Journal of Science and Technology Research

Journal homepage: www.nipesjournals.org.ng

Modified Estimators of Population Mean Using Robust Multiple Regression Methods

[a*]A. Audu, [b]O. O. Ishaq, [a]J. O. Muili, [a]A. Abubakar, [c]A. Rashida, [d]K. A. Akintola, [a]U. Isah

[a]Department of Mathematics, Usmanu Danfidiyo University, Sokoto, Nigeria.
[b]Department of Statistics, Kano State Uni. Sci. and Tech, Wudil, Nigeria
[c]State College of Basic and Remedial Studies, Sokoto, Nigeria
[d]Department of Statistics, Oyo State college of Agric.& Tech., Igboora, Nigeria
[a*]ahmed.audu@udusok.edu.ng, [b]babinton4u@gmail.com, [a]jamiunice@gmail.com, [a]daddyabduldabai@gmail.com, [c]ummuhalima2@gmial.com, [d]onikolakafayat@gmail.com, [a]usmanisahyabo@gmail.com

Article Info

Received 10 Sept. 2020
Revised 22 Sept. 2020
Accepted 23 Sept. 2020
Available online 26 Nov. 2020

Keywords: *Estimators, Auxiliary variables, Multiple Regression, Outliers, Efficiency.*

https://doi.org/10.37933/nipes/2.4.2020.2

https://nipesjournals.org.ng

Abstract

Efficiency of estimators can be improved by using the information of multi-auxiliary variables associated to the study variable [1]. [2] and [3] suggested robust estimators with single auxiliary variable which are not applicable to situation when study variable is associated with independent multi-auxiliary variables. In this paper, finite population mean modified estimator with independent multi-auxiliary variables has been proposed. The mean squared error (MSE) of the proposed estimator was derived up to second degree approximation. The empirical study was conducted and the results revealed that proposed estimators were more efficient.

1. Introduction

Supplementary variables associated with the study variables have been identified to be helpful in improving the efficiency of ratio, product and regression estimators both at planning and estimation stages. Authors such as [1], [2], [3], [4], [5] and [6] have worked extensively in this direction. However, the efficiency of these estimators may be affected when data under study is characterized by outliers or leverages. Authors like [7], [8] and [9] have studied several robust ratio estimators to solve the problem of outliers. However, none of the existing studies on robust ratio estimators considered situations when study variables are associated with independent multi-auxiliary variables like expenditure with salary and teacher-pupils ratio, GDP with inflation rate, export rate and import rate, obesity with body weight, height and blood pressure etc in estimators which use robust regression methods. Therefore, in this study some ratio estimators with multiple auxiliary independent variables using robust multiple regression methods have been suggested. [2] extended the work of [10] by inclusion of some slopes' coefficient of other robust regression estimators like [11], [12], [13] and LAD [14] in addition to Huber-M [15] used by [10] and this inclusion leads to new estimators of population mean in the presence of outliers given as follows:

$$t_{ZB1} = \frac{\bar{y} + \alpha_{rbst(zb)}\left(\bar{X} - \bar{x}\right)}{\bar{x}}\bar{X} \qquad (1)$$

$$t_{ZB2} = \frac{\bar{y} + \alpha_{rbst(zb)}\left(\bar{X} - \bar{x}\right)}{\bar{x} + C_x}\left(\bar{X} + C_x\right) \qquad (2)$$

$$t_{ZB3} = \frac{\bar{y} + \alpha_{rbst(zb)}\left(\bar{X} - \bar{x}\right)}{\left(\bar{x} + \beta_2(x)\right)}\left(\bar{X} + \beta_2(x)\right) \quad (3)$$

$$t_{ZB4} = \frac{\bar{y} + \alpha_{rbst(zb)}\left(\bar{X} - \bar{x}\right)}{\left(\bar{x}\beta_2(x) + C_x\right)}\left(\bar{X}\beta_2(x) + C_x\right) \quad (4)$$

$$t_{ZB5} = \frac{\bar{y} + \alpha_{rbst(zb)}\left(\bar{X} - \bar{x}\right)}{\left(\bar{x}C_x + \beta_2(x)\right)}\left(\bar{X}C_x + \beta_2(x)\right) \quad (5)$$

where $C_x, \beta_2(x)$ and $b_{rob(zb)}$ are population coefficients of variation, kurtosis and robust regression methods.

