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Response Surface Modelling of an Automobile Brake Pad Made from a New-Fangled Composite Material

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Abstract

This study focused on the response surface modelling of an automobile brake pad made from a new-fangled composite material. The developed brake pad was made from the mixture of cow bone, binder (Epoxy and hardener) Abrasive (iron fillings) and filler (Calcium carbonate) which were mixed till a homogenous mixture was obtained. The properties of the brake evaluated in this study were, hardness, compressive strength, water absorption, tensile strength, coefficient of friction, wear rate, and density respectively. The mixture design and the response surface methodology (RSM) were used to optimize the effects of the interaction of cow bone, epoxy, hardener, iron fillings, and calcium carbonate on the developed brake pad. Models were formulated to predict hardness, compressive strength, coefficient of friction, wear rate, and density using response surface modelling. A desirability of 0.91 was obtained which shows the adequacy of the model terms. The models were then validated using coefficient of determination R^2. The coefficient of determination (R^2) obtained ranged from 0.9213, (92.13%) to 0.98.1, (98.1%) which indicates that a substantial good fit was achieved by the models developed. The values obtained from the validation of these models were therefore found to be satisfactory, and shows good predictability of the model.

1.0 Introduction

Inadequate models for predicting the physical and mechanical properties of automobile braking components have led to failure in automobile components especially brake pad. This have led to modelling and development of brake pad using cow bone as a base material. Ji-Hoon, Choi and Lee [1] presented a paper on finite element analysis of transient thermo elastic behaviors in disk brakes. In this paper a transient analysis for thermo elastic contact problem of disk brakes with frictional heat generation was performed using the finite element method. To analyze the thermo elastic phenomenon occurring in disk brakes, the coupled heat conduction and elastic equations (Cylindrical coordinates) are solved with contact problem. Material used was carbon, carbon composite and wear was assumed negligible. The numerical simulation for the thermo elastic behavior of disk brake is obtained in the repeated brake condition. The computational results are presented for the distributions of pressure and temperature on each friction surface between the contacting bodies. It is observed that the orthotropic disc brakes can provide better brake performance than the isotropic one because of uniform and mild pressure distribution. Sterle and Klob [2] focuses on surface changes induced by repeated brake applications and tries to provide explanations on how such material modifications might affect friction and wear properties of automotive disc brakes. Surface films were investigated locally by transmission electron microscopy (TEM) after having prepared thin cross-sections with a focused ion beam instrument (FIB). Since the observed friction layers revealed a nano-crystalline structure, modeling with the method of movable cellular automata (MCA) was performed by assuming an array of linked nanometer-sized particles. Chikalthankar and Nandedkar [3] investigated the frictional and wear characteristics of non-asbestos brake pad were studied using link chase machine. The chase machine is used to perform the test as per SAE J661.The coefficient of friction and wear is an important performance measure in this process. Since long, researchers have explored a number of ways to improve and stable the coefficient of friction and wear rate which is similar to the asbestos material. A large range of different non-asbestos materials are studied by different researchers; all the research work in this area shares the same objectives of achieving the same performance from non-asbestos material as that of asbestos material. Sowjanya et al [4**]** in their research on Structural analysis of disk brake rotor. The disc brake is usually made of cast iron was selected for investigation of the effect of strength variations on the predicted stress distributions. Aluminum metal matrix composite materials were selected and analyzed. The domain was considered as axis-symmetric; inertia and body force effects are negligible during the analysis. The model of disc brake is developed by using Solid modeling software Pro/E (Cero-Parametric 1.0). Bouchetara et al [5] researched on thermo elastic analysis of disk brakes rotor. The main purpose of this study was to analyze the thermo-mechanical behavior of the dry contact between the brake disk and pads during the braking phase. The simulation strategy is based on computer code ANSYS11. The modeling of transient temperature in the disk is actually used to identify the factor of geometric design of the disk to install the ventilation system in vehicles The thermal-structural analysis is then used with coupling to determine the deformation and the Von-Mises stress established in the disk, the contact pressure distribution in pads. The results are satisfactory when compared to those of the specialized literature. Oder [6] investigated the thermal and stress analysis of brake discs in railway vehicles. This paper present work on thermal and stress analysis of brake discs in railway vehicles. Performed analysis deals with two cases of braking; the first case considers braking to a standstill; the second case considers braking on a hill and maintaining a constant speed. In both cases the main boundary condition is the heat flux on the braking surfaces and the holding force of the brake calipers. In addition the centrifugal load is considered. Finite element method (FEM) approach is been used, 3D model has been modeled for analysis. The results need to be compared with experimental results. Zaid [7] presented a paper on an investigation of disc brake rotor by Finite element analysis. In his paper, the author has conducted a study on ventilated disc brake rotor of normal passenger vehicle with full load of capacity. The study is more likely concern of heat and temperature distribution on disc brake rotor. In this study, finite element analysis approached has been conducted in order to identify the temperature distributions and behaviors of disc brake rotor in transient response. Modeling is done in CATIA & ABAQUS/CAE has been used as finite elements software to perform the thermal analysis on transient response.

