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Determination of Nuclear Radius Parameter from β^+ Transformation Energy of Mirror Nuclei

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ABSTRACT

In this study the values of the β^+ disintegration energy is calculated using the standard values of masses of mirror nuclei. These values are used to plot a graph of β^+ transformation energy against $A^{2/3}$. The nuclear radius parameter is determined from the slop of the graph as $r_0 = 1.23 \times 10^{-15}$ m. The study then continues to compute the numerical values of the Coulomb energy difference between mirror nuclei using Bethe-Weizsäcker mass formula. The nuclear radius parameter determined from the Coulomb energy difference appears to have a mean value of $r_0 = 1.2368 \times 10^{-15}$ m. These calculated values are in good agreement with $r_0 = 1.2 \times 10^{-15}$ m, measured from the experimental data by electron scattering and μ-mesonic atoms. These results have shown that the apparent discrepancy between the values for the nuclear charge parameter derived from electron scattering and μ-mesonic atoms and those derived from mirror nuclei experiments might not be attributed to the use of classical principles. Thus, these developments in the theoretical measurement for nuclear radius parameter from Coulomb energy difference and β^+ disintegration energy provide more accurate results which can be used to improve model parameters.

1. Introduction

In the march towards the new era of nuclear physics, the knowledge of nuclear extension in space, often characterized by nuclear radius, plays a very important role in understanding complex atomic nuclei. It plays a key role in studying the static properties of atomic nuclei [1, 2] in testing theoretical models of nuclei as well as in studying astrophysics and atomic physics [3]. The developments in the measurement techniques for radii of nuclei provide more accurate experimental results which can be used to improve model parameters. Thus, experimental and theoretical nuclear radii studies are one of the important topics in nuclear physics. The radius of atomic nucleus can be determined from its charge density distribution [4] which is most probably spherical and experimental studies demonstrate that the volume or radius of the nucleus is naturally proportional to the number of nucleons. Most nuclei have a nearly spherical shape and can be characterized by an effective radius $R = r_0 A^{1/3}$, where A is the nucleon number [5]. The experimental data indicate that the order of magnitude of the range of nuclear radius parameter, r_0 is not constant [6]. Its value varies within an interval depending on the nuclide and on the way the nuclear radius was measured.

Several authors investigated the apparent discrepancy between the values, $r_0 = 1.2 \times 10^{-15}$ A$^{1/3}$ m for the nuclear radii derived from electron scattering and μ-mesonic atoms carried out with heavier nuclei and those $r_0 = 1.45 \times 10^{-15}$ A$^{1/3}$ m derived from mirror nuclei experiments which are concerned with light nuclei. Wilson, ref.[7] determined the values of charge radius, $r_0 = 1.4 \times 10^{-15}$ $A^{1/3}$ m from the average nuclear radius, $R = 1.36 \times 10^{-15} A^{1/3} [1 + (3/\alpha r)]^{-1/3}$ m deduced from the measured Coulomb energy differences between mirror nuclei. Thus this value of nuclear charge is in agreement with the nuclear radii obtained for the same nuclei from experimental methods [8]. But it is slightly higher than that obtained by high energy electron scattering method. The discrepancy between these values suggested being due to the use of classical principles instead of quantum mechanical principles in calculating the Coulomb energy. Peaslee [9] provide a more rigorous expression for Coulomb energy by adding correction due to non-uniformity of the nuclear charge distribution, the requirement of the discrete arrangement of the charges on protons, effect of uncertainty in the localization of the protons, non-sphericity of the nucleus, corrections of position of the protons and the size of the nucleus and measure the correct value of nuclear charge parameter.

In this work circumvent the difficulty in calculating the charge radii from quantum mechanical principles; we determine r_0 from the β^+ transformation energy using the standard values of masses of the mirror nuclei from ref. [10] and from Coulomb energy difference calculated from Bethe-Weizsäcker formula [11-14]. The energetic in the β^+ transformation of the mirror nuclei contain an important insight on how Coulomb interaction may affect nuclear wave functions [15,16]. The nuclear charge radius can be estimated based on the study of the energetic in the β^+ transformation of the mirror nuclei. As for other methods, the nuclear charge radii from β^+ transformation energy leads to the evidence that the nuclear volume is substantially proportional to the number of nucleons in a determined nucleus.

