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Ratio Estimators for Estimating Population Mean Using Tri-mean, Median and Quartile Deviation of Auxiliary Variable

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ABSTRACT

In this paper, we proposed improved ratio estimators of Nasir's finite population mean using information of the linear combination of the values of tri-mean (TM), quartile deviation (QD) and median (M_d) of auxiliary variable. The bias and mean square errors (MSEs) of the proposed estimators have been obtained up to first order of approximation using Taylor's Series Expansion and the conditions for their efficiencies over some existing estimators have been established. The numerical illustration was also conducted to corroborate the theoretical results. The results of the empirical study show that the proposed estimators are more efficient than existing estimators.

1. Introduction

In a situation where auxiliary information is available, it is possible to devise suitable ways of using it in obtaining the sample strategies which are better than those in which no such information is used. When the information on an auxiliary variable X is known, a number of estimators such as ratio, product and linear regression estimators are available in the literature. When the correlation between the study variable and the auxiliary variable is positive, ratio method of estimation is quite effective. On the other hand, when the correlation is negative, product method of estimation can be employed effectively. [1] made an important contribution to the modern sampling theory by suggesting methods of using the auxiliary information for the purpose of estimation in order to increase the precision of the estimates. [1] developed the ratio estimator to estimate population mean or the total of the study variable. In a class of estimators, the estimator with minimum variance or mean square error is regarded as the most efficient estimator.

In recent past, this concept has been utilized by several authors to improve the efficiency of ratio and product type estimators for estimating population mean of study variable. For instance, [2] defined ratio, product and regression estimators of population mean when the prior information of population proportion of units, possessing the same attribute is available. Also, [3, 4, 5] etc.

Let U be a set of real numbers and $U=\{U_1,U_2,U_3,...,U_N\}$ be a finite population having N units and each $U_i=(X_i,Y_i)$, $i=1,2,3,...,N$ has a pair of values. Y is the study variable and X is the auxiliary variable which is correlated with Y. Let $y=\{y_1,y_2,...,y_n\}$ and $x=\{x_1,x_2,...,x_n\}$ be n sample values. $\bar{y}$ and $\bar{x}$ are the sample means of the study and auxiliary variables respectively. Let S_y^2 and S_x^2 be the finite population variance of Y and X respectively and s_y^2 and s_x^2 be respective sample variances based on the random sample of size n drawn without replacement.

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} X_i, \quad \mu_y = \frac{1}{N}\sum_{i=1}^{N} Y_i, \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \quad \gamma = \frac{1-f}{n},$$

$$TM = \frac{(Q_1 + 2Q_2 + Q_3)}{4}, s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2, \quad s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2, \; S_y^2 = \frac{1}{N-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2,$$

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{n}(X_i - \bar{X})^2, \quad QD = \frac{(Q_3 - Q_1)}{2}$$

The ratio estimator for estimating population mean $(\bar{Y})$ of the study variable (Y) is given as:

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\mu_x = \hat{R}\mu_x \tag{1}$$

Where $\hat{R} = \frac{\bar{y}}{\bar{x}}$ is the estimate of $R = \frac{\mu_y}{\mu_x}$

$$Bias\left(\hat{\bar{Y}}_R\right) = \gamma \frac{1}{\bar{X}}\left(RS_x^2 - \rho S_x S_y\right) \tag{2}$$

$$MSE\left(\hat{\bar{Y}}_R\right) = \gamma\left(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y\right) \tag{3}$$

Where $R = \frac{\bar{Y}}{\bar{X}}$

[6] proposed a class of ratio type estimators for finite population mean by imposing Coefficient of variation (C_x) and Coefficient of kurtosis (β_{2x}) of auxiliary variable as:

$$\hat{\bar{Y}}_1 = \frac{\bar{y} + b(\mu_x - \bar{x})}{\bar{x}}\mu_x \tag{4}$$

$$\hat{\bar{Y}}_2 = \frac{\bar{y} + b(\mu_x - \bar{x})}{(\bar{x} + C_x)}(\mu_x + C_x) \tag{5}$$

