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Enhanced Ratio-Type Estimator for Finite Population Mean using Auxiliary Variable in Simple Random Sampling

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Abstract

In this paper, a ratio-type estimator of finite population mean in simple random sampling without replacement by using information on an auxiliary variable has been proposed. The proposed estimator was obtained by using the strategy of power transformation and incorporated the unknown weight (α). The objective of this study is to develop a new ratio estimator that provide better precision of estimation of population mean. The properties such as bias and mean square error (MSE) of the proposed estimator are derived and tested using four real data sets. The results of the empirical study revealed that the proposed ratio type estimator performed better than the existing estimators considered in the study. Therefore, the proposed estimator is more efficient than the existing estimators based on the criteria of mean square error and percent relative efficiency.

1.0 Introduction

Auxiliary information has been frequently utilized in probability sampling to enhance the accuracy of the estimator of finite population mean or total. If utilized effectively, this information can help us come up with better sampling strategies compared to those where no auxiliary information is utilized. The idea of using auxiliary information was first introduced by [1], who proposed a ratio estimator for population mean. It is commonly accepted that the ratio estimator is more efficient than the sample mean estimator, provided that there is a positive correlation between the study and auxiliary variables. However, in cases where the correlation is negative, the product estimator becomes more efficient. Modified ratio estimators were then developed using unknown constants, and among them, the estimator with the minimum variance or mean square error is considered the most efficient estimator [2], [3], [4], [5] and [6]. In survey sampling, one of the ways to enhance and modify estimators is by using an unknown weight (α). This unknown weight is a constant that is suitable for minimizing the mean square error of an estimator of population mean through a process called partial differentiation. The value of alpha ranges from 0 to 1, and it is a non-negative value that determines the mean square error. This study used the strategy of power transformation and incorporated the unknown weight (α) to [7] estimator and developed a new estimator that is both effective and efficient in enhancing the accuracy of estimating the population mean in simple random sampling without replacement.

Several researchers have proposed various methods to develop efficient estimators for estimating population mean or variance using auxiliary information. [8] introduced a product type estimator for the estimation of population mean when the correlation coefficient is negative. [4] developed a family of estimators for the estimation of population mean using known parameters. [9] proposed a ratio estimator with non-conventional location parameters. [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24] and others have also developed various efficient estimators for population mean and variance using auxiliary information.

1.1. Symbols, Notations and Some Existing Estimators

Let $U = \{U_1, , , , , U_N\}$ be a finite population of size N and let (y_i, x_i) be the value of the study variable Y and the auxiliary variable X on ith unit $U_i, i = 1, \ldots N$. Let $\bar{Y}$ and $\bar{X}$ be population means of the study variable Y and the auxiliary variable X respectively. We assume that the population mean $\bar{X}$ and the population variance S_x^2 of the auxiliary variable are known. The following notations are defined:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i \quad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \quad C_y = \frac{S_y}{\bar{Y}}, \quad C_x = \frac{S_x}{\bar{X}}, \quad S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2,$$

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2 \text{ and } \lambda = \left(\frac{1}{n} - \frac{1}{N}\right)$$

where n is the sub-sample size; N is the number of units in the population; $\bar{Y}$ is the population mean of the study variable; $\bar{X}$ is the population mean of the auxiliary variable; λ is a known constant involving the samples and population units; ρ is the population correlation coefficient between the auxiliary and the study variables; S_x^2 and S_y^2 are the variances of the auxiliary and the study variables, respectively; C_y and C_x are the coefficients of variation of the study and auxiliary variables, respectively; M_d is the median of the auxiliary variable; β_1 is the coefficient of skewness of auxiliary variable; β_2 is the coefficient of kurtosis of auxiliary variable; e_0 is the error term associated with the study variable; e_0 is the error term associated with the auxiliary variable [16].

