	K_{19}	Rb_{37}	Sc_{55}	Fr_{87}
1	0.533435462	0.177826547	0.048554445	0.028180565	0.014624264	0.010012235	0.006652601
2	1.066862641	0.355628243	0.097017640	0.056203025	0.028936565	0.019550496	0.012507748
3	1.600291661	0.533435462	0.145501142	0.084260765	0.043319284	0.029197763	0.018555670
4	2.133721140	0.711244061	0.193989709	0.112327274	0.057719222	0.038870977	0.024646202
5	2.667150805	0.889053213	0.242480303	0.140397285	0.072126011	0.048554445	0.030753272
6	3.200580560	1.066862641	0.290971909	0.168469047	0.086536216	0.058243016	0.036868511
7	3.734010369	1.244672226	0.339464093	0.196541808	0.100948372	0.067934496	0.042988387
8	4.267440210	1.422481911	0.387956640	0.224615195	0.115361747	0.077627789	0.049111151
9	4.800869017	1.600291661	0.436449436	0.252688994	0.129775932	0.087322292	0.055235835
10	5.334299954	1.778101457	0.484942383	0.280763092	0.144190687	0.097017640	0.061361862
11	5.867730274	1.955911429	0.533435462	0.308837394	0.158605856	0.106713603	0.067488865
12	6.401159738	2.133721140	0.581928633	0.336911863	0.173021332	0.116410027	0.073616597
13	6.934589641	2.311531014	0.630421874	0.364986450	0.187437047	0.126106806	0.079744892
14	7.468019556	2.489340903	0.678915171	0.393061134	0.201852950	0.135803862	0.085873628
15	8.001449468	2.667150805	0.727408513	0.421135894	0.216269002	0.145501142	0.092002716
16	8.534879381	2.844960716	0.775901891	0.449210717	0.230685176	0.155198602	0.098132091
17	9.068311296	3.022770045	0.824395293	0.477285603	0.245101446	0.164896210	0.104261703
18	9.601743447	3.200580560	0.872888696	0.505360520	0.259517811	0.174593946	0.110391510
19	10.135169898	3.378390739	0.921382135	0.533435462	0.273934235	0.184291782	0.116521482
20	10.668599075	3.556200428	0.969875655	0.561510446	0.288350724	0.193989709	0.122651596

Table 5: The Bohr-de Broglie wavelength of electron hydrogen-like atoms

Quantum No. *n*	The Bohr-de Broglie wavelength for electron hydrogen-like atoms (10^{-10} *m*)						
	H_1	Li_3	Na_{11}	K_{19}	Rb_{37}	Sc_{55}	Fr_{87}
1	3.3251428571	1.1083809524	0.3022857143	0.1750075188	0.0898687259	0.0604571429	0.0382200328
2	6.6502857142	2.2167619047	0.6045714286	0.3500150376	0.1797374517	0.1209142857	0.0764400657
3	9.9754285713	3.3251428571	0.9068571428	0.5250225564	0.2696061776	0.1813714286	0.1146600985
4	13.3005714284	4.4335238095	1.2091428571	0.7000300752	0.3594749035	0.2418285714	0.1528801314
5	16.6257142855	5.5419047618	1.5114285714	0.8750375940	0.4493436293	0.3022857143	0.1911001642
6	19.9508571426	6.6502857142	1.8137142857	1.0500451128	0.5392123552	0.3627428571	0.2293201970
7	23.2759999997	7.7586666666	2.1160000000	1.2250526316	0.6290810811	0.4232000000	0.2675402299
8	26.6011428568	8.8670476189	2.4182857143	1.4000601504	0.7189498069	0.4836571429	0.3057602627
9	29.9262857139	9.9754285713	2.7205714285	1.5750676692	0.8088185328	0.5441142857	0.3439802956
10	33.2514285710	11.0838095237	3.0228571428	1.7500751879	0.8986872587	0.6045714286	0.3822003284
11	36.5765714281	12.1921904760	3.3251428571	1.9250827067	0.9885559845	0.6650285714	0.4204203612
12	39.9017142852	13.3005714284	3.6274285714	2.1000902255	1.0784247104	0.7254857143	0.4586403941
13	43.2268571423	14.4089523808	3.9297142857	2.2750977443	1.1682934363	0.7859428571	0.4968604269
14	46.5519999994	15.5173333331	4.2319999999	2.4501052631	1.2581621621	0.8464000000	0.5350804598
15	49.8771428565	16.6257142855	4.5342857142	2.6251127819	1.3480308880	0.9068571428	0.5733004926
16	53.2022857136	17.7340952379	4.8365714285	2.8001203007	1.4378996139	0.9673142857	0.6115205254
17	56.5274285707	18.8424761902	5.1388571428	2.9751278195	1.5277683397	1.0277714286	0.6497405583
18	59.8525714278	19.9508571426	5.4411428571	3.1501353383	1.6176370656	1.0882285714	0.6879605911
19	63.1777142849	21.0592380950	5.7434285714	3.3251428571	1.7075057915	1.1486857143	0.7261806240
20	66.5028571420	22.1676190473	6.0457142856	3.5001503759	1.7973745174	1.2091428571	0.7644006568