There are a group of nuclei, called mirror nuclei, which their stable decay products each contain just one more neutron than the number of protons and their mass number is $A = 2Z - 1$. Since the mass number A doesn't change, the nuclear radius, $R = r_0A^{1/3}$ will not change. Experimental evidence showed that nuclear forces are perfectly charge-symmetric and charge-independent symmetrical in neutrons and protons and that nuclear binding between two neutrons is the same as that between two protons. The measured energy differences in the excited analogue states between mirror nuclei are close to each other. This indicates that the "nuclear part" of the binding energies in pairs of mirror nuclei should be close to each other [17-19]. But one expects differences in the excited analogue states between mirror nuclei due to Coulomb interaction as the number of protons and neutrons are interchanged [16].

2. Methodology

2.1 The Liquid Drop Model of Nucleus

Protons inside the nucleus suffer electrostatic repulsion. This acts against the attractive binding of the nuclear forces. We know there is an electromagnetic repulsive force between protons due to their charge and so this will reduce the binding energy for nucleons with several protons. As we believe the nuclear force itself is independent of nucleon type, then the protons will on average be spread evenly throughout the nucleus, which means the charge density is uniform. The nucleus is electrically charged with total charge $+Ze$. Assume that the charge distribution is spherical and from the liquid drop model we set ρ = constant.

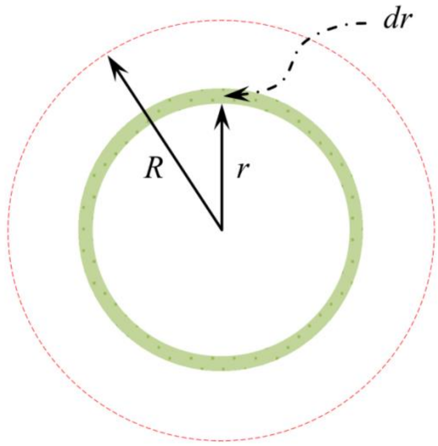

Figure 1: A charge drops of Van der Waal like fluid with a relatively thin surface layer dr.

The reduction in binding energy due to the Coulomb interaction can be computed from classical electrostatics by taking the definition of the charge density:

$$\rho = \frac{\text{charge}}{\text{volume}} = \frac{3Ze}{4\pi R^3}$$

where $R = r_0 A^{1/3}$ is the outer radius of nucleus includes self interaction of last proton with itself. From Figure 1, the electrostatic potential at the surface of a sphere of radius $r < R$:

$$U(r) = \frac{kq}{r} = \frac{k}{r}\frac{4\pi r^3 \rho}{3} = \frac{k}{r}\frac{4\pi r^3}{3}\frac{3Ze}{4\pi R^3} = \frac{Zke}{R}\left(\frac{r}{R}\right)^2 \quad (1)$$

The next layer of nuclear matter has a charge equal to

$$dq = 4\pi r^2 dr\rho \quad (2)$$

and its potential energy is

$$U(r)dq = \frac{Zke}{R}\left(\frac{r}{R}\right)^2 4\pi r^2 dr\rho = k(Ze)^2 \frac{3r^4}{R^6} dr$$

where the use of (1) and (2) have been made. Hence the total Coulomb energy is

$$E_C = \int_0^{Ze} U(r)dq = \frac{3k(Ze)^2}{R^6}\int_0^R r^4 dr$$

Changing the integral to dr, we find:

$$E_C = \frac{3k(Ze)^2}{R^6}\frac{R^5}{5} = \frac{3}{5}\frac{ke^2}{r_0}\frac{Z^2}{A^{1/3}} \quad (3)$$

Equation (3) represents the electrostatic energy required to assemble a spherical nucleus with Z protons. Assuming mirror nuclei to be of the same structure, their mass difference is caused by Coulomb energy difference and mass difference between neutron and proton. The Coulomb energy of both mirror nuclei is:

$${}^{A}_{Z}X \rightarrow E_C = \frac{3}{5}\frac{ke^2}{r_0}\frac{Z^2}{A^{1/3}}$$

$${}_{Z+1}^{A}Y \rightarrow E_{C\prime} = \frac{3}{5}\frac{ke^2}{r_0}\frac{Z(Z-1)}{A^{1/3}}$$