This research therefore focused on the response surface modelling of an automobile brake pad made from a new-fangled composite material.

2 Methodology

2.1 Experimental Procedures

2.1.1. Experimental Design

In this study, a four variable mixture design was used to plan and conduct the experiments for the production of the brake pads. The mixture design is the best experimental design used in optimising formulation processes such as the production of brake pads **[8]**. It was noted that due to the range of selected factors, a D-Optimal design was used rather than a simplex design. The D-Optimal design is very suitable for highly constrained designs. For the D-optimal design selected for the mixture design used in this work, the design points were selected to minimize the variance associated with the estimates of the coefficients in the model by maximizing the value of determinant of the information matrix **[9]**. Another benefit of the D-Optimal design is that it requires a smaller number of experimental runs compared with other types of design **[10].** The design space was characterised by the low-and high-level constraints for all four factors with their associated constraints. For the ranges of the input factors chosen, the upper bounded pseudo values (U_Pseudo) was chosen because it gives a larger design space compared to lower bounded pseudo values (L_Pseudo). The Design Expert® software version 7.0.0, (Stat-ease, Inc. Minneapolis, USA) was used to develop the mixture design as well as developing a statistical model to relate the input factors to the chosen responses. Table 1 shows the range of the factors considered in this study and they were selected after a thorough review of previous studies [11, 12]. For a mixture design of experiment, the input factors are the components or ingredients of a formulation or mixture, and as such, their levels are not independent [13]. Consequently, for a four-component mixture:

$$0 \leq X_i \leq 100 \tag{1}$$

Where i = 1, 2,3, 4

$$X_1 + X_2 + X_3 + X_4 = 100 \tag{2}$$

Four independent factors were studied: Cow bone, Binder (Epoxy and hardener), Abrasive (Iron fillings), and Filler (Calcium carbonate). In this study, the responses chosen for consideration were hardness, compressive strength, water absorption, tensile strength, coefficient of friction, wear rate and density. The four input factors generated 20 experimental runs as shown in Table 2. Of these 20 experimental runs, 10 represented actual model points while 5 points were used to estimate lack of fit and the remaining 5 were replicates.

Table 1: Coded and actual levels of the factors for brake pad formulation

Factors	Unit	Symbols	Variable levels	
			Low level	High level
Cow bone	%	X_1	30	40
Binder	%	X_2	40	50
Abrasive	%	X_3	5	10
Filler	%	X_4	10	15

Table 2: Experimental design matrix for the formulation of brake pads

Run	Actual values of factors			
	Cow bone (%)	Binder (%)	Abrasive (%)	Filler (%)
1	38	40	8	14
2	30	50	5	15
3	30	50	5	15
4	35	47	5	13
5	36	42	10	12
6	40	40	10	10
7	31	47	9	13
8	32	43	10	15
9	34	45	8	13

10	40	44	6	10
11	35	50	5	10
12	30	50	10	10
13	40	44	6	10
14	40	40	5	15
15	30	50	10	10
16	35	50	5	10
17	32	43	10	15
18	36	43	6	15
19	32	50	7	11
20	36	45	9	10

Different models were selected from the Design Expert software library and evaluated for their suitability in modelling the formulation of the brake pad. These models include linear, quadratic, special cubic and cubic. These models can be recognized and easily distinguished from response surface models by their lack of an intercept term. The linear model is shown in Equation 3 and it is usually the first model to be investigated in situations where the relationship between the factors and the responses is thought to be linear. For the linear model, the candidate points should include the vertices of the region, the edge centers, the overall centroid, and the axial points that are located halfway between the overall centroid and the vertices.

$$Y = \sum_{i=1}^{N} b_i X_i \tag{3}$$

Where Y_i is the dependent variable or predicted response, X_i is the independent variables, b_o is offset term, b_i is the regression coefficient and e_i is the error term.
For the quadratic model, the candidate points should include the vertices, the edge centers, the constraint plane centroids, the overall centroid, and the axial points as shown in Equation 4.

$$Y = \sum_{i=1}^{N} b_i X_i + \sum_{i,j=1}^{N} b_{ij} X_i X_j \tag{4}$$

X_j is the independent variables or factors while b_{ij} is the coefficient of the interaction terms.
For the special cubic and cubic model, the candidate points should include the vertices, the thirds of edges, the constraint plane centroids, the overall centroid, and the axial points as shown in Equations 5 and 6.

$$Y = \sum_{i=1}^{N} b_i X_i + \sum_{i,j=1}^{N} b_{ij} X_i X_j + \sum_{i=1}^{N} b_{ijk} X_i X_j X_k \tag{5}$$

$$Y = \sum_{i=1}^{N} b_i X_i + \sum_{i,j=1}^{N} b_{ij} X_i X_j + \sum_{i=1}^{N} b_{ijk} X_i X_j X_k + \sum_{i=1}^{N} b_{ij} X_i X_j (X_i - X_j) \tag{6}$$

By default, the built-in algorithm of the Design-Expert utilises each model to select the design points. For instance, higher order models will usually require more points. In any case, if a model with the highest degree is chosen for the experiment, the Design-Expert software will ensure that there are enough design points to evaluate that model. The optimisation of the responses and input factors was done numerically [14]**.** The steepest ascent optimisation method was used for maximisation of responses while second order models were optimised using the method of ridge analysis [14].