Table 6: The de Broglie wavelength of muonic hydrogen-like atoms

Quantum No. *n*	The de Broglie wavelength for muonic hydrogen-like atoms (10^{-12} *m*)						
	H_1	Li_3	Na_{11}	K_{19}	Rb_{37}	Sc_{55}	Fr_{87}
1	2.576982907	0.859065442	0.234562538	0.136137996	0.070648617	0.048368283	0.032138167
2	5.153925802	1.718010834	0.468684254	0.271512198	0.139790168	0.094446842	0.060423905
3	7.730877587	2.576982907	0.702904066	0.407056836	0.209271904	0.141051993	0.089640920
4	10.307831597	3.435961648	0.937148355	0.542643836	0.278836823	0.187782495	0.119063776
5	12.884786494	4.294943058	1.171402429	0.678247756	0.348434833	0.234562538	0.148566532
6	15.461741837	5.153925802	1.405661396	0.813860131	0.418049353	0.281367229	0.178108746
7	18.038697435	6.012909306	1.639923157	0.949477336	0.487673296	0.328185969	0.207673366
8	20.615653191	6.871893288	1.874186665	1.085097559	0.557303124	0.375013476	0.237251936
9	23.192603953	7.730877587	2.108451381	1.220719780	0.626936870	0.421846821	0.266839784
10	25.769564986	8.589862109	2.342716826	1.356343438	0.696573368	0.468684254	0.296434118
11	28.346523046	9.448847484	2.576982907	1.491968088	0.766211864	0.515524655	0.326033165
12	30.923477006	10.307831597	2.811249433	1.627593540	0.835851844	0.562367282	0.355635736
13	33.500433065	11.166816494	3.045516299	1.763219567	0.905492981	0.609211621	0.385241024
14	36.077389153	12.025801466	3.279783436	1.898846057	0.975135023	0.656057306	0.414848443
15	38.654345261	12.884786494	3.514050786	2.034472919	1.044777788	0.702904066	0.444457566
16	41.231301367	13.743771572	3.748318313	2.170100083	1.114421140	0.749751700	0.474068074
17	43.808267128	14.602753836	3.982585956	2.305727547	1.184064958	0.796600049	0.503679723
18	46.385234030	15.461741837	4.216853602	2.441355170	1.253709232	0.843449015	0.533292317
19	48.962173395	16.320728212	4.451121425	2.576982907	1.323353794	0.890298464	0.562905710
20	51.539125963	17.179712213	4.685389638	2.712610849	1.392998665	0.937148355	0.592519789