$$MSE\left(t_{ZBi}\right) \cong \theta\left(S_y^2 + \left(\phi_{rbst(zb)} + R\lambda_{KCi}\right)^2 S_x^2 - 2\left(\phi_{rbst(zb)} + R\lambda_{KCi}\right)S_{xy}\right) \quad (6)$$

where $i = 1, 2, ..., 5$, $\lambda = (1 - f)/n$, $f = n/N$, n is the sample size, N is the population size, $B_{rob(zb)}$ are coefficients of slope obtained from Tukey-M, Hampel-M, Huber-M, LMS and LAD methods, $S_y^2 = \frac{1}{N-1}\sum_{j=1}^{N}\left(y_j - \bar{Y}\right)^2$, $S_x^2 = \frac{1}{N-1}\sum_{j=1}^{N}\left(x_j - \bar{X}\right)^2$, $S_{xy} = \frac{1}{N-1}\sum_{j=1}^{N}\left(y_j - \bar{Y}\right)\left(x_j - \bar{X}\right)$,

$R = \frac{\bar{Y}}{\bar{X}} = \lambda_{KC(1)}, \lambda_{KC(2)} = \frac{\bar{X}}{\bar{X} + C_x}$, $\lambda_{KC(3)} = \frac{\bar{X}}{\bar{X} + \beta_2(x)}$, $\lambda_{KC(4)} = \frac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x) + C_x}$, $\lambda_{KC(5)} = \frac{\bar{X}C_x}{\bar{X}C_x + \beta_2(x)}$.

[3] adopted transformation techniques to the work of [2] and then proposed a general form of estimators as:

$$t_Z = \mu\frac{\bar{y} + \alpha_{rbst(zb)}\left(\bar{X} - \bar{x}\right)}{\bar{x}}\bar{X} + (1 - \mu)\frac{\bar{y} + \alpha_{rbst(zb)}\left(\bar{X} - \bar{x}\right)}{\left(\bar{x}w_1 + w_2\right)}\left(\bar{X}w_1 + w_2\right) \quad (7)$$

where μ is a real constant to be determined such that the MSE of t_{Zi} is minimum. $w_1 \neq 0$ and w_2 are either real number or the function of known parameters like C_x *and* $\beta_2(x)$.

$$MSE\left(t_{zm}\right) \cong \theta\left[S_y^2 + \psi_m^2 S_x^2 - 2\psi_m S_{xy}\right], \quad m = 1, 2, ..., 4 \quad (8)$$

where $\psi_m = \mu\left(\phi_{rbst(zb)} + R\right) + (1 - \mu)\left(\phi_{rbst(zb)} + \lambda_{KC(m+1)}\right), \mu = \frac{B_{reg} + \phi_{rbst(zb)} + \lambda_{KC(m+1)}}{\lambda_{KC(m+1)} - R}$

2. Methodology

2.1 Suggested estimators

Having studied the work of [3], the suggested estimator is presented in general form as:

$$t_p = \nu\frac{\left(\bar{y} + \sum_{j=1}^{r}\alpha_{rbst(zb)j}\left(\bar{X}_j - \bar{x}_j\right)\right)}{\prod_{j=1}^{r}\bar{x}_j}\prod_{j=1}^{r}\bar{X}_j + (1 - \nu)\frac{\bar{y} + \sum_{j=1}^{r}\alpha_{rbst(zb)j}\left(\bar{X}_j - \bar{x}_j\right)}{\prod_{j=1}^{r}\left(A_j\bar{x}_j + B_j\right)}\prod_{i=1}^{r}\left(A_j\bar{X}_j + B_j\right) \quad (9)$$

where A_j and B_j are either population coefficients of variation or kurtosis of j^{th} independent auxiliary variables $X_i, j = 1, 2, ..., r$, but $A_j \neq B_j$.