2.2. Statistical Analysis of Model Results

Statistical analysis of the results obtained from the experiments was carried out using the Design Expert software. Analysis of variance (ANOVA) was used to assess the significance of the statistical significance fit of the models representing the responses (hardness, compressive strength, water absorption, tensile strength, coefficient of friction, wear rate and density) to the experimental data. The ANOVA results was evaluated in terms of statistical parameters such as p value, F value, sum of squares, mean square, lack of fit, standard deviation, coefficient of variation, coefficient of determination (R^2), adjusted R^2, adequate precision, predicted residual sum of squares (PRESS).

2.3 F value

The value for a term is the test for comparing the variance associated with that term with the residual variance. It is the Mean Square for the term divided by the Mean Square for the Residual. The F-value is used to test the significance of adding new model terms to those terms already in the model. For instance, the significance of the linear terms is tested after removing the effect of the average and the blocks. Then, the significance of the quadratic terms is tested after removing the average, block and linear effects and so on.

2.4 P value (Prob>F)

This is the probability value that is associated with the F Value for a particular term model. It is the probability of getting an F Value of this size if the term did not have an effect on the response. In general, a term that has a probability value less than 0.05 would be considered a significant effect; otherwise, it is generally regarded as not significant. The lack of fit p value is the probability associated with the Lack of Fit calculation for a model. Generally, a good model should have an insignificant probability value, or $P>0.10$.

2.5 Coefficient of determination (R^2)

This is a measure of the amount of variation around the mean explained by the model. The Adjusted R-Squared is a measure of the amount of variation around the mean explained by the model, adjusted for the number of terms in the model. The adjusted R-squared decreases as the number of terms in the model increases if those additional terms don't add value to the model. Both the R-Squared and related Adjusted R-Squared values should be close to one. A value of 1.0 represents the ideal case at which 100 percent of the variation in the observed values can be explained by the chosen model. The Predicted R-Squared estimates the amount of variation in new data explained by the model. It can be negative, but this is very bad and suggests that the model consisting of only the intercept is a better predictor of the response than this model! The closer to 1.0, the better the predicted R-squared.

2.6 Standard deviation

This was used to express the deviation of the individual response values from the mean. A small value of standard deviation is generally desired.

2.7 Coefficient of variation

The coefficient of variation for this model. It is the error expressed as a percentage of the mean. It is computed by dividing the standard deviation by the mean and multiplying by 100.

2.8 Lack of fit

This is the variation of the data around the fitted model. If the model does not fit the data well, this will be significant.

2.9 Adequate Precision

This is a signal to noise ratio. It compares the range of the predicted values at the design points to the average prediction error. Ratios greater than 4 indicate adequate model discrimination.

$$\left[\frac{\max(\hat{Y})-\min(\hat{Y})}{\sqrt{\bar{V}(\hat{Y})}}\right]>4 \quad \bar{V}(\hat{Y})=\frac{1}{n}\sum_{i=1}^{n}V(\hat{Y})=\frac{p\sigma^2}{n} \tag{7}$$

p = number of model parameters (including intercept (b_0) and any block coefficients)

σ^2 = residual MS from ANOVA table
n = number of experiments

2.10 Predicted Residual Sum of Squares (PRESS)

The PRESS statistic indicates how well the model fits the data. The PRESS for the chosen model should be small relative to the other models under consideration.

2.11 Optimization of Responses

The optimum values of the responses were obtained by numerical optimisation based on the criterion of desirability. The optimization process searches for a combination of factor levels that simultaneously satisfy the criteria placed on each of the responses and factors. To include a response in the optimization criteria, it must have a model fit through analysis. For this work, the optimisation was done by choosing the desired goal for each factor and response. For this study, the goal was to maximize the chosen responses. The independent variables were kept at their natural levels while a minimum and a maximum level was set for the responses. A weight was assigned to each goal to adjust the shape of its particular desirability function. The default setting was used for the goal and this was that all goals be equally important at a setting of 3 pluses (+++). The goals were combined into an overall desirability function which was maximised by the software. Contour, 3D surface, and perturbation plots of the desirability function at each optimum were then used to explore the function in the factor space.