Table 7: The Bohr-de Broglie wavelength of muonic hydrogen-like atoms

Quantum No. *n*	The Bohr-de Broglie wavelength for muonic hydrogen-like atoms (10^{-13} *m*)						
	H_1	Li_3	Na_{11}	K_{19}	Rb_{37}	Sc_{55}	Fr_{87}
1	16.0634920633	5.3544973544	1.4603174603	0.8454469507	0.4341484341	0.2920634921	0.1846378398
2	32.1269841266	10.7089947089	2.9206349206	1.6908939014	0.8682968683	0.5841269841	0.3692756796
3	48.1904761899	16.0634920633	4.3809523809	2.5363408521	1.3024453024	0.8761904762	0.5539135194
4	64.2539682531	21.4179894177	5.8412698412	3.3817878028	1.7365937366	1.1682539682	0.7385513592
5	80.3174603164	26.7724867721	7.3015873015	4.2272347535	2.1707421707	1.4603174603	0.9231891990
6	96.3809523797	32.1269841266	8.7619047618	5.0726817042	2.6048906049	1.7523809524	1.1078270388
7	112.4444444430	37.4814814810	10.2222222221	5.9181286549	3.0390390390	2.0444444444	1.2924648787
8	128.5079365063	42.8359788354	11.6825396824	6.7635756056	3.4731874731	2.3365079365	1.4771027185
9	144.5714285696	48.1904761899	13.1428571427	7.6090225563	3.9073359073	2.6285714285	1.6617405583
10	160.6349206329	53.5449735443	14.6031746030	8.4544695070	4.3414843414	2.9206349206	1.8463783981
11	176.6984126961	58.8994708987	16.0634920633	9.2999164577	4.7756327756	3.2126984127	2.0310162379
12	192.7619047594	64.2539682531	17.5238095236	10.1453634084	5.2097812097	3.5047619047	2.2156540777
13	208.8253968227	69.6084656076	18.9841269839	10.9908103591	5.6439296439	3.7968253968	2.4002919175
14	224.8888888860	74.9629629620	20.4444444442	11.8362573098	6.0780780780	4.0888888888	2.5849297573
15	240.9523809493	80.3174603164	21.9047619045	12.6817042605	6.5122265121	4.3809523809	2.7695675971
16	257.0158730126	85.6719576709	23.3650793648	13.5271512112	6.9463749463	4.6730158730	2.9542054369
17	273.0793650758	91.0264550253	24.8253968251	14.3725981619	7.3805233804	4.9650793650	3.1388432767
18	289.1428571391	96.3809523797	26.2857142854	15.2180451126	7.8146718146	5.2571428571	3.3234811165
19	305.2063492024	101.7354497341	27.7460317457	16.0634920633	8.2488202487	5.5492063491	3.5081189563
20	321.2698412657	107.0899470886	29.2063492060	16.9089390140	8.6829686829	5.8412698412	3.6927567962

Table 8: The frequency of electron hydrogen-like atoms from Bohr Model

Quantum No. *n*	Frequency for Bohr Model (10^{15} *Hz*)						
	H_1	Li_3	Na_{11}	K_{19}	Rb_{37}	Sc_{55}	Fr_{87}
1	2.558884996	23.02805778	309.2404825	920.3151179	3453.507130	7498.325798	17850.91275
2	0.639726216	5.757416742	77.38283471	230.7260152	872.6847848	1920.028014	4747.257460
3	0.284323171	2.558884996	34.39835544	102.5981665	388.6259985	857.0862090	2133.310196
4	0.159931864	1.439379330	19.35025327	57.72195629	218.7529485	482.8473954	1204.597366
5	0.102356417	0.921204702	12.38451108	36.94515888	140.0465638	309.2404825	772.3080653
6	0.071080854	0.639726216	8.600486585	25.65753221	97.27141293	214.8326247	536.8402320
7	0.052222672	0.470003257	6.318783177	18.85095104	71.47217788	157.8726660	394.6414644
8	0.039982986	0.359846404	4.837847343	14.43301732	54.72459601	120.8893764	302.2607839
9	0.031591510	0.284323171	3.822512214	11.40400559	43.24119745	95.52734484	238.8847747
10	0.025589112	0.230301819	3.096244116	9.237325254	35.02653400	77.38283471	193.5322628
11	0.021148024	0.190332085	2.558884996	7.634202806	28.94826027	63.95623246	159.9657810
12	0.017770217	0.159931864	2.150177752	6.414882458	24.32503525	53.74322265	134.4296042
13	0.015141487	0.136273318	1.832106479	5.465956339	20.72695906	45.79451485	114.5527917
14	0.013055670	0.117500982	1.579726077	4.713007316	17.87192112	39.48709500	98.77887074
15	0.011372939	0.102356417	1.376118071	4.105563132	15.56857418	34.39835544	86.05180743
16	0.009995748	0.089961699	1.209479589	3.608412352	13.68342151	30.23343922	75.63466165
17	0.008854361	0.079689290	1.071373536	3.196384283	12.12103171	26.78155247	67.00051888
18	0.007897868	0.071080854	0.955639251	2.851099845	10.81171766	23.88876244	59.76455979
19	0.007088396	0.063795551	0.857693209	2.558884996	9.703635254	21.44054258	53.64043688

