Development of Internal Rate of Return (IRR) Calculator

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ARTICLE INFORMATION

ABSTRACT

The analytical determination of internal rate of return (IRR) is computationally demanding and the computational labour tends to be compounded as the project life becomes longer. Although some commercial software exist for computing the IRR but such exist as mere products (computing aid) that never provide the fundamental theory upon which the calculation is moored or founded. As a result the user merely inputs the necessary data in the textbox, enter and a display appears showing the results; but that is just all. This paper goes beyond user – friendliness to seek to contribute to knowledge by demonstrating the plausibility of application of binomial theory and Newton-Raphson's equation to IRR calculation, an initiative that past studies have sparsely investigated. Further, the research objective is sharpened by the need to develop an easy-to-use IRR calculator. The theoretical development of the IRR calculator is rooted on Newton-Raphson's equation of Numerical Analysis and the computational scheme was programmed with Visual Basic.net 2010. Our results prove that the method advocated is sure fire and the computational accuracy, which could be approximated to the nearest one percent, is the first rate. A numerical illustration is presented. The method proposed in this study is presented in both illustrative and instructive ways thereby making the research outcome not only relevant for academic purpose but also ideally suited to industrial engineers and practising finance managers as their guide to necessary action.

1. Introduction

 A common investment decision problem is whether or not such an investment will be profitable. The Net Present Value (NPV) and the Internal Rate of Return (IRR) are the preferred techniques among the sophisticated models available. Whereas the former is preferred by the academics because it has well established theoretical basis, the latter is more appealing to practitioners in the finance industry. This preference for IRR is attributed to the general disposition of investors and businessmen towards rates of return rather than pure currency return. Moreover, interest rates, profitability measures, among others, are often cited as annual returns and so use of IRR makes sense to corporate decisions makers [1]. Moreover since the search for IRR incorporates

the greedy solutions which locates the IRR that equates NPV with zero [2], this paper will turn the spotlight on IRR computation only.

Industrial Engineers, like financial managers, make investment decisions. Often times they are confronted with making decisions among alternative projects to undertake or equipment replacement decisions. Then, the IRR computation becomes imperative. A good understanding of the theory studying the NPV and IRR computations, particularly the existence of real roots of the characteristic equation of IRR will provide more enlightenment to investment decision makers in arriving at well informed decisions. The method advocated follows the most widely used of all root-locating techniques, the Newton-Raphson iteration of numerical analysis, which starts a progressively greedy search for the true root(s) of the IRR equation by using an assumed root.

A widely applied analytical computation of IRR is the heuristic approach, which is a trial-and-error method. See, for example, Gitman[3]. There are some weaknesses associated with this. First, heuristics are fallible and do not always guarantee a correct solution. Second, user assumptions associated with some of the steps. Fourth, analytically, convergence rate is slow in relation to the method proposed. Fifth, one needs financial table to use the heuristic method whereas the method advocated obviates the need to consult financial table. Above all, the selling point of this study is the new IRR calculator that computes as fast and accurate as the existing commercial software and at same time a theoretical basis regarding what the software does is presented.

The original formulation of the standard IRR problem came from Lorie and savage [4]. The papers, among other issues, identified the existence of multiple IRR. Since then there has been considerable interest in the field. Accordingly, extensions have been developed by a variety of studies, most notably Grant and Ireson [5-10]. Several studies have expressed their reservations about the appropriateness of the application of IRR when cash flow pattern gives rise to multiple or non-existent IRR. Representative studies include [11-20]. All on a positive note, Hazen [21] proposed four theorem and proofs to support the claim that when there are multiple (or even complex-valued) internal rates, each has a meaningful interpretation as a rate of return on its own underlying investment stream. The author is able to hold this strong view because his analysis furnishes sound mathematical theory that afforded him the enlightenment to rebut the pessimisms expressed by the foregoing studies. It goes to support the view of the current paper that IRR determination should have theoretical basis. More recent studies on IRR computation are reported in [23-27]. Taken together, the sum and substance of the foregoing review is that though past studies have focused on the problem of existence of multiple or complex-valued roots, as identified by Lorie and savage, studies dealing with application of numerical methods (Newton-Raphson and geometrical series) to IRR determination are scanty. Therefore, more studies are investigated to calculate IRR more efficiently than the traditional method [28] and the use of Newton-Raphson method with quadratic convergence is finding application in its computation [29]. The aim of this study therefore is to demonstrate the use of Newton-Raphson method and develop software (IRR calculator) based on its theory. It is the belief of the authors that professionals and students of Engineering Economics and those of Finance would find this paper insightful and hence helpful.