2.12. Model Validation

The capacity of RSM to predict the responses was evaluated by comparing the results predicted by the RSM models with those of the actual experiments. The level of fit between the two was assessed by using the coefficient of determination (R^2 value), adjusted coefficient of determination (adjusted R^2 value), predicted coefficient of determination (predicted R^2 value), standard deviation, and coefficient of variation **[14]**. It is desirable that the R^2 value be as close to unity as possible while the standard deviation should be as small as possible **[13].**

3. Results And Discussion

3.1. Response Surface Modelling

3.1.1. Determination of Most Suitable Model

The different statistical models selected were evaluated to determine their suitability for modeling the responses (hardness, compressive strength, water absorption, tensile strength, coefficient of friction, wear rate and density). The models evaluated include linear, quadratic, special cubic and cubic models and their suitability was assessed on the basis of their respective coefficient of determination (R^2 value), p value, F value etc. The results of the analysis are shown in Table 3 to 16. The results show the summary of model fit and lack of fit test for all seven responses. The statistical results obtained for (model summary and lack of fit test) for hardness, compressive strength, water absorption, tensile strength, coefficient of friction, wear rate and density are presented respectively in Tables 3 and 4, Tables 5 and 6, Tables 7 and 8, Tables 9 and 10, Tables 11 and 12, Tables 13 and 14, Tables 15 and 16. It was found that the quadratic model was suitable to represent all the responses apart from tensile strength, density and coefficient of friction which were represented by the special cubic model. The selection of the models was done on the basis of the highest R^2 value, lowest standard deviation, and lowest PRESS. The linear model was discarded because it exhibited poor statistical characteristics. Although the cubic model displayed some desirable statistical properties, it was however not chosen because it was flagged as “aliased”. This model was not chosen because it contains aliased model terms meaning that the experimental runs might be not enough to independently estimate all the terms for the model. The PRESS statistics for the special cubic and cubic models were not defined because they had a leverage value of one. These observations were also supported by the results obtained from the lack of fit test carried out as shown in Tables 4, 6, 8, 10, 12, 14 and 16 for all seven responses. The quadratic model was shown to have insignificant lack of fit, (a situation that is desirable) for all the responses apart from tensile strength, density and coefficient of friction while the same observation was recorded for the special cubic model for tensile strength, density and coefficient of friction. Thus, the quadratic model was adopted for predicting the hardness, compressive strength, water absorption, and wear rate while the special cubic model was used for predicting tensile strength, density and coefficient of friction.

Table 3: Summary of model fit results for hardness

Source	Standard deviation	R^2	Adjusted R^2	Predicted R^2	PRESS	Remark
Linear	4.89	0.9312	0.9183	0.9033	661.50	
Quadratic	4.13	0.9694	0.9419	0.9111	538.10	Suggested
Special cubic	3.90	0.9836	0.9480		+	
Cubic	1.58	0.9978	0.9915		+	Aliased

Table 4: Lack of fit test results for hardness

Source	Sum of square	degree of freedom	Mean square	F-value	p value	Remark
Linear	370.27	11	33.66	13.46	0.0050	
Quadratic	157.66	5	31.53	12.61	0.0703	Suggested
Special cubic	78.85	1	78.85	31.54	0.0025	
Cubic	0.000	0				Aliased
Pure Error	12.50	5	2.50			

Table 5: Summary of model fit results for compressive strength

Source	Standard deviation	R^2	Adjusted R^2	Predicted R^2	PRESS	Remark
Linear	0.16	0.8112	0.7758	0.6951	0.88	
Quadratic	0.14	0.9369	0.8318	0.5804	0.64	Suggested
Special cubic	0.15	0.9115	0.8000		+	
Cubic	0.16	0.9393	0.7692		+	Aliased

Table 6: Lack of fit test results for compressive strength

Source	Sum of square	degree of freedom	Mean square	F-value	p value	Remark
Linear	0.27	11	0.025	0.96	0.5598	
Quadratic	0.058	5	0.012	0.46	0.7945	Suggested
Special cubic	5.10E-3	1	5.10E-3	0.20	0.6737	
Cubic	0.000	0				Aliased
Pure Error	0.13	5	0.026			

Table 7: Summary of model fit results for water absorption

Source	Standard deviation	R^2	Adjusted R^2	Predicted R^2	PRESS	Remark

Linear	0.58	0.6649	0.6021	0.4396	9.00	
Quadratic	0.38	0.9109	0.8307	0.6829	5.09	Suggested
Special cubic	0.45	0.9240	0.7593		+	
Cubic	0.49	0.9265	0.7207		+	Aliased

Table 8: Lack of fit test results for water absorption

Source	Sum of square	degree of freedom	Mean square	F-value	p value	Remark
Linear	4.20	11	0.38	1.62	0.3109	
Quadratic	0.25	5	0.050	0.21	0.9429	Suggested
Special cubic	0.040	1	0.040	0.17	0.6976	
Cubic	0.000	0				Aliased
Pure Error	1.18	5	0.24			

Table 9: Summary of model fit results for tensile strength

Source	Standard deviation	R^2	Adjusted R^2	Predicted R^2	PRESS	Remark
Linear	0.12	0.9076	0.8903	0.8661	0.33	
Quadratic	0.11	0.9519	0.9087	0.7199	0.68	
Special cubic	0.055	0.9925	0.9763		+	Suggested
Cubic	0.035	0.9974	0.9903		+	Aliased

Table 10: Lack of fit test results for tensile strength

Source	Sum of square	degree of freedom	Mean square	F-value	p value	Remark
Linear	0.22	11	0.020	15.97	0.0034	
Quadratic	0.11	5	0.022	17.80	0.0033	
Special cubic	0.012	1	0.012	9.65	0.0607	Suggested
Cubic	0.000	0				Aliased
Pure Error	6.25E-3	5	1.25E-3			