$$\hat{\bar{Y}}_3 = \frac{\bar{y} + b(\mu_x - \bar{x})}{(\bar{x} + \beta_{2x})}(\mu_x + \beta_{2x}) \tag{6}$$

$$\hat{\bar{Y}}_4 = \frac{\bar{y} + b(\mu_x - \bar{x})}{(\bar{x}\beta_{2x} + C_x)}(\mu_x\beta_{2x} + C_x) \tag{7}$$

$$\hat{\bar{Y}}_5 = \frac{\bar{y} + b(\mu_x - \bar{x})}{(\bar{x}C_x + \beta_{2x})}(\mu_x C_x + \beta_{2x}) \tag{8}$$

$$Bias\left(\hat{\bar{Y}}_i\right) = \gamma \frac{S_x^2}{\mu_y} R_i^2, \qquad where\ i = 1,2,3,4,5 \tag{9}$$

$$MSE\left(\hat{\bar{Y}}_i\right)=\gamma\left(R_i^2S_x^2+S_y^2\left(1-\rho^2\right)\right) \qquad where\ i=1,2,3,4,5 \tag{10}$$

$$R_1=\frac{\mu_y}{\mu_x},\ R_2=\frac{\mu_y}{\mu_x+C_x},\ R_3=\frac{\mu_y}{\mu_x+\beta_{2x}},\ R_4=\frac{\mu_y\beta_{2x}}{\mu_x\beta_{2x}+C_x},\ R_5=\frac{\mu_yC_x}{\mu_xC_x+\beta_{2x}}$$

[7] proposed a class of ratio type estimators for finite population mean by using Correlation Coefficient of variation (ρ) of auxiliary variable as:

$$\hat{\bar{Y}}_6=\frac{\bar{y}+b\left(\mu_x-\bar{x}\right)}{\left(\bar{x}+\rho\right)}\left(\mu_x+\rho\right) \tag{11}$$

$$\hat{\bar{Y}}_7=\frac{\bar{y}+b\left(\mu_x-\bar{x}\right)}{\left(\bar{x}C_x+\rho\right)}\left(\mu_xC_x+\rho\right) \tag{12}$$

$$\hat{\bar{Y}}_8=\frac{\bar{y}+b\left(\mu_x-\bar{x}\right)}{\left(\bar{x}\rho+C_x\right)}\left(\mu_x\rho+C_x\right) \tag{13}$$

$$\hat{\bar{Y}}_9=\frac{\bar{y}+b\left(\mu_x-\bar{x}\right)}{\left(\bar{x}\beta_{2x}+\rho\right)}\left(\mu_x\beta_{2x}+\rho\right) \tag{14}$$

$$\hat{\bar{Y}}_{10}=\frac{\bar{y}+b\left(\mu_x-\bar{x}\right)}{\left(\bar{x}\rho+\beta_{2x}\right)}\left(\mu_x\rho+\beta_{2x}\right) \tag{15}$$

$$Bias\left(\hat{\bar{Y}}_i\right)=\gamma\frac{S_x^2}{\mu_y}R_i^2, \qquad where\ i=6,7,8,9,10 \tag{16}$$

$$MSE\left(\hat{\bar{Y}}_i\right)=\gamma\left(R_i^2S_x^2+S_y^2\left(1-\rho^2\right)\right) \qquad where\ i=6,7,8,9,10 \tag{17}$$

$$R_6=\frac{\mu_y}{\mu_x+\rho},\ R_7=\frac{\mu_yC_x}{\mu_xC_x+\rho},\ R_8=\frac{\mu_y\rho}{\mu_x\rho+C_x},\ R_9=\frac{\mu_y\beta_{2x}}{\mu_x\beta_{2x}+\rho},\ R_{10}=\frac{\mu_y\rho}{\mu_x\rho+\beta_{2x}}$$