To obtain the mean squared error of t_p, the error terms $e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$ and $e_j = \frac{\bar{x}_j - \bar{X}_j}{\bar{X}_j}$ are defined such that the expectations are given as:

$$\left.\begin{array}{l} E\left(e_0\right)=E\left(e_j\right)=0, E\left(e_0^2\right)=\theta C_y^2, E\left(e_j^2\right)=\theta C_{x_j}^2 \\ E\left(e_0 e_j\right)=\theta \rho_{yx_j} C_y C_{x_j}, E\left(e_j e_k\right)=0 \,\forall\, j \neq k=1,2,\ldots,r \end{array}\right\} \quad (10)$$

Express t_p in terms of e_0 and e_j, we have

$$\begin{array}{l} t_p=\nu\left(\bar{Y}\left(1+e_0\right)-\sum_{j=1}^{r} \alpha_{rbst(zb)j} \bar{X}_j e_j\right)\left(\prod_{j=1}^{r} \bar{X}_j\right) / \prod_{j=1}^{r}\left(1+e_j\right) \bar{X}_j \\ \quad +(1-\nu)\left(\bar{Y}\left(1+e_0\right)-\sum_{j=1}^{r} \alpha_{rbst(zb)j} \bar{X}_j e_j\right) \prod_{j=1}^{r}\left(A_j \bar{X}_j+B_j\right) / \prod_{j=1}^{r}\left(A_j\left(1+e_j\right) \bar{X}_j+B_j\right) \end{array} \quad (11)$$

$$\begin{array}{l} t_p=\nu\left(\bar{Y}\left(1+e_0\right)-\sum_{j=1}^{r} \alpha_{rbst(zb)j} \bar{X}_j e_j\right) \prod_{j=1}^{r}\left(1+e_j\right)^{-1} \\ \quad +(1-\nu)\left(\bar{Y}\left(1+e_0\right)-\sum_{j=1}^{r} \alpha_{rbst(zb)j} \bar{X}_i e_i\right) \prod_{j=1}^{r}\left(1+\varphi_j e_j\right)^{-1} \end{array} \quad (12)$$

where $\varphi_j=A_j \bar{X}_j /\left(A_j \bar{X}_j+B_j\right)$

Simplify (12) up to first order approximation, we have

$$\begin{array}{l} t_p=\nu\left(\bar{Y}-\bar{Y} \sum_{j=1}^{r} e_j+\bar{Y} \sum_{j=1}^{r} e_j^2+\bar{Y} \sum_{j \neq k=1}^{r} e_j e_k+\bar{Y} e_0-\bar{Y} \sum_{j=1}^{r} e_0 e_j-\sum_{j=1}^{r} \alpha_{rbst(zb)j} \bar{X}_j e_j\right. \\ \quad \left.+\sum_{j=1}^{r} \alpha_{rbst(zb)j} \bar{X}_j e_j^2\right)+(1-\nu)\left(\bar{Y}-\bar{Y} \sum_{j=1}^{r} \varphi_j e_j+\bar{Y} \sum_{j=1}^{r} \varphi_j^2 e_j^2+\bar{Y} \sum_{j \neq k=1}^{r} \varphi_j \varphi_k e_j e_k\right. \\ \quad \left.+\bar{Y} e_0-\bar{Y} \sum_{j=1}^{r} \varphi_j e_0 e_j-\sum_{j=1}^{r} \alpha_{rbst(zb)j} \bar{X}_j e_j+\sum_{j=1}^{r} \alpha_{rbst(zb)j} \varphi_j \bar{X}_j e_j^2\right) \end{array} \quad (13)$$

$$\begin{array}{l} t_p-\bar{Y}=\bar{Y} e_0-\sum_{j=1}^{r}\left(\bar{Y}\left(\nu+(1-\nu) \varphi_j\right)+\alpha_{rbst(zb)j} \bar{X}_j\right) e_j-\bar{Y} \sum_{j=1}^{r}\left(\nu+(1-\nu) \varphi_j\right) e_0 e_j \\ \quad +\sum_{j=1}^{r}\left(\bar{Y}\left(\nu+(1-\nu) \varphi_j^2\right)+\alpha_{rbst(zb)j} \bar{X}_j \varphi_j\right) e_j^2+\textit{terms with cross product of } X_j^s \end{array} \quad (14)$$