Table 11: Summary of model fit results for coefficient of friction

Source	Standard deviation	R^2	Adjusted R^2	Predicted R^2	PRESS	Remark
Linear	0.12	0.1271	-0.0366	-0.3932	0.39	
Quadratic	0.11	0.5452	0.1359	-1.3282	0.66	
Special cubic	0.055	0.9348	0.7934		+	Suggested
Cubic	0.058	0.9411	0.7761		+	Aliased

Table 12: Lack of fit test results for coefficient of friction

Source	Sum of square	degree of freedom	Mean square	F-value	p value	Remark
Linear	0.23	11	0.021	6.28	0.0275	
Quadratic	0.11	5	0.022	6.72	0.0284	
Special cubic	1.78E-3	1	1.78E-3	0.54	0.4971	Suggested

Cubic	0.000	0				Aliased
Pure Error	0.017	5	3.32E-3			

Table 13: Summary of model fit results for wear rate

Source	Standard deviation	R^2	Adjusted R^2	Predicted R^2	PRESS	Remark
Linear	0.45	0.5478	0.2966	-0.4410	10.35	
Quadratic	0.52	0.6298	0.4631	0.3329	4.79	Suggested
Special cubic	0.55	0.7477	0.2011		+	
Cubic	0.42	0.8746	0.5236		+	Aliased

Table 14: Lack of fit test results for wear rate

Source	Sum of square	degree of freedom	Mean square	F-value	p value	Remark
Linear	2.35	11	0.21	1.19	0.4538	
Quadratic	1.76	5	0.35	1.95	0.2401	Suggested
Special cubic	0.91	1	0.91	5.06	0.0743	
Cubic	0.000	0				Aliased
Pure Error	0.90	5	0.18			

Table 15: Summary of model fit results for density

Source	Standard deviation	R^2	Adjusted R^2	Predicted R^2	PRESS	Remark
Linear	0.28	0.2983	0.1667	-0.0232	1.79	
Quadratic	0.25	0.6299	0.2967	-0.8674	3.26	
Special cubic	0.098	0.9672	0.8961		+	Suggested
Cubic	0.085	0.9791	0.9206		+	Aliased

Table 16: Lack of fit test results for density

Source	Sum of square	degree of freedom	Mean square	F-value	p value	Remark
Linear	1.19	11	0.11	14.82	0.0040	
Quadratic	0.61	5	0.12	16.72	0.0039	
Special cubic	0.021	1	0.021	2.85	0.1520	Suggested
Cubic	0.000	0				Aliased
Pure Error	0.036	5	7.30E-3			

3.2. Analysis of Statistical Models

The statistical models were analysed by fitting the selected models to the respective experimental data which was obtained from the 20 experiments carried out according to the D-Optimal mixture design. The quadratic model was fitted to the experimental data for hardness, compressive strength, water absorption, and wear rate while the special cubic model was fitted to the experimental data for tensile strength, density and coefficient of friction. This process was done using multiple regression analysis and resulted in the estimation of the unknown model parameters. The estimated model parameters were then fixed into the general quadratic and special cubic equations to obtain the final models for hardness, compressive strength, water absorption, tensile strength and wear rate, density and coefficient of friction in terms of actual values of the input factors. The equations represent hardness, compressive strength, water absorption, tensile strength, coefficient of friction, wear rate and density as a function of Cow bone (X_1), Binder (X_2), Abrasive (X_3), and Filler (X_4).

These equations (Equation 8 to 14) were used to predict their corresponding responses and the results are shown in Table 17 to 23.

$$\begin{aligned} Hardness &= 4.02X_1 - 3.02X_2 - 95.17X_3 + 79.95X_4 + 0.14X_1X_2 + 1.11X_1X_3 \\ &\quad - 1.15X_1X_4 + 1.25X_2X_3 - 0.89X_2X_4 - 0.054X_3X_4 \end{aligned} \tag{8}$$

$$\begin{aligned} Compressive\ strength &= 0.18X_1 - 0.013X_2 - 0.58X_3 + 1.32X_4 - 0.0011X_1X_2 + 0.0052X_1X_3 \\ &\quad - 0.019X_1X_4 + 0.012X_2X_3 - 0.014X_2X_4 - 0.024X_3X_4 \end{aligned} \tag{9}$$

$$\begin{aligned} Water\ absorption &= -1.95X_1 - 0.58X_2 - 0.26X_3 + 1.28X_4 + 0.049X_1X_2 + 0.043X_1X_3 \\ &\quad + 0.015X_1X_4 + 0.0012X_2X_3 - 0.014X_2X_4 - 0.030X_3X_4 \end{aligned} \tag{10}$$

$$\begin{aligned} Tensile\ strength &= 0.76X_1 + 0.26X_2 + 1.02X_3 + 4.07X_4 - 0.016X_1X_2 - 0.055X_1X_3 \\ &\quad - 0.11X_1X_4 + 0.0065X_2X_3 - 0.064X_2X_4 - 0.15X_3X_4 \\ &\quad + 0.00012X_1X_2X_3 + 0.0013X_1X_2X_3 + 0.0054X_1X_3X_4 - 0.00096X_2X_3X_4 \end{aligned} \tag{11}$$