For a pair of mirror nuclei ${}^{A}X_Z - {}^{A}X_{Z-1}$ of radius R, charges Z and (Z - 1), the Coulomb energy difference is

$$\Delta E_C = E_C - E_{C'} = \frac{3}{5}\frac{ke^2}{r_0 A^{1/3}}[Z^2 - (Z-1)^2]$$

$$= \frac{3}{5}\frac{ke^2}{r_0}A^{2/3} \qquad (4)$$

2.2 The Coulomb Energy Difference from Transformation Energy

As is well known, experimentally, there exists a difference in the mass of the constituents of an atom, Zm_p and Nm_n, and its atomic mass, $m_{nucleus}$. That difference is called the Nuclear Binding Energy (E_B), which is considered the energy necessary to keep the nucleons bound together, or the energy required to separate the nucleus into nucleons. For a nucleus with A nucleons, Z protons and N neutrons the Binding energy is given as

$$E_B = a_V A - a_S A^{\frac{2}{3}} - \frac{a_C Z(Z-1)}{A^{1/3}} - \frac{a_A(A-2Z)^2}{A} + \frac{\delta}{A^{1/2}} \qquad (5)$$

$$\delta = \begin{cases} +a_P, & \text{if } N \text{ is even and } Z \text{ is even} \\ 0, & \text{if } A \text{ is odd} \\ -a_P, & \text{if } N \text{ is odd and } Z \text{ is odd} \end{cases}$$

where a_V, a_S, a_C and a_P are fit parameters. Equation (5) includes both empirical and theoretical parts; the theoretical part of this formula is obtained from the “liquid drop” model as proposed by George Gamow [20] containing some terms which were later developed by Niels Bohr and John Archibald Wheeler [12]. The atomic mass of elements as a function of mass number and atomic number can be estimated in terms of the binding energy (E_B) written in Bethe–Weizsäcker formula [11,21], as below:

$$M({}^{A}_{Z}X) = Zm_H + Nm_n - \frac{1}{c^2}\, E_B \qquad (6)$$

where Zm_H and Nm_n are mass of hydrogen atom, and neutron mass respectively, c is the speed of light in a vacuum, c^2 is the mass-energy equivalence factor [18]. Equation (6) estimates appropriately the atomic masses, binding energy data of stable and near-stable nuclei and other properties of the nuclei [21]. Using the Bethe-Weizsäcker mass formula and whiting for odd A nuclei; δ = 0 and a_3 is semi-empirical parameters. Some values have been found by several authors. The Coulomb energy ΔE_C can be calculated using SEMF. Therefore the difference in binding energy of both mirror nuclei, $M({}^{A}_{Z}X)$ and $M({}_{Z-1}^{A}Y)$ is given by

$$\Delta E_B = M({}^{A}_{Z}X) - M({}_{Z-1}^{A}Y)$$

$$= [Z-(Z-1)]M_H + (N-Z)M_n + a_C[Z^2-(Z-1)^2]A^{-\frac{1}{3}}$$

$$= M_H - M_n + a_C A^{\frac{2}{3}} = M_H - M_n + \Delta E_C$$

$$= 1.007825 - 1.008665 + \Delta E_C$$

$$= \Delta E_C - 0.782455\ MeV \qquad (7)$$

2.3 The β^+ Disintegration Energy

The positron β-decay involves the transformation of one proton in the parent nucleus turns into a neutron in the product nucleus, via the weak interaction. Simultaneously a neutrino and a positron (the β-ray) are expelled from the nucleus the process can be represented as:

$$^1_1p \rightarrow {}^1_0n + {}^{\ 0}_{+1}e + v_e + E(\beta^+)$$

For the decay product, the nuclear charge Z of the parent positron β-decaying nucleus decreases to Z - 1 and the mass number A doesn't change [22]. The first member of the pair of the mirror nuclear is usually β^+ active and undergoes β^+ transformation into the second as