Take expectation of (14) and apply the results of (10), we obtained of $Bias\left(t_p\right)$ as;

$$Bias\left(t_p\right)=\theta\left(\sum_{j=1}^{r} C_{xj}^2\left(\bar{Y}\left(\nu+(1-\nu) \varphi_j^2\right)+\alpha_{rbst(zb)j} \bar{X}_j \varphi_j\right)-\bar{Y} \sum_{j=1}^{r} \rho_{yx_j} C_y C_{x_j}\left(\nu+(1-\nu) \varphi_j\right)\right) \quad (15)$$

Similarly, square both sides of (14), take expectation and apply the results of (10), we obtained of $MSE\left(t_p\right)$ as;

$$MSE\left(t_p\right)=\theta\left(S_y^2+\sum_{j=1}^{r} S_{xj}^2\left(R_j\left(\nu+(1-\nu) \varphi_j\right)+\alpha_{rbst(zb)j}\right)^2-2 \sum_{j=1}^{r} S_{yxi}\left(R_j\left(\nu+(1-\nu) \varphi_j\right)+\alpha_{rbst(zb)j}\right)\right) \quad (16)$$

where $R_j=\bar{Y} / \bar{X}_j$

To obtain the expression for ν for which $MSE\left(t_p\right)$ is at minimum, we differentiate partially (16) with respect to ν, equate to zero and solve for ν. That is,

$$\frac{\partial\left(MSE\left(t_p\right)\right)}{\partial \nu}=0 \quad (17)$$

$$\nu=-\frac{\sum_{j=1}^{r}R_j\left(1-\varphi_j\right)\left(S_{x_j}^2\left(\alpha_{rbst(zb)j}+R_j\varphi_j\right)-S_{yx_j}\right)}{\sum_{j=1}^{r}S_{x_j}^2R_j^2\left(1-\varphi_j\right)^2}=-\frac{D_{yx}}{D_x} \tag{18}$$

Substitute (18) in (16), we obtain the minimum MSE of t_p as

$$MSE\left(t_p\right)_{\min}=\theta\left(S_y^2+\sum_{j=1}^{r}\left(\alpha_{rbst(zb)j}+R_j\varphi_j\right)\left(\left(\alpha_{rbst(zb)j}+R_j\varphi_j\right)S_{x_j}^2-2S_{yx_j}\right)-\frac{D_{yx}^2}{D_x}\right) \tag{19}$$

If $A_j=1$ and $B_j=C_{xj}$, then the suggested estimator t_{p1} becomes;

$$t_{p1}=\nu\frac{\left(\bar{y}+\sum_{j=1}^{r}\alpha_{rbst(zb)j}\left(\bar{X}_j-\bar{x}_j\right)\right)}{\prod_{j=1}^{r}\bar{x}_j}\prod_{j=1}^{r}\bar{X}_j+\left(1-\nu\right)\frac{\bar{y}+\sum_{j=1}^{r}\alpha_{rbst(zb)j}\left(\bar{X}_j-\bar{x}_j\right)}{\prod_{j=1}^{r}\left(\bar{x}_j+C_{xj}\right)}\prod_{i=1}^{r}\left(\bar{X}_j+C_{xj}\right) \tag{20}$$

The MSE of t_{p1} is equivalent to $MSE\left(t_p\right)$ but φ_j is replaced by $\bar{X}_j/\left(\bar{X}_j+C_{x_j}\right)$ and the expression for optimal value of ν denoted by ν_1 is obtained as in (21)

$$\nu_1=-\frac{\sum_{j=1}^{r}R_j\frac{C_{x_j}}{\bar{X}_j+C_{x_j}}\left(S_{x_j}^2\left(\alpha_{rbst(zb)j}+R_j\frac{\bar{X}_j}{\bar{X}_j+C_{x_j}}\right)-S_{yx_j}\right)}{\sum_{j=1}^{r}S_{x_j}^2R_j^2\left(\frac{C_{x_j}}{\bar{X}_j+C_{x_j}}\right)^2} \tag{21}$$