[8] proposed ratio type estimators based on the values of coefficient of skewness $\left(\beta_{1x}\right)$ and coefficient of Kurtosis $\left(\beta_{2x}\right)$ of variable as:

$$\hat{\bar{Y}}_{11}=\frac{\bar{y}+b\left(\mu_x-\bar{x}\right)}{\left(\bar{x}+\beta_{1x}\right)}\left(\mu_x+\beta_{1x}\right) \tag{18}$$

$$\hat{\bar{Y}}_{12}=\frac{\bar{y}+b\left(\mu_x-\bar{x}\right)}{\left(\bar{x}\beta_{1x}+\beta_{2x}\right)}\left(\mu_x\beta_{1x}+\beta_{2x}\right) \tag{19}$$

$$Bias\left(\hat{\bar{Y}}_i\right)=\gamma\frac{S_x^2}{\mu_y}R_i^2, \qquad where\ i=11,12 \tag{20}$$

$$MSE\left(\hat{\bar{Y}}_i\right)=\gamma\left(R_i^2S_x^2+S_y^2\left(1-\rho^2\right)\right) \qquad where\ i=11,12 \tag{21}$$

[9, 10,11 and12] proposed ratio type estimators for finite population mean using quartile and functions of quartiles of auxiliary variable as:

$$\hat{\bar{Y}}_{13} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x} + M_d\right)}\left(\mu_x + M_d\right) \tag{22}$$

$$\hat{\bar{Y}}_{14} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(C_x\bar{x} + M_d\right)}\left(C_x\mu_x + M_d\right) \tag{23}$$

$$\hat{\bar{Y}}_{15} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\beta_{1x}\bar{x} + M_d\right)}\left(\beta_{1x}\mu_x + M_d\right) \tag{24}$$

$$\hat{\bar{Y}}_{16} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\beta_{2x}\bar{x} + M_d\right)}\left(\beta_{2x}\mu_x + M_d\right) \tag{25}$$

$$\hat{\bar{Y}}_{17} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x} + D_1\right)}\left(\mu_x + D_1\right) \tag{26}$$

$$\hat{\bar{Y}}_{18} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x} + D_2\right)}\left(\mu_x + D_2\right) \tag{27}$$

$$\hat{\bar{Y}}_{19} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x} + D_3\right)}\left(\mu_x + D_3\right) \tag{28}$$

$$\hat{\bar{Y}}_{20} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x} + D_4\right)}\left(\mu_x + D_4\right) \tag{29}$$

$$\hat{\bar{Y}}_{21} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x} + D_5\right)}\left(\mu_x + D_5\right) \tag{30}$$

$$\hat{\bar{Y}}_{22} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x} + D_6\right)}\left(\mu_x + D_6\right) \tag{31}$$

$$\hat{\bar{Y}}_{23} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x} + D_7\right)}\left(\mu_x + D_7\right) \tag{32}$$

$$\hat{\bar{Y}}_{24} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x} + D_9\right)}\left(\mu_x + D_9\right) \tag{33}$$

$$\hat{\bar{Y}}_{25} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x} + D_9\right)}\left(\mu_x + D_9\right) \tag{34}$$

$$\hat{\bar{Y}}_{26} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x} + D_{10}\right)}\left(\mu_x + D_{10}\right) \tag{35}$$

$$Bias\left(\hat{\bar{Y}}_i\right) = \gamma \frac{S_x^2}{\mu_y} R_i^2, \qquad \left(i = 13, 14, \ldots, 26\right) \tag{36}$$

$$MSE\left(\hat{\bar{Y}}_i\right)=\gamma\left(R_i^2S_x^2+S_y^2\left(1-\rho^2\right)\right) \qquad \left(i=13,14,...,26\right) \tag{37}$$

$$R_{13}=\frac{\mu_y}{\mu_x+M_d},\ R_{14}=\frac{\mu_yC_x}{\mu_xC_x+M_d},\ R_{15}=\frac{\mu_y\beta_{1x}}{\mu_x\beta_{1x}+M_d},\ R_{16}=\frac{\mu_y\beta_{2x}}{\mu_x\beta_{2x}+M_d},\ R_{17}=\frac{\mu_y}{\mu_x+D_1},$$

$$R_{18}=\frac{\mu_y}{\mu_x+D_2},\ R_{19}=\frac{\mu_y}{\mu_x+D_3},R_{20}=\frac{\mu_y}{\mu_x+D_4},R_{21}=\frac{\mu_y}{\mu_x+D_5},R_{22}=\frac{\mu_y}{\mu_x+D_6},R_{23}=\frac{\mu_y}{\mu_x+D_7},$$

$$R_{24}=\frac{\mu_y}{\mu_x+D_8},R_{25}=\frac{\mu_y}{\mu_x+D_9},R_{26}=\frac{\mu_y}{\mu_x+D_{10}}$$