2. Methodology

We present here a thumbnail sketch of the Newton-Raphsons analytical technique adopted in this paper.

Convergence Analysis of the Newton Raphson's Iteration.
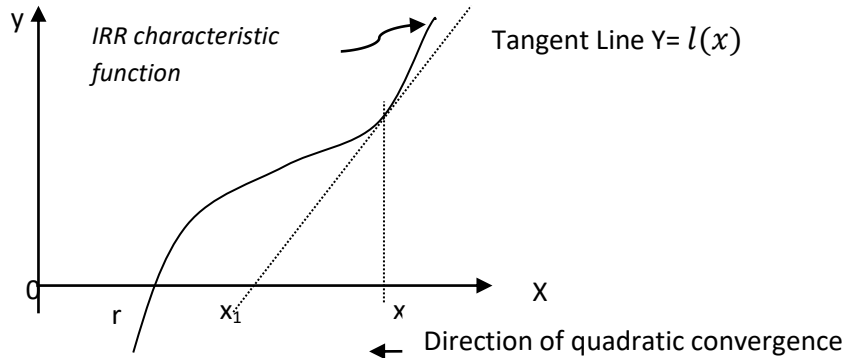
Figure 1 depicts the geometry of the method.

Figure 1- Method Geometry

2.1 Lemma

If f, f', f'' are continuous in a neighborhood of roots r of f and if $f'(r) \neq 0$ because r is simple, then a basin of attraction represented by $(x_0, x_1, ..., x_n)$ converges quadractically to r.

Clearly, $y = l(x)$ is tangent to $y = f(x)$ at point A (x_0), $f(x)$. Take an arbitrary point $L(x, l(x))$ on the line $y = l(x)$

Considering the straight line (tangent at A), and taking two points L and A on the line $l(x)$ from analytic geometry;

$$l(x) - f(x_0) = f'(x_0)(x-x_0)$$

$$l(x) = f'(x_0)(x-x_0) + f(x_0) \qquad (1)$$

Equation (1) suggest that $l(x)$ and $y = f(x)$ are in the neighborhood of x_0, and at exactly x_0, $y = f(x)$ and $l(x)$ coincide. We seek the zero of $l(x)$

$$l(x) - f(x_0) = f'(x_0)(x - x_0)$$

$$x - x_0 = \frac{-f(x_0)}{f'(x)}$$

Or $x = x_0 - \dfrac{f(x_0)}{f'(x)}$ \qquad (2)

Since the two curves $l(x)$ and $y = f(x)$ are in the same neighborhood. It means that this value of x in (2) that located the root of $l(x)$ is close to the real root, r of $y = f(x)$. Therefore by choosing sequence of values $(x_0, x_1, x_2, ..., x_n, x_{n+1})$ along the x-axis towards r, we can hit the required root r of the function $y = f(x)$. This was basically the original idea that Isaac Newton and

Raphson used in the famous, classical root location problem, Generalizing Equation (2), we have:

$$x_{n+1} = x_n - \frac{f(x_0)}{f'(x)} \qquad (3)$$

If the function is well behaved and x_0 properly selected

$lim_{n \to \infty} x_n \to r$, where r is the desired root

In this case, r = IRR and

$$f(k) = CF_t \sum_{t=1}^{n}[(1+k)^{-t}] - II = 0 \qquad (4)$$

Where k = discount rate, t = time in years, CF_t = stream of cash inflows and II = initial investment.