If $A_j=1$ and $B_j=\beta_2\left(x\right)_j$, then the suggested estimator t_p becomes;

$$t_{p2}=\nu\frac{\left(\bar{y}+\sum_{j=1}^{r}\alpha_{rbst(zb)j}\left(\bar{X}_j-\bar{x}_j\right)\right)}{\prod_{j=1}^{r}\bar{x}_j}\prod_{j=1}^{r}\bar{X}_j+\left(1-\nu\right)\frac{\bar{y}+\sum_{j=1}^{r}\alpha_{rbst(zb)j}\left(\bar{X}_j-\bar{x}_j\right)}{\prod_{j=1}^{r}\left(\bar{x}_j+\beta_2\left(x\right)_j\right)}\prod_{i=1}^{r}\left(\bar{X}_j+\beta_2\left(x\right)_j\right) \tag{22}$$

The MSE of t_{p2} is equivalent to $MSE\left(t_p\right)$ but φ_j is replaced by $\bar{X}_j/\left(\bar{X}_j+\beta_2\left(x\right)_j\right)$ and the expression for optimal value of ν denoted by ν_2 is obtained as in (23)

$$\nu_2=-\frac{\sum_{j=1}^{r}R_j\frac{\beta_2\left(x\right)_j}{\bar{X}_j+\beta_2\left(x\right)_j}\left(S_{x_j}^2\left(\alpha_{rbst(zb)j}+R_j\frac{\bar{X}_j}{\bar{X}_j+\beta_2\left(x\right)_j}\right)-S_{yx_j}\right)}{\sum_{j=1}^{r}S_{x_j}^2R_j^2\left(\frac{\beta_2\left(x\right)_j}{\bar{X}_j+\beta_2\left(x\right)_j}\right)^2} \tag{23}$$

If $A(x)_j=\beta_2\left(x\right)_j$ and $B(x)_j=C_{x_j}$, then the suggested estimator t_p becomes;

$$t_{p3}=\nu\frac{\left(\bar{y}+\sum_{j=1}^{r}\alpha_{rbst(zb)j}\left(\bar{X}_j-\bar{x}_j\right)\right)}{\prod_{j=1}^{r}\bar{x}_j}\prod_{j=1}^{r}\bar{X}_j+\left(1-\nu\right)\frac{\bar{y}+\sum_{j=1}^{r}\alpha_{rbst(zb)j}\left(\bar{X}_j-\bar{x}_j\right)}{\prod_{j=1}^{r}\left(\beta_2\left(x\right)_j\bar{x}_j+C_{xj}\right)}\prod_{i=1}^{r}\left(\beta_2\left(x\right)_j\bar{X}_j+C_{xj}\right) \tag{24}$$

The MSE of t_{p3} is equivalent to $MSE\left(t_p\right)$ but φ_j is replaced by $\beta_2\left(x\right)_j \bar{X}_j / \left(\beta_2\left(x\right)_j \bar{X}_j + C_{x_j}\right)$ and the expression for optimal value of ν denoted by ν_3 is obtained as in (25)

$$\nu_3 = -\frac{\sum_{j=1}^{r} R_j \frac{C_{x_j}}{\beta_2\left(x\right)_j \bar{X}_j + C_{x_j}} \left(S_{x_j}^2 \left(\alpha_{rbst(zb)j} + R_j \frac{\beta_2\left(x\right)_j \bar{X}_j}{\beta_2\left(x\right)_j \bar{X}_j + C_{x_j}} \right) - S_{yx_j} \right)}{\sum_{j=1}^{r} S_{x_j}^2 R_j^2 \left(\frac{C_{x_j}}{\beta_2\left(x\right)_j \bar{X}_j + C_{x_j}} \right)^2} \tag{25}$$