[5] proposed a ratio-type estimator using quartile deviation (QD) and coefficient of skewness $\left(\beta_{1x}\right)$ of auxiliary variable as:

$$\hat{\bar{Y}}_{27}=\frac{\bar{y}+b\left(\mu_x-\bar{x}\right)}{\left(\bar{x}\beta_{1x}+QD\right)}\left(\mu_x\beta_{1x}+QD\right) \tag{38}$$

$$Bias\left(\hat{\bar{Y}}_{27}\right)=\gamma\frac{S_x^2}{\mu_y}R_{27}^2 \tag{39}$$

$$MSE\left(\hat{\bar{Y}}_{27}\right)=\gamma\left(R_{27}^2S_x^2+S_y^2\left(1-\rho^2\right)\right) \tag{40}$$

Where $R_{27}=\dfrac{\mu_y\beta_{1x}}{\left(\mu_x\beta_{1x}+QD\right)}$

[13] proposed ratio estimators for estimating population mean using information of the maximum value $\left(M_{(x)}\right)$ with the linear combination of the coefficient of variation $\left(C_x\right)$ and coefficient of correlation $\left(\rho\right)$ of auxiliary variable as:

$$\hat{\bar{Y}}_{28}=\frac{\bar{y}+b\left(\mu_x-\bar{x}\right)}{\left(\bar{x}+M_{(x)}\right)}\left(\mu_x+M_{(x)}\right) \tag{41}$$

$$\hat{\bar{Y}}_{29}=\frac{\bar{y}+b\left(\mu_x-\bar{x}\right)}{\left(\bar{x}C_x+M_{(x)}\right)}\left(\mu_xC_x+M_{(x)}\right) \tag{42}$$

$$\hat{\bar{Y}}_{30}=\frac{\bar{y}+b\left(\mu_x-\bar{x}\right)}{\left(\bar{x}\rho+M_{(x)}\right)}\left(\mu_x\rho+M_{(x)}\right) \tag{43}$$

$$Bias\left(\hat{\bar{Y}}_i\right)=\gamma\frac{S_x^2}{\mu_y}R_i^2, \qquad \left(i=28,29,30\right) \tag{44}$$

$$MSE\left(\hat{\bar{Y}}_i\right)=\gamma\left(R_i^2S_x^2+S_y^2\left(1-\rho^2\right)\right), \qquad \left(i=28,29,30\right) \tag{45}$$

Where $R_{28}=\dfrac{\mu_y}{\left(\mu_x+M_{(x)}\right)},\quad R_{29}=\dfrac{\mu_yC_x}{\left(\mu_xC_x+M_{(x)}\right)},\quad R_{30}=\dfrac{\mu_y\rho}{\left(\mu_x\rho+M_{(x)}\right)}$

In this paper, an improved linear combination of ratio estimators for estimating finite population mean has been proposed with objective to produce efficient estimators and their properties have been established.

2. Materials and Method

2.1 Proposed Estimators

Motivated by the work of [13], we proposed ratio estimators for estimating population mean using information of the linear combination of the values of tri-mean (TM), quartile deviation (QD) and median (M_d) of auxiliary variable under the following assumptions:

i. Tri-mean (TM), quartile deviation (QD) and median (M_d) of auxiliary variable are known.

ii. Population of study is finite.

iii. Information about the study variable (y) and auxiliary variable (x) in sample are completely available.

$$\hat{\bar{Y}}_{M1} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x} + \alpha_1\right)}\left(\mu_x + \alpha_1\right) \quad (46)$$

$$\hat{\bar{Y}}_{M2} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x}C_x + \alpha_2\right)}\left(\mu_x C_x + \alpha_2\right) \quad (47)$$

$$\hat{\bar{Y}}_{M3} = \frac{\bar{y} + b\left(\mu_x - \bar{x}\right)}{\left(\bar{x}\rho + \alpha_3\right)}\left(\mu_x \rho + \alpha_3\right) \quad (48)$$