$$^A_ZX \rightarrow {}_{Z-1}^{\ \ A}Y + \beta^+ + v_e + E(\beta^+) \qquad (8)$$

The expression of the β^+ disintegration energy is therefore:

$$E(\beta^+) = [\Delta m(^A_ZX) - \Delta m(_{Z-1}^{\ \ A}Y) - 2m_e]c^2 - \Delta E_{enl}(e^-)$$

$$= [M(^A_ZX) - M(_{Z-1}^{\ \ A}Y) - 2m_e]c^2$$

$$= \Delta E_B - 0.10220\ MeV \qquad (9)$$

Where $\Delta E_B = M(^A_ZX) - M(_{Z-1}^{\ \ A}Y)$. The difference in binding energy can be defined in terms of transformation energy as

$$\Delta E_B = E(\beta^+) + 0.10220\ MeV \qquad (10)$$

Therefore the Coulomb energy difference in Equation (7) can be calculated by substituting the values of binding energy (10) as

$$\Delta E_C = \Delta E_B + 0.782455\ MeV$$

$$= E(\beta^+) + 0.884655\ MeV \qquad (11)$$

The numerical values of Equation (11) are computed in Table 1.

2.4 The Binding Energy Difference

Here, the binding energy difference, ΔE_B in (1) is calculated by substituting the readily available and highly accurate values of masses of the mirror nuclei.

For the mirror nuclei, $^{13}N_7 - {}^{13}C_6$, the disintegration scheme of $^{13}N_7$ is:

$$^{13}_{\ 7}N \rightarrow {}^{13}_{\ 6}C + \beta^+ + v$$

The atomic masses of the nuclei are: $^{13}N_7$ = 13.005739 u; $^{13}C_6$ = 13.003355 u.

$$\Delta m({}^{13}_{7}N) = Zm_p + (A - Z)m_n - m_{nucleus}$$

$$= 0.10103\, u = 94.10977\, MeV$$

$$\Delta m({}^{13}_{6}C) = Zm_p + (A - Z)m_n - m_{nucleus}$$

$$= 0.10426\, u = 97.11291\, MeV$$

$$\Delta E_B = \Delta m({}^{13}_{7}N) - \Delta m({}^{13}_{6}C) = 3.00314\, MeV \qquad (12a)$$

For the mirror nuclei, ${}^{15}O_8 - {}^{15}N_7$, the disintegration scheme of ${}^{15}O_8$ is:

$${}^{15}_{8}O \to {}^{15}_{7}N + \beta^+ + v$$

The expression of the β^+ disintegration energy is:

$$E(\beta^+) = [\Delta m({}^{15}_{8}O) - \Delta m({}^{15}_{7}N) - 2m_e]c^2$$

The atomic masses of the nuclei are: ${}^{15}O_8 = 15.003065\, u$; ${}^{15}N_7 = 15.000109\, u$.

$$\Delta m({}^{15}_{8}O) = Zm_p + (A - Z)m_n - m_{nucleus}$$

$$= 0.12020\, u = 111.96090\, MeV$$

$$\Delta m({}^{15}_{7}N) = Zm_p + (A - Z)m_n - m_{nucleus}$$

$$= 0.12399\, u = 115.49690\, MeV$$

$$\Delta E_B = \Delta m({}^{15}_{8}O) - \Delta m({}^{15}_{7}N) = 3.53600\, MeV \qquad (12b)$$

For the mirror nuclei, ${}^{23}Mg_{12} - {}^{23}Na_{11}$, the disintegration scheme of ${}^{23}Mg_{12}$ is:

$${}^{23}_{12}Mg \to {}^{23}_{11}Na + \beta^+ + v$$

The expression of the β^+ disintegration energy is:

$$E(\beta^+) = [\Delta m({}^{23}_{12}M) - \Delta m({}^{23}_{11}Na) - 2m_e]c^2$$

The atomic masses of the nuclei are: ${}^{23}Mg_{12} = 22.994125\, u$; ${}^{23}Na_{11} = 22.989770\, u$.