2.2 Flow Chart for the Software Development
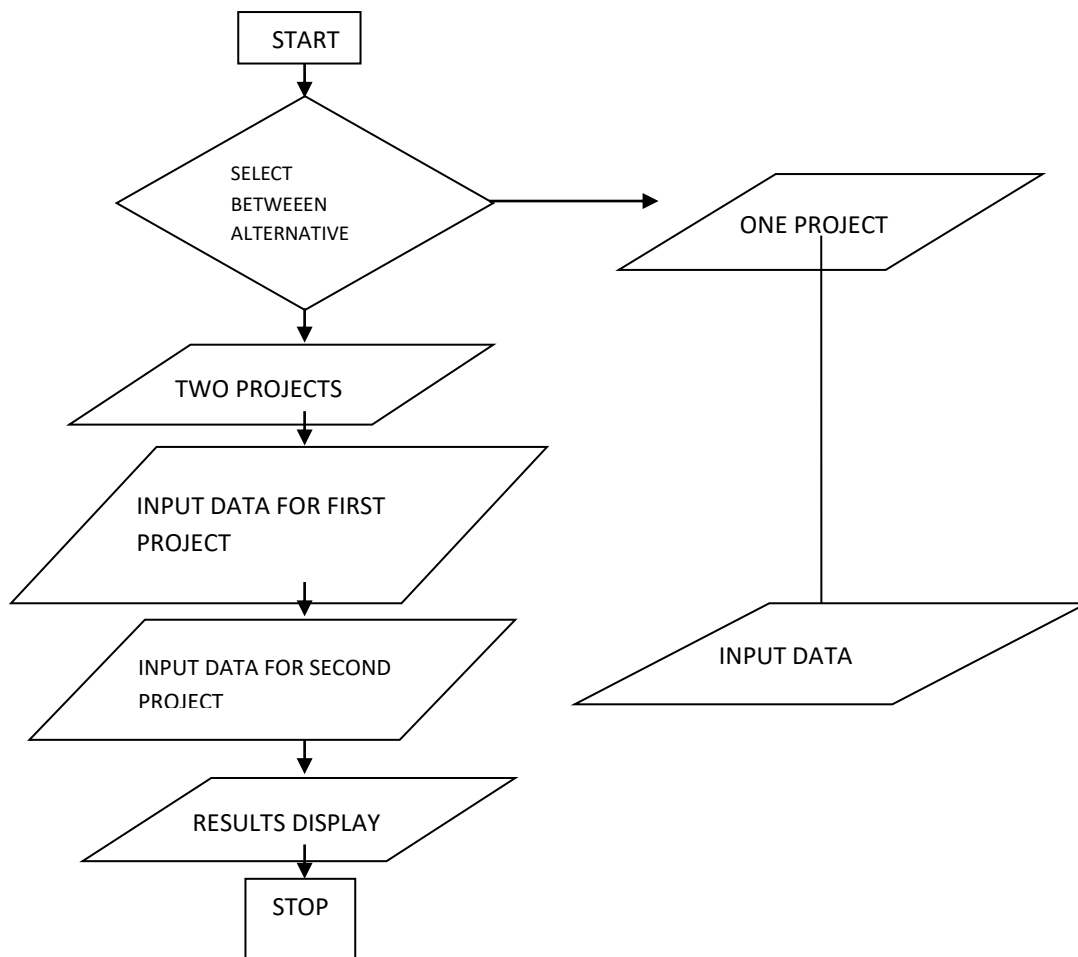
Figure 2 depicts the flow chart

Figure 2- Software flow chart

2.3 Software development

The IRR calculator was programmed using visual basic.net 2010. This is a very powerful alternative as results are realized within split seconds. The program is user interactive and the user can easily select whether to choose between two projects or to find the IRR foe a single project. This user interactivity is one of the reasons why visual basic is the most widely used programming language by engineers today. The user can easily correct errors and errors of entry are also debugged by this software developed. Results can be obtained within split seconds for any number of years.

2.4 Program pseudocode

Select one or two projects alternative

If one project

      Enter Values for Internal investment, Cost of Capital and Number of years of investment

      Enter Value for annual Cash Flows

      Use data to get first trial value for the Newton-Raphson equation

      Obtain Internal Rate of Return using Newton-Raphson's equation

      Compare IRR with Cost or Capital

      Display results

If two projects

      Enter value for Initial investment, Cost of Capital and Number of years of investment

      Enter Value for annual Cash Flows for project A

      Enter Value for annual Cash Flows for project B

      For each Project

      Use data to get first trial value for the Newton-Raphson equation

      Obtain Internal Rate of Return using Newton-Raphson's equation

      Compare IRR for both projects with the cost of capital

      Display results

End sub

2.5 Numerical Example

A Local Government Council, as part of the bid to fulfill its electoral premises to the governed intends to grant micro-credit facilities to their citizens with a view to stimulating entrepreneurial culture in them. Table 1 shows a 5-year cash flow projection as contained in a feasibility study

report submitted by a potential beneficiary of the scheme. It is required to evaluate which of the two projects namely Fishery (A) or Pineapple farm (B) that would bring more return on investment at the end of 5 years. The evaluation between the alternatives is to be effected with internal rate of return (IRR) criterion. Cost of Capital (discount rate = 10%)

We are required to illustrate the use of Newton-Raphson's iteration method to obtain the IRR. In addition, we are to use the software developed in the study to compute IRR.