Where $\alpha_1 = M_{(x)} \times TM,\ \alpha_2 = M_{(x)} \times QD,\ \alpha_3 = M_{(x)} \times M_d$

2.3Properties of the Proposed Estimators

In order to obtain the bias and MSE, we define $e_0 = \frac{\bar{y} - \mu_y}{\mu_y}$ and $e_1 = \frac{\bar{x} - \mu_x}{\mu_x}$ such that

$\bar{y} = \mu_y\left(1 + e_0\right)$ *and* $\bar{x} = \mu_x\left(1 + e_1\right)$, from the definition of e_0 and e_1, we obtain

$$\left.\begin{aligned} &E\left(e_0\right) = E\left(e_1\right) = 0,\ E\left(e_0^2\right) = \gamma C_y^2 \\ &E\left(e_1^2\right) = \gamma C_x^2,\ E\left(e_0 e_1\right) = \gamma C_{yx} = \gamma\rho C_y C_x \end{aligned}\right\} \quad (49)$$

$$Bias\left(\hat{\bar{Y}}_{Mi}\right) = \gamma \frac{S_x^2}{\mu_y} R_{Mi}^2, \qquad \left(i = 1,2,3\right) \quad (50)$$

$$MSE\left(\hat{\bar{Y}}_{Mi}\right) = \gamma\left(R_{Mi}^2 S_x^2 + S_y^2\left(1 - \rho^2\right)\right), \qquad \left(i = 1,2,3\right) \quad (51)$$

Where $R_{M1} = \frac{\mu_y}{\left(\mu_x + \alpha_1\right)},\quad R_{M2} = \frac{\mu_y C_x}{\left(\mu_x C_x + \alpha_2\right)},\quad R_{M3} = \frac{\mu_y \rho}{\left(\mu_x \rho + \alpha_3\right)}$

2.2 Efficiency and Comparison

In this section efficiencies of the proposed estimators are compared with efficiencies of some estimators in the literature

The $\hat{\bar{Y}}_{Mi} - family$ of estimators of the finite population mean is more efficient than $\hat{\bar{Y}}_R$ if,

$$MSE\left(\hat{\bar{Y}}_{Mi}\right) < MSE\left(\hat{\bar{Y}}_{R}\right) \qquad i = 1,2,3$$

$$\left(R_i^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right) < \left(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y\right) \tag{52}$$

The $\hat{S}^2_{MJi} - family$ of proposed estimators of the population mean is more efficient than $\hat{\bar{Y}}_j$ if,

$$MSE\left(\hat{\bar{Y}}_{Mi}\right) < MSE\left(\hat{\bar{Y}}_{j}\right) \qquad i = 1,2,3, \;\; j = 1,2,3,\ldots 30$$

$$\left(R_i^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right) < \left(R_j^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right) \qquad i = 1,2,3 \quad j = 1,2,3,\ldots 30 \tag{53}$$

When conditions (52), and (53) are satisfied, we can conclude that the proposed estimators are more efficient than some selected existing estimators.

3. Results and Discussion

3.1 Numerical Illustration

In order to investigate the merits of the proposed estimators, we have considered the real populations as:

Data: Populations I and II: [14]. Populations III: [15]

Table 1: Values of Populations Characteristics

Parameter	Population I	Population II	Population III
N	34	34	80
n	20	20	20
μ_y	856.411	856.411	5182.637
μ_x	208.883	199.441	1126.463
ρ	0.449	0.44556	0.941
S_y	733.141	733.141	1835.659
C_y	0.857	0.856	0.354
S_x	150.506	150.215	845.610
C_x	0.721	0.753	0.751
β_{2x}	0.098	1.045	-0.063
β_{1x}	0.978	1.182	1.050
M_d	150	142.5	757.500
QD	80.25	89.375	588.125
TM	162.25	165.562	931.562

Table 1 shows the numerical values of the parameters used in computing Bias, Mean Square Error (MSE) and Percentage Relative Efficiency (PRE) of the proposed and other related estimators considered in the study and the results are presented in Tables 2 and 3.