If $A_j = C_{x_j}$ and $B_j = \beta_2\left(x\right)_j$, then the suggested estimator t_p becomes;

$$t_{p4} = \nu \frac{\left(\bar{y} + \sum_{j=1}^{r} \alpha_{rbst(zb)j} \left(\bar{X}_j - \bar{x}_j \right) \right)}{\prod_{j=1}^{r} \bar{x}_j} \prod_{j=1}^{r} \bar{X}_j + (1-\nu) \frac{\bar{y} + \sum_{j=1}^{r} \alpha_{rbst(zb)j} \left(\bar{X}_j - \bar{x}_j \right)}{\prod_{j=1}^{r} \left(C_{x_j} \bar{x}_j + \beta_2\left(x\right)_j \right)} \prod_{i=1}^{r} \left(C_{x_j} \bar{X}_j + \beta_2\left(x\right)_j \right) \tag{26}$$

The MSE of t_{p4} is equivalent to $MSE\left(t_p\right)$ but φ_j is replaced by $C_{x_j} \bar{X}_j / \left(C_{x_j} \bar{X}_j + \beta_2\left(x\right)_j \right)$ and the expression for optimal value of ν denoted by ν_4 is obtained as in (27)

$$\nu_4 = -\frac{\sum_{j=1}^{r} R_j \frac{\beta_2\left(x\right)_j}{C_{x_j} \bar{X}_j + \beta_2\left(x\right)_j} \left(S_{x_j}^2 \left(\alpha_{rbst(zb)j} + R_j \frac{C_{x_j} \bar{X}_j}{C_{x_j} \bar{X}_j + \beta_2\left(x\right)_j} \right) - S_{yx_j} \right)}{\sum_{j=1}^{r} S_{x_j}^2 R_j^2 \left(\frac{\beta_2\left(x\right)_j}{C_{x_j} \bar{X}_j + \beta_2\left(x\right)_j} \right)^2} \tag{27}$$

2.2 Efficiency Comparisons

In this section, efficiency of t_p is compared to the efficiency of t_{Zi} and t_{ZBi} theoretically and the following conditions were established.

$$MSE\left(t_{ZBi}\right) - MSE\left(t_p\right) > 0 \tag{28}$$

$$\begin{aligned} D_{yx}^2 / D_x > & \left(\alpha_{rbst(zb)} + R\lambda_{KCi} \right)^2 S_x^2 - \sum_{j=1}^{r} \left(\alpha_{rbst(zb)j} + R_j \varphi_j \right)^2 S_{x_j}^2 \\ & - 2\left(\alpha_{rbst(zb)} + R\lambda_{KCi} \right) S_{yx} + 2\sum_{j=1}^{r} \left(\alpha_{rbst(zb)j} + R_j \varphi_j \right) S_{yx_j} \end{aligned} \tag{29}$$

$$MSE\left(t_{Zm}\right) - MSE\left(t_p\right) > 0 \tag{30}$$

$$\begin{aligned} \frac{D_{yx}^2}{D_x} - \frac{D_{yx}^2}{D_x} > & \left(\alpha_{rbst(zb)} + R\lambda_{KC(m+1)} \right)^2 S_x^2 - \sum_{j=1}^{r} \left(\alpha_{rbst(zb)j} + R_j \varphi_j \right)^2 S_{x_j}^2 \\ & - 2\left(\alpha_{rbst(zb)} + R\lambda_{KC(m+1)} \right) S_{yx} + 2\sum_{j=1}^{r} \left(\alpha_{rbst(zb)j} + R_j \varphi_j \right) S_{yx_j} \end{aligned} \tag{31}$$

If conditions (29) and (31) are satisfied, t_p is be more efficiency than t_{ZBi} and t_{Zi} respectively.