$$\Delta m({}^{23}_{12}Mg) = Zm_p + (A - Z)m_n - m_{nucleus}$$

$$= 0.19510\, u = 181.72980\, MeV$$

$$\Delta m({}^{23}_{11}Na) = Zm_p + (A - Z)m_n - m_{nucleus}$$

$$= 0.20029\, u = 186.56890\, MeV$$

$$\Delta E_B = \Delta m({}^{23}_{12}M\mathrm{g}) - \Delta m({}^{23}_{11}Na) = 4.83910\, MeV \qquad (12c)$$

For the mirror nuclei, ${}^{31}S_{16} - {}^{31}P_{15}$, the disintegration scheme of ${}^{31}S_{16}$ is:

$$^{31}_{16}S \rightarrow {}^{31}_{15}P + \beta^+ + v$$

The expression of the β^+ disintegration energy is:

$$E(\beta^+) = [\Delta m(^{31}_{16}S) - \Delta m(^{31}_{15}P) - 2m_e]c^2$$

The atomic masses of the nuclei are: $^{31}S_{16}$ = 30.972071 *u*; $^{31}P_{15}$ = 30.973726 *u*.

$$\Delta m(^{31}_{16}S) = Zm_p + (A - Z)m_n - m_{nucleus}$$

$$= 0.27563\, u = 186.56890\, MeV$$

$$\Delta m(^{31}_{15}P) = Zm_p + (A - Z)m_n - m_{nucleus}$$

$$= 0.28226\, u = 262.92350\, MeV$$

$$\Delta E_B = \Delta m(^{31}_{16}S) - \Delta m(^{31}_{15}P) = 6.17580\, MeV \qquad (12d)$$

For the mirror nuclei, $^{39}Ca_{20} - {}^{39}K_{19}$, the disintegration scheme of $^{39}Ca_{20}$ is:

$$^{39}_{20}Ca \rightarrow {}^{39}_{19}K + \beta^+ + v$$

The expression of the β^+ disintegration energy is:

$$E(\beta^+) = [\Delta m(^{39}_{20}Ca) - \Delta m(^{39}_{19}K) - 2m_e]c^2$$

The atomic masses of the nuclei are: $^{39}Ca_{20}$ = 38.970718 *u*; $^{39}K_{19}$ = 38.963707 *u*.

$$\Delta m(^{39}_{20}Ca) = Zm_p + (A - Z)m_n - m_{nucleus}$$

$$= 0.35042\, u = 326.41600\, MeV$$

$$\Delta m(^{39}_{19}K) = Zm_p + (A - Z)m_n - m_{nucleus}$$

$$= 0.35827\, u = 333.72910\, MeV$$

$$\Delta E_B = \Delta m(^{39}_{20}Ca) - \Delta m(^{39}_{19}K) = 7.31310\, MeV \qquad (12e)$$

where the use of M_H = 1.007825 *u*, M_n = 1.008655 *u* and 1*u* = 931.494 *MeV*, have been made.

The size of a nucleus is characterized by the root mean square R_{rms} or by the radius *R* of the uniform sphere [23]. It is well known that the mean squared radii of neutron, proton, charge and mass distribution can be defined as follows:

$$\langle r_c^2 \rangle = \frac{\int_0^\infty r^2 4\pi r^2 \rho(r) dr}{\int_0^\infty 4\pi r^2 \rho(r) dr} \qquad (13)$$

where $\rho(r)$ is the nuclear charge density [24]. For a uniformly charged sphere [$\rho(r)$ = constant] of radius *R*, (13) takes the form:

$$\langle r^2 \rangle = \frac{\int_0^R r^4 dr}{\int_0^R r^2 dr} = \frac{3}{5} R^2$$

Thus, the two quantities, R_{rms} and R are related through the following equation:

$$R_{rms} = \langle r_c^2 \rangle^{1/2} = \sqrt{\frac{3}{5}} R \qquad (14)$$

The Coulomb energy difference for mirror nuclei and β^+ transformation energy measured the root mean square radius R_{rms} of the electrical charge distribution [8;25-28].