Table 2: Constant and Bias of some selected existing and proposed estimators

Estimator	Constant			Bias		
	Pop-I	Pop-II	Pop-III	Pop-I	Pop-II	Pop-III
$\hat{Y}_R$	4.100	4.294	4.601	4.271	4.943	60.89
$\hat{Y}_1$	4.100	4.294	4.601	9.154	10.002	109.52
$\hat{Y}_2$	4.086	4.278	4.598	9.091	9.927	109.37
$\hat{Y}_3$	4.098	4.272	4.601	9.145	9.898	109.53
$\hat{Y}_4$	3.960	4.279	4.650	8.539	9.930	111.86
$\hat{Y}_5$	4.097	4.264	4.601	9.142	9.865	09.53
$\hat{Y}_6$	4.091	4.285	4.597	9.115	9.957	109.34
$\hat{Y}_7$	4.088	4.281	4.596	9.099	9.943	109.27
$\hat{Y}_8$	4.069	4.258	4.598	9.015	9.834	109.36
$\hat{Y}_9$	4.012	4.285	4.662	8.763	9.960	112.46
$\hat{Y}_{10}$	4.96	4.244	4.601	9.135	9.771	109.53
$\hat{Y}_{11}$	4.081	4.269	4.597	9.069	9.885	109.31
$\hat{Y}_{12}$	4.098	4.275	4.601	9.145	9.914	109.53
$\hat{Y}_{13}$	2.386	2.505	2.751	3.101	3.403	39.15
$\hat{Y}_{14}$	2.053	2.204	2.427	2.296	2.634	30.47
$\hat{Y}_{15}$	2.364	2.676	2.804	3.043	3.886	40.69
$\hat{Y}_{16}$	0.489	2.550	-0.478	0.130	3.526	1.186
$\hat{Y}_{17}$	3.068	3.293	3.464	5.124	5.886	62.082
$\hat{Y}_{18}$	2.998	3.032	3.266	4.893	4.987	55.188
$\hat{Y}_{19}$	2.701	2.834	3.007	3.973	4.358	46.786
$\hat{Y}_{20}$	2.532	2.757	2.874	3.490	4.122	42.737
$\hat{Y}_{21}$	2.386	2.505	2.751	3.101	3.402	39.154
$\hat{Y}_{22}$	1.964	2.091	2.622	2.100	2.370	35.567
$\hat{Y}_{23}$	1.865	1.846	1.985	1.893	1.848	20.385
$\hat{Y}_{24}$	1.573	1.700	1.765	1.347	1.567	16.117

$\hat{\bar{Y}}_{25}$	1.328	1.496	1.429	0.960	1.213	10.567
$\hat{\bar{Y}}_{26}$	1.108	1.028	1.124	0.669	0.573	6.535
$\hat{\bar{Y}}_{27}$	2.944	3.114	3.073	4.719	5.260	48.850
$\hat{\bar{Y}}_{28}$	1.108	1.028	1.124	0.669	0.573	6.535
$\hat{\bar{Y}}_{29}$	0.864	0.822	0.898	0.406	0.367	4.176
$\hat{\bar{Y}}_{30}$	0.585	0.528	1.073	0.186	0.151	5.960
$\hat{\bar{Y}}_{M1}$	**0.0093**	**0.0081**	**0.0016**	**0.000047**	**0.000036**	**0.000013**
$\hat{\bar{Y}}_{M2}$	**0.1772**	**0.1654**	**0.1814**	**0.017093**	**0.01484**	**0.1703357**
$\hat{\bar{Y}}_{M3}$	**0.0045**	**0.0042**	**0.0018**	**0.000011**	**0.0000096**	**0.000018**

Table 2 shows the constants and biases values of the existing and proposed estimators. The results revealed that the proposed estimators have minimum biases for all the three data sets considered in the study. This result implies that the proposed estimators have higher chances in giving estimates of population mean that are closer to actual value of the population mean than other estimators considered in the study. From the results of the constants in the table 2, it is observed that the smaller the values of the constants, the smaller the results of the biases.