Table1: Capital Expenditure Data

	Project A	Project B
Initial Investment (11)	₦ 39,000	₦ 40,000
Year (t)	CF_t in Nigeria Currency (₦)	CF_t in Nigeria Currency(₦)
1	13,000	20,000
2	13,000	15,000
3	13,000	10,000
4	13,000	10,000
5	13,000	10,000

From (4):

$$f(k) = CF_t \sum_{t=1}^{n} [(1+k)^{-t}] - \text{II} = 0 \qquad (5)$$

Substitute cash flow CF_t, t=1, 2,….,5, then

$$CF_1(1+k)^{-1} + CF_2(1+k)^{-2} + CF_3(1+K)^{-3} + CF_4(1+K)^{-4} + CF_5(1+K)^{-5} - \text{II} = 0 \qquad (6)$$

2.5.1 Project A

$CF_1 = CF_2 = \ldots = CF_3 = 13,000$

$$13,000\left[\frac{1}{(1+k)} + \frac{1}{(1+k)^2} + \frac{1}{(1+k)^3} + \frac{1}{(1+k)^4} + \frac{1}{(1+k)^5}\right] - 39,000 = 0$$

Clearing terms and simplifying:

$$(1 + k)^4 + (1 + k)^3 + (1 + k)^2 + (1 + k) + 1 = 3(1 + k)^5 \tag{7}$$

Expand each term by binomial series:

$$(1 + k)^5 = 1 + 5k + 10k^2 + 10k^3 + 5k^4 + k^5 \tag{8}$$

$$(1 + K)^4 = 1 + 4k + 6k^2 + 4k^3 + k^4 \tag{9}$$

$$(1 + k)^3 = 1 + 3k + 3k^2 + k^3 \tag{10}$$

$$(1 + k)^2 = 1 + 2k + k^2 \tag{11}$$

Substitute (8)-(11) into (7) and simplifying

$$3k^5 + 14k^4 + 25k^3 + 20k^2 + 5k - 2 = 0 \tag{12}$$

Apply Newton-Raphson equation:

$$k_{n+1} = k_n - \frac{F(k_n)}{F'(k_n)} \tag{13}$$

$F(k) = F(0) = 3(0)^5 + 14(0)^4 + 25(0)^3 + 20(0)^2 + 5(0) - 2 = -2$

$F(0) < 0$

Take next approximate F(1):

$$F(k) = F(1) = 3(1)^5 + 14(1)^4 + 25(1)^3 + 20(1)^2 + 5(1) - 2 = 65 \tag{14}$$

$F(0) > 0$

This suggests that there is a solution to the equation which lies between k = 0 and k = 1. Its therefore seems reasonable to start iteration with

$k_0 = \left[\frac{0+1}{2}\right] = \frac{1}{2} = 0.5$, i.e. midway.

Substitute k = 0.5 into (14):

$F(0.5) = 3(0.5)^5 + 14(0.5)^4 + 25(0.5)^3 + 20(0.5)^2 + 5(0.5) - 2 = 9.59375$

From (12): F'(0.5) = $15(0.5)^4 + 56(0.5)^3 + 40(0.5) - 5 = 51.6875$

We now use these sets of values F(0.5) and F'(0.5) to estimate the new value X_1 in the Equation (13).

$$K_1 = 0.5 - \frac{9.5938}{51.6875} = 0.3144$$

And for n = 1, we have

$$K_2 = K_1 - \frac{F(K_1)}{F'(K_1)} \tag{15}$$

$F(K_1) = F (0.3144) = 2.4719$

$$F^1 (K_1) = F^1(0.3144) = 26.8765$$

Hence, from (15):

$$K_2 = 0.3144 - \frac{2.4719}{26.8765}$$

$$K_2 = 0.2224$$

Similarly, for n = 2

$$K_3 = K_2 - \frac{F(K_2)}{F'(K_2)} \tag{16}$$

$F((K_2) = F(0.2224) = 0.4121$

$$F(K_2) = F (0.2224) = 0.4121$$

$$F^1(K_2) = F^1 (0.2224) = 18.2583$$

and substituting the values of K_2 into (16):