3. Results and Discussion

The computed values of the binding energy difference (12*a* – 12*e*) are presented in Table 1. It can be seen from Table 1 that the calculated values of β^+ transformation energy (which is through weak interaction) of mirror nuclei is closed to the difference in the binding energy of the nuclei.

Table 1: The calculated values of the β^+ transition energy between the mirror nuclei

$^AX_Z - {}^AX_{Z-1}$	$A^{2/3}$	ΔE_B *MeV*	$E(\beta^+)$ *MeV*
$^{13}N_7 - {}^{13}C_6$	5.52878	3.00314	2.90094
$^{15}O_8 - {}^{15}N_7$	6.08220	3.53600	3.43380
$^{23}Mg_{12} - {}^{23}Na_{11}$	8.08758	4.83910	4.73690
$^{31}S_{16} - {}^{31}P_{15}$	9.86827	6.17580	6.07360
$^{39}Ca_{20} - {}^{39}K_{19}$	11.50032	7.31310	7.21090
$^{51}Fe_{26} - {}^{51}Mn_{25}$	13.75245	8.80634	8.70414

Table 2 shows the computed values of Coulomb energy difference from which nuclear radius parameter, r_0 is evaluated from the equation:

$$r_0 = \frac{3}{5}\frac{ke^2}{\Delta E_C} A^{\frac{2}{3}} = 0.8640 \times 10^{-11} \frac{A^{\frac{2}{3}}}{\Delta E_C}$$

$$= 0.8640\, MeV \frac{A^{2/3}}{\Delta E_C} \times 10^{-17}$$

The mean value of r_0 from Table 2 is 1.2368×10^{-15} *m*. this value is in good agreement with those calculated by electron scattering and μ-mesonic atoms, $r_0 = 1.2 \times 10^{-15}$ *m*.

Table 2: will be used to evaluate the r_0 parameter of the nuclear radii

$^AX_Z - {}^AX_{Z-1}$	$A^{2/3}$	ΔE_B *MeV*	ΔE_C (*MeV*)	r_0 (*fm*)
$^{13}N_7 - {}^{13}C_6$	5.52878	3.00314	3.78559	1.26185
$^{15}O_8 - {}^{15}N_7$	6.08220	3.53600	4.31845	1.21687
$^{23}Mg_{12} - {}^{23}Na_{11}$	8.08758	4.83910	5.62155	1.24301
$^{31}S_{16} - {}^{31}P_{15}$	9.86827	6.17580	6.95825	1.22533
$^{39}Ca_{20} - {}^{39}K_{19}$	11.50032	7.31310	8.09555	1.22737
$^{51}Fe_{26} - {}^{51}Mn_{25}$	13.75245	8.80634	9.48660	1.24672

The information represented in Table 1 is extended further by plotting a graph of β^+ disintegration energy against $A^{2/3}$ (Figure 2).