Table 3: MSE and PRE of Some Selected Existing and Proposed Estimators

Estimator	**MSE**			**PRE**		
	Pop-I	**Pop-II**	**Pop-III**	**Pop-I**	**Pop-II**	**Pop-III**
$\hat{\bar{Y}}_{R}$	16865.50	17614.20	584407.34	100	100	100
$\hat{\bar{Y}}_{1}$	16673.45	17437.65	581994.20	101.1518	101.0125	100.414633
$\hat{\bar{Y}}_{2}$	16619.64	17373.31	581238.50	101.4793	101.3866	100.545188
$\hat{\bar{Y}}_{3}$	16666.14	17348.62	582058.10	101.1962	101.5308	100.403609
$\hat{\bar{Y}}_{4}$	16146.61	17376.04	594119.80	104.4523	101.3706	98.3652354
$\hat{\bar{Y}}_{5}$	16663.31	17319.75	582079.30	101.2134	101.7001	100.399952
$\hat{\bar{Y}}_{6}$	16639.85	17399.52	581046.80	101.3561	101.2338	100.57836
$\hat{\bar{Y}}_{7}$	16626.87	17387.08	580732.70	101.4352	101.3063	100.632759
$\hat{\bar{Y}}_{8}$	16554.40	17294.19	581191.40	101.8793	101.8504	100.553336
$\hat{\bar{Y}}_{9}$	16338.65	17401.14	597260.90	103.2246	101.2244	97.8479154
$\hat{\bar{Y}}_{10}$	16657.19	17239.66	582062.10	101.2506	102.1725	100.402919

$\hat{\bar{Y}}_{11}$	16600.54	17336.98	580937.6	101.5961	101.599	100.597266
$\hat{\bar{Y}}_{12}$	16665.98	17362.26	582055.1	101.1972	101.4511	100.404127
$\hat{\bar{Y}}_{13}$	11489.70	11785.7	217319.8	146.788	149.454	268.915828
$\hat{\bar{Y}}_{14}$	10800.40	11127.47	172323.8	156.1563	158.2947	339.133271
$\hat{\bar{Y}}_{15}$	11440.80	12199.76	225319.5	147.4154	144.3815	259.368293
$\hat{\bar{Y}}_{16}$	8945.90	11892.07	20545.47	188.5277	148.1172	2844.45836
$\hat{\bar{Y}}_{17}$	13222.50	13910.4	336145.8	127.5515	126.6261	173.855315
$\hat{\bar{Y}}_{18}$	13025.00	13142.8	300416.7	129.4856	134.0217	194.532241
$\hat{\bar{Y}}_{19}$	12236.10	12604.0	256874.4	137.834	139.7509	227.507038
$\hat{\bar{Y}}_{20}$	11823.00	12402.5	235888.6	142.6499	142.0214	247.747174
$\hat{\bar{Y}}_{21}$	11489.70	11785.7	217319.8	146.788	149.454	268.915828
$\hat{\bar{Y}}_{22}$	10632.60	10902.1	198733.1	158.6207	161.567	294.066434
$\hat{\bar{Y}}_{23}$	10455.5	10454.6	120048.6	161.3074	168.4828	486.808959
$\hat{\bar{Y}}_{24}$	9987.70	10213.8	97924.6	168.8627	172.4549	596.793186
$\hat{\bar{Y}}_{25}$	9656.20	9910.7	69163.6	174.6598	177.7291	844.963738
$\hat{\bar{Y}}_{26}$	9406.60	9362.1	48266.6	179.2943	188.1437	1210.79036
$\hat{\bar{Y}}_{27}$	12875.40	13376.0	267595.2	130.9901	131.6851	218.39231
$\hat{\bar{Y}}_{28}$	9406.6	9362.1	48266.6	179.2943	188.1437	1210.79036
$\hat{\bar{Y}}_{29}$	9181.8	9185.9	36039.3	183.684	191.7526	1621.58349
$\hat{\bar{Y}}_{30}$	8993.4	9000.9	45286.9	187.532	195.6938	1290.45561
$\hat{\bar{Y}}_{M1}$	**8835.20**	**8874.76**	**14470.88**	**190.8898**	**198.4752**	**4038.50588**
$\hat{\bar{Y}}_{M2}$	**8849.79**	**8887.44**	**15353.60**	**190.5751**	**198.1921**	**3806.32125**
$\hat{\bar{Y}}_{M3}$	**8835.16**	**8874.74**	**14470.90**	**190.8907**	**198.4757**	**4038.5003**