$$K_3 = 0.2224 - \frac{0.4121}{18.2583}$$

$$K_3 = 0.1998$$

Similarly, for n = 4,

$$K_4 = K_3 - \frac{F(K_3)}{F^1(K_3)} \tag{17}$$

$F(K_3) = F(0.1998) = 0.0201$

$F^1(K_3) = F^1(0.1998) = 16.4566$

and substituting for K_3 into (17)

$$K_4 = 0.1998 - \frac{0.0201}{16.4566}$$

$K_4 = 0.1986$

Further for n = 4, the Newton Raphson's equation becomes

$$K_5 = K_4 - \frac{F(K_4)}{F^1(K_4)} \tag{18}$$

Clearly,
$F(K_4) = F(0.1986) = 0.0004$

$F^1(K_4) = F^1(0.1986) = 16.3641$

$$K_5 = 0.1986 - \frac{0.0004}{16.3641}$$

$$K_5 = 0.1986$$

We now collate the sequence of K value,

$$K = \{ 0.3144, \ 0.2224, \ 0.1998, \ 0.1986, \ 0.1986 \}$$

There is quadratic convergence to K = 0.1986.

The Internal rate of return for project A is therefore:

K = IRR = 19.86%≈20% to the nearest one percent

2.5.2 Project B

Initial investment II = ₦ 40,000

From (5):

$$CF_t \sum_{t=1}^{n} [(1 + k)^{-t}] - II = 0$$

Recall that this is a mixed stream, hence CF_t is distinct for each year so that:

$$(CF_1(1+k)^{-1} + CF_2(1+k)^{-2} + CF_3(1+K)^{-3} + CF_4(1+K)^{-4} + CF_5(1+K)^{-5}) - II = 0 \qquad (19)$$

Substituting the values for yearly cash inflows:

$$\left[20 \frac{1}{(1+k)} + 15 \frac{1}{(1+k)^2} + 10 \frac{1}{(1+k)^3} + 10 \frac{1}{(1+k)^4} + 10 \frac{1}{(1+k)^5} \right] = 40 \qquad (20)$$

By clearing terms we obtain:

$$20(1 + k)^4 + 15(1 + k)^3 + 10(1 + k)^2 + 10(1 + k) + 10 = 40(1 + K)^5$$

As before, expanding the terms in bracket by binomial and simplifying; we obtain:

$$40k^5 + 180k^4 + 305k^3 + 225k^2 + 45k - 25 = 0 \qquad (21)$$

As usual, we evoke equation (13):

$$k_{n+1} = k_n - \frac{F(k_n)}{F'(k_n)}$$

As before, substitute K = 0 in (21)

F(k) = F(0) = $40(0)^5 + 180(0)^4 + 305(0)^3 + 225(0)^2 + 45(0) - 25$ = -25

Therefore, $F(0) < 0$

Similarly,

$F(k) = F(1) = 40(1)^5 + 180(1)^4 + 305(1)^3 + 225(1)^2 + 45(1) - 25 = 770$

Therefore, $F(1) > 0$

Implying that the solution lies between $F(0)$ and $F(1)$. And as before, we shall take a trail value to be in-between, i.e 0.5.

Again, from (21):

$$F^1(k) = 200K^4 + 720K^3 + 915K^2 + 450K + 45 \tag{22}$$

Since our seed value is $K = 0.5$, from (21):

$F(K_0) = F(0.5) = 40(0.5)^5 + 180(0.5)^4 + 305(0.5)^3 + 225(0.5)^2 + 45(0.5) -25 = 104.38$

And from (22):

$F^1(K_0) = F^1(0.5) = 200(0.5)^4 + 720(0.5)^3 + 915(0.5)^2 + 450(0.5) + 45 = 601.25$

For $n = 0$,

$$k_1 = k_0 - \frac{F(k_0)}{F'(k_0)} \tag{23}$$

Substituting the values for $F(0.5)$ and $F^1(0.5)$ into equation (23), we have:

$$k_1 = 0.5 - \frac{104.38}{601.25}$$

$$k_1 = 0.3264$$

Similarly, for $n = 1$

$$k_2 = k_1 - \frac{F(k_1)}{F'(k_1)} \tag{24}$$

$F(k_1) = 40(0.3264)^5 + 180(0.3264)^4 + 305(0.3264)^3 + 225(0.3264)^2 + 45(0.3264) - 25 = 26.4559$