It is well known that the sample mean $\bar{y}$ is an unbiased estimator of population mean $\bar{Y}$, and its estimator and variance under simple random sample without replacement (SRSWOR) are, respectively, given as

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{1}$$

$$Var\left(\bar{y}\right) = \bar{Y}^2 \lambda C_y^2 \tag{2}$$

[1] considered the use of supplementary information (X) and suggested the usual ratio estimator for estimating population mean ($\bar{Y}$) of the study variable (Y). The suggested estimator, its bias and mean square error are, respectively, given as:

$$\hat{\bar{Y}}_r = \frac{\bar{y}}{\bar{x}}\bar{X} \tag{3}$$

$$Bias\left(\hat{\bar{Y}}_r\right) = \lambda \bar{Y}\left[C_x^2 - \rho C_y C_x\right] \tag{4}$$

$$MSE\left(\hat{\bar{Y}}_r\right) = \lambda \bar{Y}^2\left[C_y^2 + C_x^2 - 2\rho C_y C_x\right] \tag{5}$$

[25] considered the use of known value of coefficient of variation (C_x) of the auxiliary variable and proposed ratio type estimator of population mean. The proposed estimator, its bias and mean square error are, respectively, given as:

$$\hat{\bar{Y}}_1 = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right) \tag{6}$$

$$Bias\left(\hat{\bar{Y}}_1\right) = \lambda\bar{Y}\left[\delta_1^2 C_x^2 - 2\delta_1\rho C_y C_x\right] \tag{7}$$

$$MSE\left(\hat{\bar{Y}}_1\right) = \lambda\bar{Y}^2\left[C_y^2 + \delta_1^2 C_x^2 - 2\delta_1\rho C_y C_x\right] \tag{8}$$

where $\delta_1 = \left(\frac{\bar{X}}{\bar{x}+C_x}\right)$

[26] proposed ratio type estimator of population mean by imposing coefficient of variation (C_x) and coefficient of kurtosis (β_2) of auxiliary variable on the work of [25]. The proposed estimator, its bias and mean square error are, respectively, given as:

$$\hat{\bar{Y}}_2 = \bar{y}\left(\frac{\bar{X}C_x+\beta_2}{\bar{x}C_x+\beta_2}\right) \tag{9}$$

$$Bias\left(\hat{\bar{Y}}_2\right) = \lambda\bar{Y}\left[\delta_2^2 C_x^2 - 2\delta_2\rho C_y C_x\right] \tag{10}$$

$$MSE\left(\hat{\bar{Y}}_2\right) = \lambda\bar{Y}^2\left[C_y^2 + \delta_2^2 C_x^2 - 2\delta_2\rho C_y C_x\right] \tag{11}$$

where $\delta_2 = \left(\frac{\bar{X}C_x}{\bar{x}C_x+\beta_2}\right)$

[27] suggested a new estimator of population mean by incorporating coefficient of correlation (ρ) of auxiliary variable in the work of [1]. The proposed estimator, its bias and mean square error are, respectively, given as:

$$\hat{\bar{Y}}_3 = \bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right) \tag{12}$$

$$Bias\left(\hat{\bar{Y}}_3\right) = \lambda\bar{Y}\left[\delta_3^2 C_x^2 - 2\delta_3\rho C_y C_x\right] \tag{13}$$

$$MSE\left(\hat{\bar{Y}}_3\right) = \lambda\bar{Y}^2\left[C_y^2 + \delta_3^2 C_x^2 - 2\delta_3\rho C_y C_x\right] \tag{14}$$

where $\delta_3 = \left(\frac{\bar{X}}{\bar{x}+\rho}\right)$

[28] consider the work of [27] and proposed a new ratio estimator of population mean by incorporating coefficient of kurtosis (β_2) of the auxiliary variable. The proposed estimator, its bias and mean square error are, respectively, given as:

$$\hat{\bar{Y}}_4 = \bar{y}\left(\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right) \tag{15}$$

$$Bias\left(\hat{\bar{Y}}_4\right) = \lambda\bar{Y}\left[\delta_4^2 C_x^2 - 2\delta_4\rho C_y C_x\right] \tag{16}$$

$$MSE\left(\hat{\bar{Y}}_4\right)=\lambda\bar{Y}^2\left[C_y^2+\delta_4^2C_x^2-2\delta_4\rho C_yC_x\right] \tag{17}$$

where $\delta_4=\left(\frac{\bar{X}}{\bar{x}+\beta_2}\right)$

[13] defined a new ratio type estimator of population mean by incorporating known value of coefficient of skewness (β_1) of the auxiliary variable. The proposed estimator, its bias and mean square error are, respectively, given as:

$$\hat{\bar{Y}}_5=\bar{y}\left(\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right) \tag{18}$$

$$Bias\left(\hat{\bar{Y}}_5\right)=\lambda\bar{Y}\left[\delta_5^2C_x^2-2\delta_5\rho C_yC_x\right] \tag{19}$$

$$MSE\left(\hat{\bar{Y}}_5\right)=\lambda\bar{Y}^2\left[C_y^2+\delta_5^2C_x^2-2\delta_5\rho C_yC_x\right] \tag{20}$$

where $\delta_5=\left(\frac{\bar{X}}{\bar{x}+\beta_1}\right)$

[14] defined a new ratio type estimator of population mean by incorporating known value of median (M_d) of the auxiliary variable. The proposed estimator, its bias and mean square error are, respectively, given as:

$$\hat{\bar{Y}}_6=\bar{y}\left(\frac{\bar{X}+M_d}{\bar{x}+M_d}\right) \tag{21}$$

$$Bias\left(\hat{\bar{Y}}_6\right)=\lambda\bar{Y}\left[\delta_6^2C_x^2-2\delta_6\rho C_yC_x\right] \tag{22}$$

$$MSE\left(\hat{\bar{Y}}_6\right)=\lambda\bar{Y}^2\left[C_y^2+\delta_6^2C_x^2-2\delta_6\rho C_yC_x\right] \tag{23}$$

where $\delta_6=\left(\frac{\bar{X}}{\bar{x}+M_d}\right)$

The sample size (n) was first used by [7] to enhance the precision of [1] estimator of population mean. The proposed estimator, its bias and mean square error are, respectively, given as:

$$\hat{\bar{Y}}_7=\bar{y}\left(\frac{\bar{X}+n}{\bar{x}+n}\right) \tag{24}$$

$$Bias\left(\hat{\bar{Y}}_7\right)=\lambda\bar{Y}\left[\delta_7^2C_x^2-2\delta_7\rho C_yC_x\right] \tag{25}$$

$$MSE\left(\hat{\bar{Y}}_7\right)=\lambda\bar{Y}^2\left[C_y^2+\delta_7^2C_x^2-2\delta_7\rho C_yC_x\right] \tag{26}$$

where $\delta_7=\left(\frac{\bar{X}}{\bar{x}+n}\right)$

[4] consider the use of unknown weight and defined a new ratio type estimator of population mean. The proposed estimator, its bias and mean square error are, respectively, given as:

$$\hat{\bar{Y}}_8 = \bar{y}\alpha\left(\frac{\bar{X}+n}{\bar{x}+n}\right) \tag{27}$$

$$Bias\left(\hat{\bar{Y}}_8\right) = \bar{Y}\left[\left(\alpha-1\right)+\lambda\delta_7^2C_x^2-\lambda\delta_7\rho C_yC_x\right] \tag{28}$$

where $\alpha = \dfrac{1+\lambda\delta_7^2C_x^2-\lambda\delta_7\rho C_yC_x}{1+\lambda C_y^2+3\lambda\delta_7^2C_x^2-4\lambda\delta_7\rho C_yC_x}$

$$MSE\left(\hat{\bar{Y}}_8\right) = \bar{Y}^2\left[1-\frac{\left(1+\lambda\delta_7^2C_x^2-\lambda\delta_7\rho C_yC_x\right)^2}{1+\lambda C_y^2+3\lambda\delta_7^2C_x^2-4\lambda\delta_7\rho C_yC_x}\right] \tag{29}$$

2.0. Methodology

2.1. Proposed Estimator

In this section, we propose a new ratio type estimator of population mean by using the strategy of power transformation in order to enhance the precision of [7] ratio type estimator. The proposed estimator is given as:

$$\hat{\bar{Y}}_{IR} = \bar{y}\left(\frac{\bar{X}+n}{\bar{x}+n}\right)^{\omega} \tag{30}$$

where ω is a suitable chosen unknown weight to be determined such that the mean square error of the proposed estimator is minimum.