20	0.006397279	0.057575495	0.774068301	2.309396039	8.757564278	19.35025327	48.41151843

Table 9: The frequency of muon hydrogen-like atoms using de Broglie Model

Quantum No. *n*	Frequency for de Broglie Model (10^{18} *Hz*)						
	H_1	Li_3	Na_{11}	K_{19}	Rb_{37}	Sc_{55}	Fr_{87}
1	0.529689194	4.766807975	64.01277940	190.5052282	714.8759897	1552.153505	3695.139178
2	0.132423327	1.191785267	16.01824669	47.76028516	180.6457519	397.4457928	982.6822679
3	0.058854897	0.529689194	7.120459593	21.23782046	80.44558146	177.4168480	441.5952000
4	0.033105896	0.297951521	4.005502416	11.94844495	45.28186006	99.94941222	249.3516584
5	0.021187778	0.190689373	2.563593796	7.647647860	28.98963893	64.01277940	159.8677689
6	0.014713737	0.132423327	1.780300723	5.311109164	20.13518246	44.47035301	111.1259298
7	0.010810093	0.097290674	1.307988116	3.902146854	14.79474078	32.67964207	81.69078359
8	0.008276478	0.074488206	1.001434401	2.987634589	11.32799141	25.02410073	62.56798259
9	0.006539443	0.058854897	0.791260029	2.360629153	8.950927866	19.77416039	49.44914811
10	0.005296946	0.047672477	0.640922532	1.912126330	7.250492528	16.01824669	40.06117811
11	0.004377641	0.039398742	0.529689194	1.580279980	5.992289882	13.23894005	33.11291660
12	0.003678435	0.033105896	0.445086795	1.327880670	5.035282305	11.12484705	27.82692794
13	0.003134288	0.028208577	0.379246041	1.131452961	4.290480524	9.479464608	23.71242789
14	0.002702524	0.024322703	0.327003298	0.975592515	3.699487676	8.173828644	20.44722631
15	0.002354199	0.021187778	0.284856441	0.849851568	3.222694853	7.120459593	17.81272411
16	0.002069120	0.018622072	0.250362275	0.746941357	2.832468253	6.258321922	15.65637491
17	0.001832853	0.016495683	0.221774322	0.661651548	2.509053562	5.543781356	13.86910745
18	0.001634859	0.014713737	0.197817325	0.590177668	2.238025555	4.944973823	12.37126392
19	0.001467298	0.013205679	0.177542494	0.529689194	2.008652495	4.438192314	11.10357044
20	0.001324237	0.011918128	0.160232138	0.478044980	1.812815808	4.005502416	10.02118429

Table 10: The energy of electron hydrogen-like atoms from the modified model

Quantum No. *n*	The Energy for an Electron, $E_n^l(\gamma)$ in different orbit (*eV*)						
	H_1	Li_3	Na_{11}	K_{19}	Rb_{37}	Sc_{55}	Fr_{87}
1	13.6360382246	122.7141811111	1647.9111053261	4904.2657497675	18403.3889256673	39957.8181507759	95125.7058932699
2	3.4090360220	30.6806890157	412.3652636534	1229.5155008092	4650.4486011881	10231.6341236534	25297.6519365066
3	1.5151292995	13.6360382246	183.3053424728	546.7352072373	2070.9484623017	4567.3254202275	11368.1930795350
4	0.8522606597	7.6703062372	103.1155338738	307.5944416546	1165.7122359300	2573.0447550498	6419.1770748672
5	0.5454469487	4.9090062826	65.9958016202	196.8769981775	746.2939175924	1647.9111053237	4115.5512105187
6	0.3787826510	3.4090360220	45.8311194127	136.7263828387	518.3494781648	1144.8212264936	2860.7671025876
7	0.2782893159	2.5045996135	33.6721536525	100.4548029874	380.8679750367	841.2874067072	2103.0042928722
8	0.2130652680	1.9175849336	25.7803970905	76.9120833213	291.6218143274	644.2072027710	1610.7170243437
9	0.1683478724	1.5151292992	20.3697801325	60.7707858743	230.4279460571	509.0555139130	1272.9926902875
10	0.1363617793	1.2272550006	16.4995703838	49.2247680438	186.6528415812	412.3652636543	1031.3137632711
11	0.1126956876	1.0142604949	13.6360382244	40.6818903250	154.2623411336	340.8162647041	852.4414116540
12	0.0946956834	0.8522606595	11.4580788406	34.1842570229	129.6256421928	286.3921735870	716.3617027103
13	0.0806874470	0.7261866669	9.7631093260	29.1275260856	110.4518593850	244.0343187173	610.4401915345
14	0.0695723402	0.6261507960	8.4181997986	25.1151372572	95.2376523853	210.4227178013	526.3825706875
15	0.0606052387	0.5454469491	7.3331934195	21.8781288837	82.9633503474	183.3053424724	458.5613401371
16	0.0532663234	0.4793967567	6.4451938738	19.2288628859	72.9175632929	161.1109265629	403.0494283239
17	0.0471840098	0.4246559680	5.7092406699	17.0332079721	64.5917446341	142.7161706532	357.0389639788
18	0.0420869720	0.3787826515	5.0925040927	15.1932221600	57.6145465476	127.3007904771	318.4792692757
19	0.0377733491	0.3399600628	4.5705595007	13.6360382243	51.7096863277	114.2544728615	285.8444357609
20	0.0340904475	0.3068139644	4.1249313444	12.3065369171	46.6681705300	103.1155338738	257.9800633718