$$\begin{aligned} Coefficient\ of\ friction &= 2.36X_1 + 1.73X_2 + 5.91X_3 + 2.35X_4 - 0.087X_1X_2 - 0.23X_1X_3 \\ &\quad - 0.11X_1X_4 - 0.19X_2X_3 - 0.098X_2X_4 + 0.16X_3X_4 \\ &\quad + 0.0062X_1X_2X_3 + 0.0031X_1X_2X_3 - 0.0030X_1X_3X_4 - 0.0014X_2X_3X_4 \end{aligned} \tag{12}$$

$$\begin{aligned} Wear\ rate &= -0.18X_1 - 0.43X_2 + 4.65X_3 - 0.50X_4 + 0.015X_1X_2 - 0.060X_1X_3 \\ &\quad + 0.0021X_1X_4 - 0.044X_2X_3 + 0.015X_2X_4 - 0.048X_3X_4 \end{aligned} \tag{13}$$

$$\begin{aligned} Density &= -1.82X_1 - 0.70X_2 + 8.20X_3 + 4.22X_4 + 0.061X_1X_2 + 0.014X_1X_3 \\ &\quad + 0.072X_1X_4 - 0.13X_2X_3 - 0.031X_2X_4 - 1.06X_3X_4 \\ &\quad - 0.0018X_1X_2X_3 - 0.0033X_1X_2X_3 + 0.0051X_1X_3X_4 + 0.019X_2X_3X_4 \end{aligned} \tag{14}$$

Table 17: Experimental and RSM predicted results for hardness

Run	Actual values of factors				Response (N/mm^2)	
	Cow bone (%)	Binder (%)	Abrasive (%)	Filler (%)	Actual Experiment	RSM Predicted
1	38	40	8	14	215	219
2	30	50	5	15	190	191
3	30	50	5	15	190	191
4	35	47	5	13	205	207
5	36	42	10	12	210	205
6	40	40	10	10	230	230
7	31	47	9	13	193	192
8	32	43	10	15	195	194
9	34	45	8	13	200	206
10	40	44	6	10	240	237
11	35	50	5	10	210	211
12	30	50	10	10	185	187

13	40	44	6	10	235	237
14	40	40	5	15	230	228
15	30	50	10	10	185	187
16	35	50	5	10	210	211
17	32	43	10	15	195	194
18	36	43	6	15	220	219
19	32	50	7	11	205	197
20	36	45	9	10	225	224

Table 18: Experimental and RSM predicted results for compressive strength

Run	Actual values of factors				Response (MPa)	
	Cow bone (%)	Binder (%)	Abrasive (%)	Filler (%)	Actual Experiment	RSM Predicted
14	38	40	8	14	3.50	3.49
2	30	50	5	15	3.10	3.09
3	30	50	5	15	3.10	3.09
4	35	47	5	13	3.30	3.30
5	36	42	10	12	3.30	3.28
6	40	40	10	10	3.80	3.83
7	31	47	9	13	3.00	3.02
8	32	43	10	15	3.10	3.00
9	34	45	8	13	3.20	3.22
10	40	44	6	10	3.80	3.80
11	35	50	5	10	3.30	3.26
12	30	50	10	10	3.29	3.15
13	40	44	6	10	3.80	3.80
14	40	40	5	15	3.88	3.87
15	30	50	10	10	3.00	3.15
16	35	50	5	10	3.20	3.26
17	32	43	10	15	2.90	3.00
18	36	43	6	15	3.50	3.54
19	32	50	7	11	3.10	3.09
20	36	45	9	10	3.60	3.56

Table 19: Experimental and RSM predicted results for water absorption

Run	Actual values of factors				Response (%)	
	Cow bone (%)	Binder (%)	Abrasive (%)	Filler (%)	Actual Experiment	RSM Predicted
1	38	40	8	14	2.1	2.2
2	30	50	5	15	3.5	3.5
3	30	50	5	15	3.5	3.5
4	35	47	5	13	3.4	3.4
5	36	42	10	12	3.2	3.2
6	40	40	10	10	1.5	1.5
7	31	47	9	13	3.5	3.5
8	32	43	10	15	3.6	3.6
9	34	45	8	13	3.5	3.5

10	40	44	6	10	1.5	1.5
11	35	50	5	10	3.4	3.5
12	30	50	10	10	3.2	3.2
13	40	44	6	10	1.4	1.5
14	40	40	5	15	1.4	1.3
15	30	50	10	10	3.2	3.2
16	35	50	5	10	3.5	3.5
17	32	43	10	15	3.7	3.6
18	36	43	6	15	3.2	3.2
19	32	50	7	11	3.6	3.5
20	36	45	9	10	3.3	3.3