To obtain the asymptotic properties of the estimator, we define the following error terms, as in [3]:

Let $e_0 = \dfrac{\bar{y}-\bar{Y}}{\bar{Y}}$ and $e_1 = \dfrac{\bar{x}-\bar{X}}{\bar{X}}$ such that $\bar{y} = \bar{Y}\left(1+e_0\right)$ and $\bar{x} = \bar{X}\left(1+e_1\right)$

$$\left.\begin{aligned}&E\left(e_0\right)=E\left(e_1\right)=0\\&E\left(e_0^2\right)=\lambda C_y^2, E\left(e_1^2\right)=\lambda C_x^2, E\left(e_0e_1\right)=\lambda\rho C_yC_x\end{aligned}\right\} \tag{31}$$

where $\lambda = \left(\dfrac{1}{n}-\dfrac{1}{N}\right)$

Expressing the estimator $\hat{\bar{Y}}_{IR}$ in terms of e_i $(i = 0, 1)$ we can write (30) as

$$\bar{Y}_{IR} = \bar{Y}\left(1+e_0\right)\left[\frac{\bar{X}+n}{\bar{X}\left(1+e_1\right)+n}\right]^{\omega} \tag{32}$$

By factorizing the second component of the RHS of (31), it gives

$$\bar{Y}_{IR} = \bar{Y}\left(1+e_0\right)\left[\frac{\bar{X}+n}{\bar{X}+n\left(1+\frac{\bar{X}e_1}{\bar{X}+n}\right)}\right]^{\omega} \tag{33}$$

Thus, it follows

$$\bar{Y}_{IR} = \bar{Y}\left(1+e_0\right)\left[\frac{1}{\left(1+\frac{\bar{X}e_1}{\bar{X}+n}\right)}\right]^{\omega} \tag{34}$$

$$\bar{Y}_{IR} = \bar{Y}\left(1+e_0\right)\left[\frac{1}{\left(1+\delta e_1\right)}\right]^{\omega} \tag{35}$$

where $\delta = \frac{\bar{X}}{\bar{X}+n}$

$$\bar{Y}_{IR} = \bar{Y}\left(1+e_0\right)\left[\left(1+\delta e_1\right)\right]^{-\omega} \tag{36}$$

$$\bar{Y}_{IR} = \bar{Y}\left(1+e_0\right)\left[1-\omega\delta e_1 + \frac{\omega(\omega-1)}{2}\delta^2 e_1^2\right] \tag{37}$$

Expanding the RHS of (36) to the first order of approximation, multiplying out and neglecting the terms of e's greater than two, we get

$$\bar{Y}_{IR} = \bar{Y}\left[1+e_0-\omega\delta e_1 + \frac{\omega(\omega-1)}{2}\delta^2 e_1^2 - \omega\delta e_0 e_1\right] \tag{38}$$

Subtracting $\bar{Y}$ and taking expectation to both sides of (37), it gives

$$E\left(\bar{Y}_{IR}\right)-\bar{Y} = \bar{Y}E\left[e_0-\omega\delta e_1 + \frac{\omega(\omega-1)}{2}\delta^2 e_1^2 - \omega\delta e_0 e_1\right] \tag{39}$$

Applying the results of (31), the bias of the estimator $\bar{Y}_{IR}$ is obtained as

$$Bias\left(\bar{Y}_{IR}\right) = \bar{Y}\left[\frac{\omega(\omega-1)}{2}\delta^2\lambda C_x^2 - \omega\delta\lambda\rho C_y C_x\right] \tag{40}$$

Similarly, subtracting $\bar{Y}$, taking expectation, and squaring both sides of (40), it gives

$$E\left(\bar{Y}_{IR}-\bar{Y}\right)^2 = \bar{Y}^2 E\left[e_0-\omega\delta e_1 + \frac{\omega(\omega-1)}{2}\delta^2 e_1^2 - \omega\delta e_0 e_1\right]^2 \tag{41}$$

Neglecting the terms of e's greater than two, (41) becomes

$$MSE\left(\bar{Y}_{IR}\right) = \bar{Y}^2 E\left[e_0^{\,2} + \omega^2\delta^2 e_1^{\,2} - 2\omega\delta e_0 e_1\right] \tag{42}$$