$F^1(k_1) = 200(0.3264)^4 + 720(0.3264)^3 + 915(0.3264)^2 + 450(0.3264) + 45 = 316.6683$

Substituting this set of values into (24):

$$k_2 = 0.3264 - \frac{26.4559}{316.6683}$$

$$k_2 = 0.2429$$

Further, for $n = 2$,

$$k_3 = k_2 - \frac{F(k_2)}{F'(k_2)} \tag{25}$$

$F(k_2) = 40(0.2429)^5 + 180(0.2429)^4 + 305(0.2429)^3 + 225(0.2429)^2 + 45(0.2429) - 25 = 4.237$

$F^1(k_2) = 200(0.2429)^4 + 720(0.2429)^3 + 915(0.2429)^2 + 450(0.2429) + 45 = 219.3051$

Hence, plugging in all values into (25)

$$k_3 = 0.2429 - \frac{4.237}{219.3051}$$

$$k_3 = 0.2236$$

Again for n = 3

$$k_4 = k_3 - \frac{F(k_3)}{F'(k_3)} \tag{26}$$

$F(k_3) = 40(0.2236)^5 + 180(0.2236)^4 + 305(0.2236)^3 + 225(0.2236)^2 + 45(0.2236) - 25 = 0.1933$

Also,

$F^1(k_3) = 200(0.2236)^4 + 720(0.2236)^3 + 915(0.2236)^2 + 450(0.2236) + 45 = 199.9162$

$$k_4 = 0.2236 - \frac{0.1933}{199.9162}$$

$$k_4 = 0.2226$$

And for n=4.

$$k_5 = k_4 - \frac{F(k_4)}{F'(k_4)} \tag{27}$$

$F(k_4) = 40(0.2226)^5 + 180(0.2226)^4 + 305(0.2226)^3 + 225(0.2226)^2 + 45(0.2226) - 25 = -0.0061$

And

$F^1(k_4) = 200(0.2226)^4 + 720(0.2226)^3 + 915(0.2226)^2 + 450(0.2226) + 45 = 198.9416$

Hence substituting these set of values into (27), we have:

$$k_5 = 0.2226 - \frac{-0.0061}{198.9416}$$

$$k_5 = 0.2226$$

And the basin of attraction is represented as:

$$\{ 0.3264, \ 0.2429, \ 0.2236, \ 0.2226, \ 0.2226\}$$

There is a quadratic convergence to 0.2226.

Hence, the IRR= K= 0.2226 or 22.26% which is K= 22% to the nearest one percent.

Thus on the basis of IRR, project B is preferred to project A. Although the two projects have similar initial investment and apparently the same average cash inflows (₦ 13,000) over the five year period, project B has early-year higher cash inflows namely ₦ 20,000 and ₦ 15,000 which are intermediate cash inflows that are reinvested at a rate equal to the project's IRR, whereupon project B has turned up trumps.

2.6 Steps to Software Solution

Step 1: Select Checkbox to "Select between two projects" and enter values for initial investments and cost of capital. Figure 3 displays the initial values entered.