3. Results and Discussion

In this section, simulation study is conducted to assess the performance of the suggested estimators with respect to [2] and [3] estimators. The steps for stimulation are as follows;

Step1: n sample of size 30,000 from normal population is drawn without replacement using simple random sampling scheme as

$X_1 \sim N(12,2), X_2 \sim N(18,4)$ and $\varepsilon \sim N(0,1)$

Step2: construct regression models as:

$$Y_{HUBM} = \alpha_0 + \alpha_{robst1} X_1 + \alpha_{rbst2} X_2 + \varepsilon \quad (32)$$

where $\alpha_{rbst}, i = 1,2$ are regression coefficient of Huber-M, Tukey-M, Hampel-M, LTS and LAD robust estimators.

Step 3: calculate MSE as given below;

$$MSE(\hat{\theta}) = \frac{1}{30000} \sum_{j=1}^{30000} (\hat{\theta}_j - \theta)^2 \quad (33)$$

where $\hat{\theta}_j$ is the estimated mean with sample sizes $n = 20, 50, 100$ and θ is the population mean.

Table 1: MSE of $t_{ZB1}, t_{ZB2}, t_{ZB3}, t_{ZB4}, t_{ZB5}, t_{Zi}$ and t_{AIJi} under Huber-M, Hampel-M, LTS and LAD

Estimators	Huber-M	Hampel-M	LTS	LAD
	$n = 20$			
t_{ZB1}	0.05001653	0.05001497	0.05001094	0.05002385
t_{ZB2}	0.05001781	0.05001614	0.05001177	0.05002554
t_{ZB3}	0.05001696	0.05001536	0.05001122	0.05002442
t_{ZB4}	0.05001721	0.05001559	0.05001138	0.05002475
t_{ZB5}	0.05001656	0.050015	0.05001096	0.05002389
t_Z	0.05000719	0.05000719	0.05000719	0.05000719
t_p	0.05000365	0.05000365	0.05000365	0.05000365
	$n = 50$			
t_{ZB1}	0.01998659	0.01998597	0.01998436	0.01998952
t_{ZB2}	0.0199871	0.01998643	0.01998469	0.01999019
t_{ZB3}	0.01998677	0.01998613	0.01998447	0.01998975
t_{ZB4}	0.01998686	0.01998622	0.01998453	0.01998988
t_{ZB5}	0.01998661	0.01998598	0.01998437	0.01998953
t_Z	0.01998286	0.01998286	0.01998286	0.01998286
t_p	0.01998144	0.01998144	0.01998144	0.01998144
	$n = 100$			
t_{ZB1}	0.009976613	0.009976301	0.009975498	0.009978072
t_{ZB2}	0.009976867	0.009976534	0.009975662	0.009978409
t_{ZB3}	0.009976699	0.00997638	0.009975553	0.009978187
t_{ZB4}	0.009976749	0.009976425	0.009975585	0.009978253
t_{ZB5}	0.009976619	0.009976307	0.009975502	0.00997808
t_Z	0.009974749	0.009974749	0.009974749	0.009974749
t_p	0.009974043	0.009974043	0.009974043	0.009974043

Table 2: PRE of $t_{ZB1}, t_{ZB2}, t_{ZB3}, t_{ZB4}, t_{ZB5}, t_{Zi}$ and t_{AIJi} under Huber-M, Hampel-M, LTS and LAD

Estimators	Huber-M	Hampel-M	LTS	LAD
	$n=20$			
t_{ZB1}	100	100	100	100
t_{ZB2}	99.9974409	99.9976608	99.9983404	99.9966217
t_{ZB3}	99.9991403	99.9992202	99.9994401	99.9988606
t_{ZB4}	99.9986405	99.9987604	99.9991202	99.9982009
t_{ZB5}	99.99994	99.99994	99.99996	99.99992
t_Z	100.018677	100.015558	100.007499	100.033315
t_p	100.025758	100.022638	100.014579	100.040397
	$n=50$			
t_{ZB1}	250.250443	250.250401	250.250396	250.250381
t_{ZB2}	250.244057	250.244641	250.246264	250.241994
t_{ZB3}	250.248189	250.248397	250.249018	250.247502
t_{ZB4}	250.247062	250.24727	250.248267	250.245874
t_{ZB5}	250.250193	250.250275	250.250271	250.250256
t_{Zi}	250.297155	250.289348	250.269181	250.333786
t_p	250.314942	250.307135	250.286966	250.351576
	$n=100$			
t_{ZB1}	501.337779	501.337821	501.337778	501.337834
t_{ZB2}	501.325015	501.326112	501.329536	501.320902
t_{ZB3}	501.333457	501.333851	501.335014	501.332056
t_{ZB4}	501.330945	501.331589	501.333406	501.32874
t_{ZB5}	501.337477	501.337519	501.337577	501.337432
t_Z	501.431465	501.415825	501.375423	501.50485
t_p	501.466958	501.451317	501.410912	501.540348