Table 11: The energy of muon hydrogen-like atoms using the modified model

Quantum No. *n*	The Energy for muon, $E_n^{\mu}(\gamma)$ in different orbit (*keV*)						
	H_1	Li_3	Na_{11}	K_{19}	Rb_{37}	Sc_{55}	Fr_{87}
1	2.8226599125	25.4018354900	341.1175988025	1015.1830102019	3809.5015076131	8271.2683572106	19691.0211199069
2	0.7056704566	6.3509026263	85.3596095763	254.5097086675	962.6428604459	2117.9482635963	5236.6139508569
3	0.3136317650	2.8226599125	37.9442058919	113.1741878981	428.6863316965	945.4363619871	2353.2159674638
4	0.1764179566	1.5877533911	21.3449155119	63.6720494225	241.3024328375	532.6202642953	1328.7696544975
5	0.1129075184	1.0161643005	13.6611309354	40.7535386228	154.4828409416	341.1175988020	851.9191005774
6	0.0784080088	0.7056704566	9.4870417184	28.3023612476	107.2983419801	236.9779938842	592.1787902356
7	0.0576058884	0.5184521200	6.9701358061	20.7941442184	78.8396708326	174.1464931884	435.3218886246
8	0.0441045105	0.3969400813	5.3365421977	15.9208012475	60.3657155658	133.3508909736	333.4184240391
9	0.0348480096	0.3136317649	4.2165444874	12.5795526760	47.6985848338	105.3744913800	263.5094868895

10	0.0282268883	0.2540417851	3.4154110694	10.1895269851	38.6371382073	85.3596095764	213.4819489971
11	0.0233280073	0.2099519224	2.8226599124	8.4211512973	31.9323046147	70.5489667938	176.4553722124
12	0.0196020065	0.1764179565	2.3718223200	7.0761412038	26.8325079339	59.2831799325	148.2868724610
13	0.0167023015	0.1503206400	2.0209636305	6.0293978997	22.8635348927	50.5151039745	126.3611196476
14	0.0144014744	0.1296132148	1.7425673583	5.1988334122	19.7141940438	43.5575025849	108.9611921323
15	0.0125452844	0.1129075185	1.5179710378	4.5287726789	17.1734135219	37.9442058918	94.9221974084
16	0.0110261289	0.0992351286	1.3341551319	3.9803746174	15.0939356016	33.3499617985	83.4312316630
17	0.0097670900	0.0879037854	1.1818128187	3.5258740502	13.3704911393	29.5422473252	73.9070655436
18	0.0087120032	0.0784080089	1.0541483472	3.1449969871	11.9262111353	26.3512636288	65.9252087401
19	0.0078190833	0.0703717330	0.9461058166	2.8226599124	10.7039050698	23.6506758823	59.1697982025
20	0.0070567226	0.0635104906	0.8538607883	2.5474531418	9.6603112997	21.3449155119	53.4018731180