Figure 3 – Program interface for initial values

Step 2: Input Annual Income for Project A

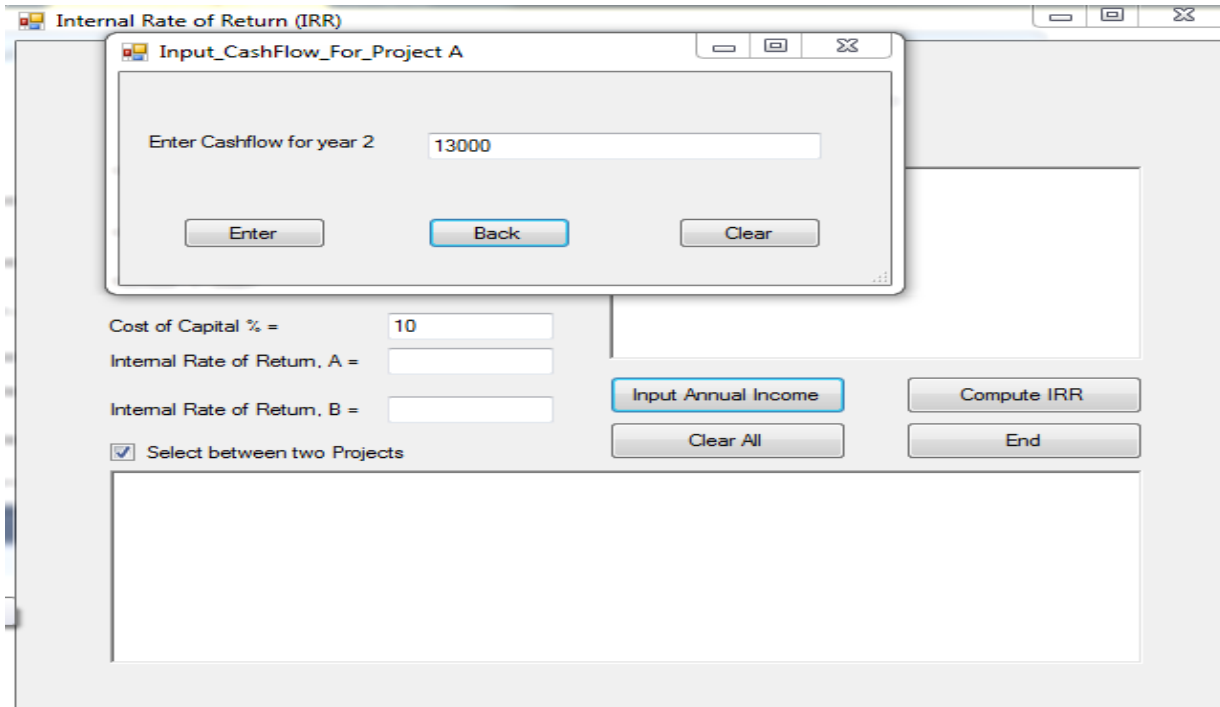Figure 4 depicts dialogue box for making cash flow entries for project A

Figure 4 Dialog box to enter Annual Cash Flow for Project A

Step 3: Input Annual **Income for Project B**

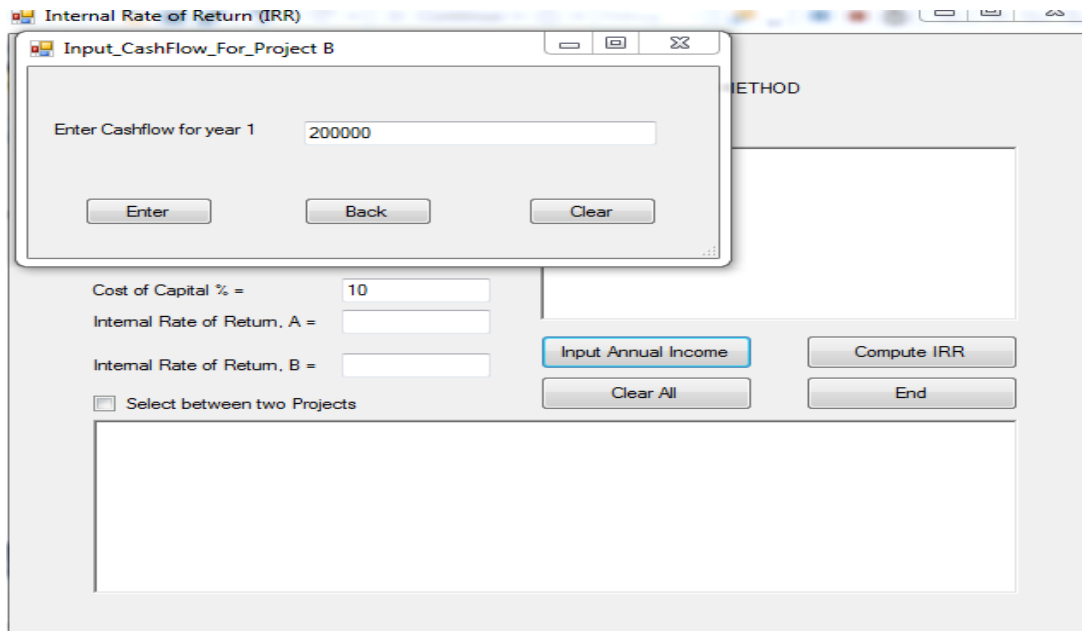Figure 5 depicts dialogue box for project B cash flow entries