Applying the results of (31), the mean square error of the estimator $\bar{Y}_{IR}$ is obtained as

$$MSE\left(\bar{Y}_{IR}\right) = \bar{Y}^2\left[\lambda C_y^2 + \omega^2\delta^2\lambda C_x^2 - 2\omega\delta\lambda\rho C_y C_x\right] \tag{43}$$

Differentiating (43) partially with respect to ω, we have

$$\frac{\partial MSE\left(\bar{Y}_{IR}\right)}{\partial\omega} = \bar{Y}^2\left[2\omega\delta^2\lambda C_x^2 - 2\delta\lambda\rho C_y C_x\right] = 0 \tag{44}$$

The optimum value of ω is obtained by solving (44) linearly and given as

$$\omega^{opt} = \frac{\rho C_y}{\delta C_x}$$

Substituting the value of ω into (43) we obtained the minimum mean square error as

$$MSE\left(\bar{Y}_{IR}\right)_{\min} = \bar{Y}^2\left[\lambda C_y^2 + \left(\frac{\rho C_y}{\delta C_x}\right)^2 \delta^2 \lambda C_x^2 - 2\left(\frac{\rho C_y}{\delta C_x}\right)\delta\lambda\rho C_y C_x\right] \quad (45)$$

$$MSE\left(\bar{Y}_{IR}\right)_{\min} = \bar{Y}^2 \lambda C_y^2\left(1-\rho^2\right) \quad (46)$$

3.2 Theoretical Efficiency Comparisons

In this section, efficiency of the proposed estimator is compared with some of the commonly used estimators in the literature. Thus, the conditions under which the proposed estimator is more efficient are given below:

i. The proposed estimator $\bar{Y}_{IR}$ is more efficient that the estimator of the sample mean if the following conditions holds:

$MSE\left(\bar{Y}_{IR}\right)_{\min} < MSE\left(\bar{Y}_r\right)$;

ii. The proposed estimator $\bar{Y}_{IR}$ is more efficient that the usual ratio estimator defined by Cochran (1940) if the following conditions holds:

$MSE\left(\bar{Y}_{IR}\right)_{\min} < MSE\left(\bar{Y}_r\right)$;

iii. The proposed estimator $\bar{Y}_{IR}$ is more efficient that the ratio type estimators defined [25], [26], [27], [28], [13], [14], [7] and [4] if the following conditions holds:

$MSE\left(\bar{Y}_{IR}\right)_{\min} < MSE\left(\bar{Y}_i\right)$, $i = 1, 2, ..., 8$

3.0. Results and Discussion

In this section, four real data sets namely; Populations I, II, III and IV were used to examine the performance of the proposed estimator over some existing ratio type estimators in the literature. The Population Statistics for the real data sets are given in Table 1.

Table 1: Population Statistics for the Real Data Sets

Parameters	Population I	Population II	Population III	Population IV
N	80	40	40	34
n	20	8	8	20
ρ	51.8264	50.7858	50.7858	856.4117
$\bar{Y}$	11.2646	2.3033	9.4543	208.8823
$\bar{X}$	0.9413	0.8006	0.8349	0.4491
C_y	0.3542	0.3295	0.3295	0.8561
C_x	0.7507	0.8406	0.6756	0.7205
β_1	1.0500	0.9740	0.8799	0.9782
β_2	-0.0634	-0.5344	-0.4622	0.0978
M_d	7.5750	1.2500	7.0700	150

Population I and II [Source: [8]]
Population III [Source: [29]]
Population IV [Source: [30]]

Table 1 shows the numerical values of the parameters used in computing Bias, Mean Square Error (MSE) and Percentage Relative Efficiency (PRE) of the proposed and other existing ratio type estimators considered in the study and the results are presented in tables 3 and 4. Table 3 shows the bias of the proposed and other existing estimators. Table 2 presents the bias values of the estimators based on the first order of approximation. Table 3 presents the mean square error values of the proposed estimator and some existing ratio type estimators. While Table 4 gives Percent Relative Efficiency of the proposed estimator as compared to the existing ratio type estimators considered in this study.