Table 20: Experimental and RSM predicted results for tensile strength

Run	Actual values of factors				Response (MPa)	
	Cow bone (%)	Binder (%)	Abrasive (%)	Filler (%)	Actual Experiment	RSM Predicted
1	38	40	8	14	3.30	3.30
2	30	50	5	15	2.20	2.20
3	30	50	5	15	2.20	2.20
4	35	47	5	13	2.60	2.60
5	36	42	10	12	2.90	2.93
6	40	40	10	10	3.30	3.29
7	31	47	9	13	2.10	2.13
8	32	43	10	15	2.40	2.45
9	34	45	8	13	2.70	2.61
10	40	44	6	10	3.10	3.10
11	35	50	5	10	2.80	2.79
12	30	50	10	10	2.45	2.47
13	40	44	6	10	3.10	3.10
14	40	40	5	15	3.00	2.99
15	30	50	10	10	2.50	2.47
16	35	50	5	10	2.80	2.79
17	32	43	10	15	2.50	2.45
18	36	43	6	15	2.85	2.88
19	32	50	7	11	2.40	2.43
20	36	45	9	10	2.90	2.91

Table 21: Experimental and RSM predicted results for coefficient of friction

Run	Actual values of factors				Response	
	Cow bone (%)	Binder (%)	Abrasive (%)	Filler (%)	Actual Experiment	RSM Predicted
1	38	40	8	14	0.42	0.42
2	30	50	5	15	0.51	0.53
3	30	50	5	15	0.55	0.53
4	35	47	5	13	0.38	0.38
5	36	42	10	12	0.55	0.54
6	40	40	10	10	0.70	0.70
7	31	47	9	13	0.30	0.29
8	32	43	10	15	0.40	0.43

9	34	45	8	13	0.42	0.45
10	40	44	6	10	0.45	0.45
11	35	50	5	10	0.40	0.37
12	30	50	10	10	0.35	0.33
13	40	44	6	10	0.46	0.45
14	40	40	5	15	0.60	0.60
15	30	50	10	10	0.31	0.33
16	35	50	5	10	0.33	0.37
17	32	43	10	15	0.46	0.43
18	36	43	6	15	0.40	0.39
19	32	50	7	11	0.63	0.62
20	36	45	9	10	0.64	0.64

Table 22: Experimental and RSM predicted results for wear rate

Run	Actual values of factors				Response (mg/m)	
	Cow bone (%)	Binder (%)	Abrasive (%)	Filler (%)	Actual Experiment	RSM Predicted
1	38	40	8	14	1.10	1.11
2	30	50	5	15	0.30	0.28
3	30	50	5	15	0.31	0.28
4	35	47	5	13	1.40	1.43
5	36	42	10	12	1.50	1.45
6	40	40	10	10	1.65	1.70
7	31	47	9	13	0.60	0.62
8	32	43	10	15	0.95	1.02
9	34	45	8	13	1.00	1.01
10	40	44	6	10	2.00	2.00
11	35	50	5	10	1.30	1.31
12	30	50	10	10	0.60	0.62
13	40	44	6	10	2.00	2.00
14	40	40	5	15	1.60	1.54
15	30	50	10	10	0.60	0.62
16	35	50	5	10	1.30	1.31
17	32	43	10	15	1.10	1.02
18	36	43	6	15	1.10	1.20
19	32	50	7	11	0.50	0.49
20	36	45	9	10	1.50	1.41

Table 23: Experimental and RSM predicted results for density

Run	Actual values of factors				Response (g/cm^3)	
	Cow bone (%)	Binder (%)	Abrasive (%)	Filler (%)	Actual Experiment	RSM Predicted
1	38	40	8	14	1.01	1.01
2	30	50	5	15	1.25	1.17
3	30	50	5	15	1.08	1.17
4	35	47	5	13	1.28	1.28
5	36	42	10	12	1.30	1.26
6	40	40	10	10	1.60	1.61

7	31	47	9	13	2.00	1.96
8	32	43	10	15	1.27	1.26
9	34	45	8	13	1.50	1.62
10	40	44	6	10	1.85	1.92
11	35	50	5	10	1.65	1.61
12	30	50	10	10	1.35	1.41
13	40	44	6	10	2.01	1.92
14	40	40	5	15	1.95	1.96
15	30	50	10	10	1.45	1.41
16	35	50	5	10	1.56	1.61
17	32	43	10	15	1.25	1.26
18	36	43	6	15	1.75	1.71
19	32	50	7	11	1.36	1.32
20	36	45	9	10	1.80	1.79