Figure 5: Dialog box to enter Annual Cash Flow for Project B

Step 4: Click on Compute Internal Rate of Return to display results

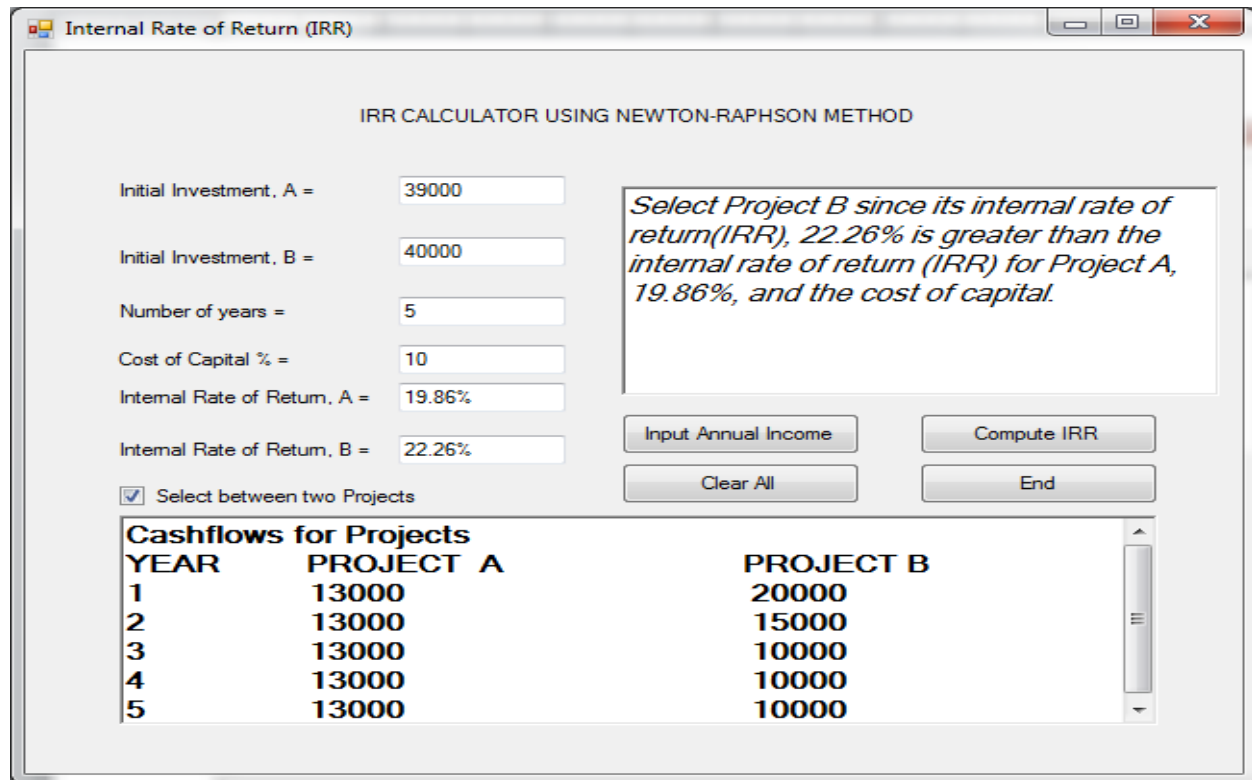Figure 6 depicts the final results of IRR competitions.

Figure 6 : Results display

3. Results and Discussion

The computed internal rate of return for projects A and B using Newton-Raphson's method converged rapidly to 20% and 35% respectively. Effort was not made to search for other roots because the results obtained are consistent with the ones obtained with heuristic approach and are therefore considered realistic in light of the structure of the cash flow used. Literature remarks indicate a 75-100% preference for IRR than NPV as tools for evaluating projects investment attractiveness. According to some of the authors – IRR is preferred to NPV on account of its intuitive appeal. Besides, the authors note that executives apparently feel more comfortable dealing in percentages, Burns and Walker [22], again Gitman and Forrester [1].

Figure 3-5 shows the textbox and typical output for the internal rate of return calculator. As can be observed in figure 3, the basic data of the sample problem were inputted into the textbox and when entered, the output display showed results consistent with those obtained with Newton-Raphson's technique. That goes to attest to the fact that the accuracy of the software is as good as those commercial types that are not easily obtained overseas. The product thus raises hope eternal.

4. Conclusion

This study has presented a thumbnail sketch of the Newton-Raphson's root location theory, ably applied to the analytical computation of IRR and finally successfully adopted the theory to develop an IRR calculator. It is hoped that the result will be quite useful in solving problems of choosing between alternatives in capital budgeting decisions, especially under capital rationing situations.

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