Table 2: Bias Estimates of the Proposed and Existing Estimators

Estimators	Population I	Population II	Population III	Population IV
$\bar{Y}_r$	0.6088	2.4624	1.3742	4.268817
$\bar{Y}_1$	0.0506	0.2760	0.2573	-0.6447285
$\bar{Y}_2$	0.1134	3.7353	0.6588	-0.6210296
$\bar{Y}_3$	0.0350	0.3047	0.2225	-0.6337635
$\bar{Y}_4$	0.1156	3.1516	0.5777	-0.6194826
$\bar{Y}_5$	0.1170	0.1896	0.2131	-0.6550856
$\bar{Y}_6$	-0.1901	0.0479	-0.3213	-2.584918
$\bar{Y}_7$	-0.2083	-0.3242	-0.3424	-1.291627
$\bar{Y}_8$	-0.035482	-0.2267399	-0.2061702	-11.38059
$\bar{Y}_{proposed}$	-0.1956514	-0.3025844	-0.447807	-3.531944

Table 2 shows the computed numerical values of bias estimates of the proposed estimator and some existing ratio type estimators considered using datasets; 1 and 2; 3 and 4, respectively.

Table 3: Mean Square Error (MSE) Values of the Proposed and Existing Estimators

Estimator	Population I	Population II	Population III	Population IV
$\bar{Y}_r$	18.9793	95.8641	49.8536	10539.96
$\bar{Y}_1$	15.2581	42.0198	41.0685	10514.92
$\bar{Y}_2$	18.5128	217.7034	61.4577	10535.21
$\bar{Y}_3$	14.4503	43.47728	39.3031	10524.31
$\bar{Y}_4$	18.6279	188.0582	57.3389	10536.54
$\bar{Y}_5$	18.7015	37.6296	38.8230	10506.05
$\bar{Y}_6$	2.7825	30.4328	11.6863	8853.316
$\bar{Y}_7$	1.8289	11.5391	10.6117	9960.905
$\bar{Y}_8$	1.838903	11.51517	10.47052	9746.47
$\bar{Y}_{proposed}$	1.439996	10.05398	8.483106	8834.957

Table 3 shows the computed numerical values of mean square error (MSE) of the proposed estimator and some existing estimators considered using populations; I, II, III and IV, respectively. Evidence from the result signifies that the proposed estimator possessed minimum mean square error values than the existing ratio type estimators; $\hat{\bar{Y}}_r$ $\hat{\bar{Y}}_1$, $\hat{\bar{Y}}_2$, $\hat{\bar{Y}}_3$, $\hat{\bar{Y}}_4$, $\hat{\bar{Y}}_5$, $\hat{\bar{Y}}_6$, $\hat{\bar{Y}}_7$ and $\hat{\bar{Y}}_8$. Therefore, based on the criteria of mean square error, the proposed ratio type estimator performed better and is more efficient than the existing ratio type estimators considered.

Table 4: Percent Relative Efficiency of Proposed and Existing Estimators with Respect to $\bar{Y}_r$

Estimator	Population I	Population II	Population III	Population IV
$\bar{Y}_r$	100	100	100	100
$\bar{Y}_1$	124.388	228.1403	121.3913	100.2381
$\bar{Y}_2$	102.5199	44.0343	81.1186	100.0451
$\bar{Y}_3$	131.3419	220.4924	126.8439	100.1487
$\bar{Y}_4$	101.8864	50.9758	86.9455	100.0325
$\bar{Y}_5$	101.4854	254.7572	128.4125	100.3228
$\bar{Y}_6$	682.0952	315.0026	426.5987	119.051
$\bar{Y}_7$	1037.744	830.7762	469.7984	105.8133
$\bar{Y}_8$	1032.0990	832.5027	476.1330	108.1413
$\bar{Y}_{proposed}$	1318.0106	953.4940	587.6810	119.2984