Table 3 shows the results of Mean Square Errors (MSEs) and Percentage Relative Efficiency (PRE) of the proposed and some related estimators considered in the study for all the data sets I, II, and III. The results revealed that the proposed estimators has minimum MSEs and higher PRE than other estimators. This results implies that the average of dispersion of the proposed estimators from the population mean is smaller and this indicate that the proposed estimators give better estimates on the average than other estimators in the study.

Percentage Relative Efficiency (PRE) of the estimators were computed using the formula

$$PRE = \frac{MSE\left(\hat{\bar{Y}}_R\right)}{MSE\left(\hat{\bar{Y}}_i\right)} X\ 100$$

Where $\hat{\bar{Y}}_i$ are the existing and proposed estimators in this paper.

In this study, we have proposed a class of ratio type population mean estimators using known population parameters of the auxiliary variable. The performance of the proposed estimators over the usual ratio estimator and some selected existing estimators using three natural populations whereby their properties (Bias and Mean Square Errors (MSEs)) were established and comparing their PREs. Table 2 shows the constants and biases values of the existing and proposed estimators. The results revealed that the proposed estimators have minimum biases for all the three data sets considered in the study. This result implies that the proposed estimators have higher chances in giving estimates of population mean that are closer to actual value of the population mean than other estimators considered in the study. From the results of the constants in the table, it is observed that the smaller the values of the constants, the smaller the results of the biases. Table 3 shows the results of Mean Square Errors (MSEs) and Percentage Relative Efficiency (PRE) of the proposed and some related estimators considered in the study for all the data sets I, II, and III. The results revealed that the proposed estimators has minimum MSEs and higher PRE than other estimators. This results implies that the average of dispersion of the proposed estimators from the population mean is smaller and this indicate that the proposed estimators give better estimates on the average than other estimators in the study.

4. Conclusion

Based on the results in Table 3, the proposed estimator $\hat{\bar{Y}}_{M1}$ outperformed other existing estimators in the study by PRE ranging from (3.3578%, 98.732%), (2.7814%, 97.4627%) and (2748.05027%, 3938.09125%) for data sets I, II and III respectively. Also the proposed estimator $\hat{\bar{Y}}_{M2}$ outperformed other existing estimators in the study by PRE ranging from (3.0431%, 89.4233%), (2.4983%, 97.1796%) and (2515.86564%, 3705.90662%) for data sets I, II and III respectively. $\hat{\bar{Y}}_{M3}$ outperformed other existing estimators in the study by PRE ranging from (3.3587%, 97.3239%), (2.7919%, 97.4632%) and (2748.04469%, 3938.08567%) for data sets I, II and III respectively. Therefore, it is clear that the proposed estimators works better than the other existing estimators having the minimum Mean Square Error (MSE) and the highest Percentage Relative Error (PRE). We therefore recommend for use in practical application in estimating population mean.

Nomenclature

N: Population size
n: Sample size
Y: Study variable
X: Auxiliary variable
$\bar{y}, \bar{x}$: Sample means of study and auxiliary variables
μ_y, μ_x: Population means of study and auxiliary variables
ρ: Coefficient of correlation

C_y, C_x : Coefficient of variations of study and auxiliary variables

D_i : Deciles of auxiliary variable i=1,2,3,…,10

Q_3 : The upper quartile

Q_r : Inter-quartile range

β_{1x} : Coefficient of skewness of auxiliary variable

β_{2x} : Coefficient of kurtosis of auxiliary variable

TM : Tri-Mean

M_d : Median of the auxiliary

QD : Quartile deviation

5. Conflict of Interest

There is no conflict of interest associated with this work.

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