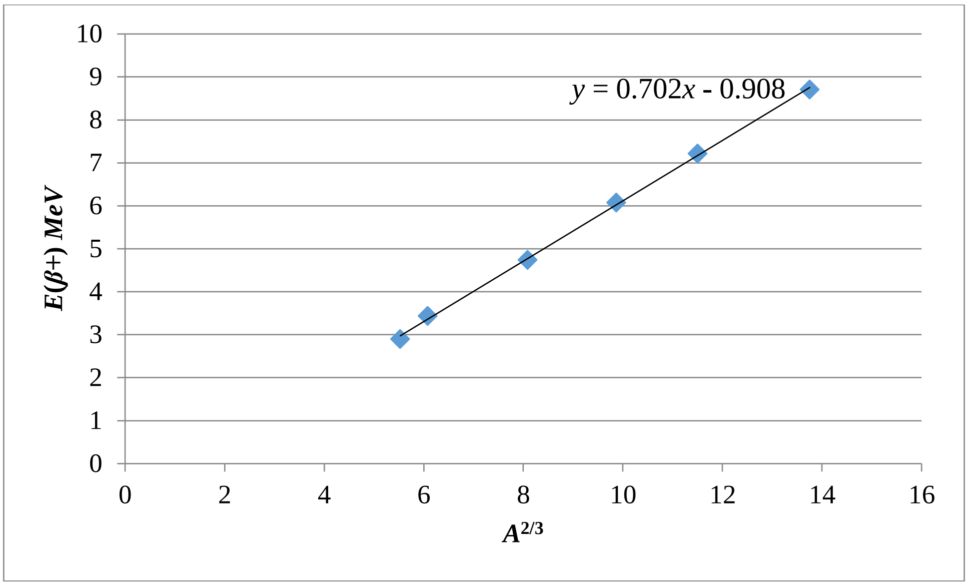

Figure 2: The plot of β^+ transition energy against $A^{2/3}$ with the intersect on 0.908 *MeV* on $E(\beta^+)$ – axis

Figure 1 shows the plot of the β^+ disintegration energy against $A^{2/3}$. This is a straight line graph with equation:

$$y = 0.702x - 0.908$$

where the slop of the graph is $a_C = 0.702\ MeV$.

$$a_C = \frac{3}{5}\frac{ke^2}{r_0} = 0.702\ MeV$$

With the value of a_C the nuclear radius parameter r_0 can be determined as:.

$$r_0 = \frac{3}{5}\frac{ke^2}{a_C} = \frac{0.8640}{a_C} \times 10^{-11} eV = \frac{0.8640}{0.7020} \times 10^{-17}\ m$$

$$= 1.23 \times 10^{-15}\ m$$

In Figure 1 the fact that the experimental values tend to lie on a straight line indicates that these nuclei have Coulomb-energy radii which correspond to a constant-density model $R = r_0 A^{1/3}$, with the slope of the data giving the particular value $r_0 = 1.23 \times 10^{-15}\ m$ for the nuclear unit radius. Thus, the mean value of r_0 for the remaining mirror nuclei will be about $1.23 \times 10^{-15}\ m$. this value is in good agreement with those calculated by electron scattering and μ-mesonic atoms, $r_0 = 1.2 \times 10^{-15}\ m$.

The root-mean-square nuclear matter radii (R_{rms}) contain an important insight on nuclear potentials and nuclear wave functions. Therefore, these nuclear radius parameters can be applied to determine R_{rms} for various atomic nuclei. The effective radii for various nuclei are calculated by substituting the nuclear radius parameter obtained from the Coulomb energy difference as:

$$R = \sqrt{\frac{5}{3}}\langle r_c^2\rangle^{1/2} = 1.23 \times 10^{-15} A^{1/5}\, m \tag{15a}$$

and from β^+ transformation energy as

$$R' = \sqrt{\frac{5}{3}}\langle r_c^2\rangle^{1/2} = 1.2368 \times 10^{-15} A^{1/5}\, m \tag{15b}$$

Now, the root mean square radii R_{rms} of nuclei which is related with R by Equation (14), is given by:

$$R_{rms} = \sqrt{\frac{3}{5}}R \tag{16a}$$

and for β^+ transformation energy it can take the form:

$$R'_{rms} = \sqrt{\frac{3}{5}}R' \tag{16b}$$

Table 3 showed the numerical values of root mean square radii R'_{rms} and R_{rms} for β^+ transformation energy and Coulomb energy difference respectively are calculated using Equations (15*a*), (15*b*), (16*a*) and (16*b*).

Table 3: The numerical values of effective radii and root mean square radii obtained from Equations (15*a*), (15*b*), (16*a*) and (16*b*) all values are in *fm* (1 *fm* = 10^{-15} *m*).

^{A}X	$R = 1.2368\ A^{1/3}$	$R' = 1.23\ A^{1/3}$	$R_{rms} = 1.2368\ A^{1/3}$	$R'_{rms} = 1.23\ A^{1/3}$
^{12}C	2.83157	2.81600	2.19333	2.18127
^{14}N	2.98086	2.96447	2.30897	2.29627
^{16}O	3.11654	3.09941	2.41406	2.40079
^{19}F	3.30028	3.28213	2.55639	2.54233
^{20}Ne	3.35719	3.33873	2.60047	2.58617
^{23}Na	3.51729	3.49796	2.72448	2.70951
^{24}Mg	3.56755	3.54793	2.76341	2.74822
^{31}P	3.88526	3.86390	3.00951	2.99297
^{32}S	3.92660	3.90501	3.04153	3.02481
^{35}Cl	4.04565	4.02341	3.13375	3.11652
^{39}K	4.19425	4.17119	3.24885	3.23099
^{40}Ar	4.22980	4.20654	3.27639	3.25837
^{55}Mn	4.70349	4.67763	3.64331	3.62328
^{56}Fe	4.73183	4.70581	3.66526	3.64511