Table 4 shows the computed numerical values of percentage relative efficiency (PRE) of the proposed estimator and some existing estimators considered using four datasets. The results obtained by using population; I, II, III and IV, signifies that the proposed estimator ($\hat{\bar{Y}}_{proposed}$) possessed the supreme percentage relative efficiency in comparison with the existing ratio type estimators; $\hat{\bar{Y}}_r$, $\hat{\bar{Y}}_1$, $\hat{\bar{Y}}_2$, $\hat{\bar{Y}}_3$, $\hat{\bar{Y}}_4$, $\hat{\bar{Y}}_5$, $\hat{\bar{Y}}_6$, $\hat{\bar{Y}}_7$ and $\hat{\bar{Y}}_8$. In population I, the proposed estimator ($\hat{\bar{Y}}_{proposed}$) has improved with; 285.91 relative efficiency compared to estimator $\hat{\bar{Y}}_8$; 280.27 relative efficiency compared to estimator $\hat{\bar{Y}}_7$; 635.92 relative efficiency compared to estimator $\hat{\bar{Y}}_6$; 1216.53 relative efficiency compared to estimator $\hat{\bar{Y}}_5$; 1216.12 relative efficiency compared to estimator $\hat{\bar{Y}}_4$; 1186.67 relative efficiency compared to estimator $\hat{\bar{Y}}_3$; 1215.49 relative efficiency compared to estimator $\hat{\bar{Y}}_2$; 1193.62 relative efficiency compared to estimator $\hat{\bar{Y}}_1$. In population II, the proposed estimator ($\hat{\bar{Y}}_{proposed}$) has improved with; 120.99 relative efficiency compared to estimator $\hat{\bar{Y}}_8$; 122.72 relative efficiency compared to estimator $\hat{\bar{Y}}_7$; 638.49 relative efficiency compared to estimator $\hat{\bar{Y}}_6$; 698.74 relative efficiency compared to estimator $\hat{\bar{Y}}_5$; 902.52 relative efficiency compared to estimator $\hat{\bar{Y}}_4$; 733.00 relative efficiency compared to estimator $\hat{\bar{Y}}_3$; 909.46 relative efficiency compared to estimator $\hat{\bar{Y}}_2$; 725.35 relative efficiency compared to estimator $\hat{\bar{Y}}_1$. In population III, the proposed estimator ($\hat{\bar{Y}}_{proposed}$) has improved with; 111.55 relative efficiency compared to estimator $\hat{\bar{Y}}_8$; 117.88 relative efficiency compared to estimator $\hat{\bar{Y}}_7$; 161.08 relative efficiency compared to estimator $\hat{\bar{Y}}_6$; 459.27 relative efficiency compared to estimator $\hat{\bar{Y}}_5$; 500.74 relative efficiency compared to estimator $\hat{\bar{Y}}_4$; 460.84 relative efficiency compared to estimator $\hat{\bar{Y}}_3$; 506.56 relative efficiency compared to estimator $\hat{\bar{Y}}_2$; 466.29 relative efficiency compared to estimator $\hat{\bar{Y}}_1$. In population IV, the proposed estimator ($\hat{\bar{Y}}_{proposed}$) has improved with; 11.16 relative efficiency compared to estimator $\hat{\bar{Y}}_8$; 13.49 relative efficiency compared to estimator $\hat{\bar{Y}}_7$; 0.25 relative efficiency compared to estimator $\hat{\bar{Y}}_6$; 18.98 relative efficiency compared to estimator $\hat{\bar{Y}}_5$; 19.27 relative efficiency compared to estimator $\hat{\bar{Y}}_4$; 19.15 relative efficiency compared to estimator $\hat{\bar{Y}}_3$;

19.25 relative efficiency compared to estimator $\hat{\bar{Y}}_2$; 19.06 relative efficiency compared to estimator $\hat{\bar{Y}}_1$. Therefore, based on the criteria of percentage relative efficiency, the proposed ratio type estimator performed better and is more efficient than the existing ratio type estimators considered.

4.0. Conclusion

A new ratio type estimator in simple random sampling without replacement by using information on an auxiliary variable is proposed in this study. A power transformation with unknown weight was used in the formation of the proposed estimator. The properties such as bias and mean square error (MSE) expression of the proposed estimator are derived and tested using four real data sets. Based on the result obtained in Table 4, the proposed ratio type estimator performed better than some existing ratio type estimators considered in this study. Therefore, the proposed estimator is recommended for use in practical application in estimating population mean of the study variable.

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