3.3 Discussion on Predictability of Model

The design model was evaluated using the standard error of each model term to determine its suitability and the results are shown in Table 17-24. The standard error should be as small as possible for the model to be considered useful. Furthermore, the standard errors should be similar within type of coefficient. This was indeed the case with the results presented in Table 17-24. As shown in Table 17-24, the VIF values obtained were equal to one. The variation inflation factor (VIF) is used as a measure of the increase in the variance of a model coefficient as a result of lack of orthogonality in the design **[15]**. A design that lacks orthogonality is a sign that the model terms exhibit collinearity and this is usually characterised by high values of VIF, a situation that is not desirable. VIFs above 10 are cause for concern and VIFs above 100 are cause for alarm, indicating coefficients are poorly estimated due to multicollinearity. If the VIFs get above 1000, and there are no built-in extra constraints to the design, then it might not be possible to get a useful model. The ideal situation is that in which the VIF values are all unity **[14].** A VIF value of one for a model coefficient indicates that the coefficient is orthogonal to the remaining model terms. The R^i squared value (R_i^2) for a model term is the multiple correlation coefficient, and it shows the extent to which the coefficient of that model term is correlated to the others. For the ideal case of an orthogonal design, the R_i^2 value is usually zero. High R_i^2 values are not desirable because it is usually an indication that the model terms are correlated with each other and this could result in a bad model. For the results presented in Table 24, it can be seen that the R_i^2 value was in the range 0.0000 to 0.0779. This is acceptable as these values are close to the ideal value of R_i^2 i.e. 0.0000 (Abdi, 2007). Models were formulated to predict hardness, compressive strength, coefficient of friction, wear rate, and density using response surface modelling. A desirability of 0.91 was obtained which shows the adequacy of the model terms. The models were then validated using coefficient of determination R^2 the coefficient of determination (R^2) obtained ranged from 0.9213, (92.13%) to 0.98.1, (98.1%) which indicates that a substantial good fit was achieved by the models developed. The values obtained from the validation of these models were therefore found to be satisfactory, and shows good predictability of the model.

4 Conclusion

Modelling of an automobile brake pad using cow bone as base material has been achieved. The models were validated using coefficient of determination (R^2). The coefficient of determination (R^2) obtained ranged from 0.9213 (92.13%) to 0.981 (98.1%) which indicates that a substantial good fit was achieved by the models developed. A desirability of 0.951 was obtained which shows the adequacy of the model terms. The values obtained from the validation of these models were therefore found to be satisfactory, and shows good predictability of the model and its adequacy.

References

[1] Ji-Hoon Chio (2004) "Finite Element Analysis of Transient Thermo Elastic-Behaviors in Disk Brakes", Wear, vol.5, pp47-58.
[2] Sterle W.O, Klob H. (2007) "Towards a better understanding of brake friction materials", Wear. Vol.2. pp 263-270.
[3] Chikalthankar S.B, Nandedkar V.M (2014) "A Review & Literature of Frictional & Wear Characteristics of Non-Asbestos Brake Pad Using Link Chase Machine", International Journal of Mechanical Engineering and Technology (IJMET), Issue1, vol. 5, pp140-145.
[4] Sowjanya K., (2013) Structural analysis of disk brake rotor, International Journal of compute trends and technology, 4(7), pp 20-28
[5] Bouchetara Mostefa, Belhocine Ali (2014) "Thermo Elastic Analysis of Disk Brakes Rotor", American Journal of Mechanical Engineering, issue 2, vol. 4,pp. 103-113.
[6] Oder G, (2009) "Thermal and stress analysis of brake discs in railway vehicles", Advance Engineering, vol. 3, pp230-237.
[7] Ziad, (2009) "An investigation of disc brake rotor by FEA, Wear"; (3)2, pp. 34-41.
[8] Zen, M., Izzati, N., Abd Gani, S. S., Shamsudin, R. and Fard Masoumi, H. R. (2015). The use of D-optimal mixture design in optimizing development of okara tablet formulation as a dietary supplement. *The Scientific World Journal*. 12(2), pp 65-73.
[9] Esbensen, K.H., Guyot, D., Westad, F. and Houmoller, L.P. (2002). Multivariate data analysis. In: *Practice: An Introduction to Multivariate Data Analysis and Experimental Design*, Aalborg University, Esbjerg, Denmark.
[10] [10] Elakhame, Z.U., Alhassan, O.A., Samuel, A.E. (2014) Development and Production of Brake Pads from Palm Kernel Shl Composites, International Journal of Scientific and Engineering Research, 5 (10), pp. 734-744.
[11] Kim, S.J., Kim, K.S., Jang, H.: Optimization of manufacturing parameters for brake lining using Taguchi method, Journal of Material Processing Technology, no.136, 2003, pp.202-208, 2003.
[12] Abutu, J., Lawal, S.A., Ndaiman, M.B., Lafia-Araga, R.A., Adedipe, O. and Choudhury, I.A. (2018). Study on mechanical properties of fly ash impregnated glass fiber reinforced polymer composites using mixture design analysis. *Engineering Science and Technology, an International Journal*, 21, pp. 787-797.
[13] Hirata, M., Takayama, K. and Nagai, T. (1992). Formulation optimization of sustained-release tablet of chlorpheniramine maleate by means of extreme vertices design and simultaneous optimization technique. *Chemical and pharmaceutical bulletin*, *40*(3), pp. 741-746.
[14] Myers, R. H., Montgomery, D. C. and Anderson-Cook, C. M. (2009). *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*. Wiley Series in Probability and Statistics.
[15] Kumar S, Panda A.K, and Singh R.K, (2011) "A review on tertiary recycling of high-density polyethylene to fuel," Resources, Conservation and Recycling, vol. 55, No. 11, pp. 893–910.