These results (Table 3) are compared with the data obtained from three theoretical approaches: optical approximation; rigid target approximation and the exact Glauber Theory, performed using a Monte Carlo simulation technique (Table 4). It can be seen from Table 4 that the values obtained from optical approximation and the rigid target approximation result in smaller values of root mean square radius when compared with the values extracted from the electron scattering data, ref. [29]. The values of root mean square radius obtained in the framework of the Glauber Theory and those from Table 3, are in better agreement with the electron scattering data.

Table 4: The values of nuclear charge radius extracted from the three theoretical approaches: optical approximation [30]; rigid target approximation [31] and Glauber Theory [32].

Nuclide: ^{A}X	**Without *NN* range**		**With *NN* range**		**Glauber Theory**
	HO, optical	**WS, optical**	**WS, optical**	**WS, rigid target**	
^{12}C	2.31 ± 0.02	2.25 ± 0.01	2.09 ± 0.01	2.18 ± 0.01	2.49 ± 0.01
^{14}N	2.47 ± 0.03	2.42 ± 0.03	2.23 ± 0.03	2.35 ± 0.04	2.64 ± 0.03
^{16}O	2.54 ± 0.02	2.48 ± 0.02	2.29 ± 0.02	2.41 ± 0.03	2.69 ± 0.02
^{19}F	2.61 ± 0.07	2.55 ± 0.08	2.34 ± 0.08	2.44 ± 0.09	2.75 ± 0.07
^{20}Ne	2.87 ± 0.03	2.84 ± 0.04	2.63 ± 0.03	2.75 ± 0.04	2.99 ± 0.03
^{23}Na	2.83 ± 0.03	2.73 ± 0.04	2.52 ± 0.04	2.62 ± 0.04	2.91 ± 0.03
^{24}Mg	2.79 ± 0.15	2.65 ± 0.23	2.44 ± 0.22	2.53 ± 0.24	2.85 ± 0.20
^{35}Cl	3.045 ± 0.037	2.92 ± 0.04	2.68 ± 0.04	2.76 ± 0.04	3.08 ± 0.04
^{40}Ar	3.282 ± 0.036	3.16 ± 0.04	2.90 ± 0.03	2.98 ± 0.04	3.30 ± 0.03

Thus, the values of root mean square radius obtained in the framework of the Glauber Theory and from the Coulomb energy difference and from β^+ transformation energy, are in better agreement with the electron scattering data than the data obtained from optical approximation and rigid target approximation.

4. Conclusion

As a conclusion, we may say that the apparent discrepancy between the values, $r_0 = 1.2 \times 10^{-15}$ $A^{1/3}$ *m* for the nuclear charge parameter derived from electron scattering and μ-mesonic atoms and those $r_0 = 1.45 \times 10^{-15}$ $A^{1/3}$ *m* derived from mirror nuclei experiments might not be attributed to the use of classical principles instead of quantum mechanical principles in calculating the Coulomb energy, as our calculated values of nuclear radius parameter using classical principle are in good agreement with those measured from the experimental data by electron scattering and μ-mesonic atoms, $r_0 = 1.2 \times 10^{-15}$ *m*.

The values of root mean square radius obtained in the framework of the Glauber Theory and from the Coulomb energy difference and from β^+ transformation energy, are in better agreement with the electron scattering data than the data obtained from optical approximation and rigid target approximation. These new developments in the theoretical measurement for nuclear radius parameter from Coulomb energy difference and β^+ disintegration energy provide more accurate results which can be used to improve model parameters.

5. Conflict of Interest

There is no conflict of interest associated with this work.

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