Table 3: Efficiency conditions of t_p over $t_{ZB1}, t_{ZB2}, t_{ZB3}, t_{ZB4}, t_{ZB5}, t_Z$ under Huber-M, Hampel-M, LTS and LAD

Estimators	Huber-M	Hampel-M	LTS	LAD
	$n=20$			
t_{ZB1}	$\varpi > 1.3984e\text{-}4$	$\varpi > 1.1859e\text{-}4$	$\varpi > 2.8067e\text{-}5$	$\varpi > 2.8610e\text{-}4$
t_{ZB2}	$\varpi > 1.2653e\text{-}4$	$\varpi > 1.3194e\text{-}4$	$\varpi > 4.4535e\text{-}5$	$\varpi > 3.1989e\text{-}4$
t_{ZB3}	$\varpi > 1.4848e\text{-}4$	$\varpi > 1.1648e\text{-}4$	$\varpi > 3.3578e\text{-}5$	$\varpi > 2.9760e\text{-}4$
t_{ZB4}	$\varpi > 1.5346e\text{-}4$	$\varpi > 1.2104e\text{-}4$	$\varpi > 3.6789e\text{-}5$	$\varpi > 3.0421e\text{-}4$
t_{ZB5}	$\varpi > 1.4047e\text{-}4$	$\varpi > 1.1917e\text{-}4$	$\varpi > 2.8468e\text{-}5$	$\varpi > 2.8695e\text{-}4$
t_Z	$\varpi > 4.7015e\text{-}5$	$\varpi > 4.7015e\text{-}5$	$\varpi > 4.7015e\text{-}5$	$\varpi > 4.7015e\text{-}5$
t_p	$\varpi = 1.1778e\text{-}4$	$\varpi = 1.1778e\text{-}4$	$\varpi = 1.1778e\text{-}4$	$\varpi = 1.1778e\text{-}4$
	$n=50$			

t_{ZB1}	$\varpi > 1.3984e-4$	$\varpi > 1.1859e-4$	$\varpi > 2.8067e-5$	$\varpi > 2.8610e-4$
t_{ZB2}	$\varpi > 1.6535e-4$	$\varpi > 1.3194e-4$	$\varpi > 4.4535e-5$	$\varpi > 3.1989e-4$
t_{ZB3}	$\varpi > 1.4848e-4$	$\varpi > 1.2648e-4$	$\varpi > 3.3578e-4$	$\varpi > 2.9760e-4$
t_{ZB4}	$\varpi > 1.5346e-4$	$\varpi > 1.2104e-4$	$\varpi > 3.6789e-5$	$\varpi > 3.0421e-4$
t_{ZB5}	$\varpi > 1.4047e-4$	$\varpi > 1.1916e-4$	$\varpi > 2.8468e-5$	$\varpi > 2.8695e-4$
t_Z	$\varpi > 4.7015e-5$	$\varpi > 4.7015e-5$	$\varpi > 4.7015e-5$	$\varpi > 4.7015e-5$
t_p	$\varpi = 1.1778e-4$	$\varpi = 1.1778e-4$	$\varpi = 1.1778e-4$	$\varpi = 1.1778e-4$
	$n = 100$			
t_{ZB1}	$\varpi > 1.3984e-4$	$\varpi > 1.1858e-4$	$\varpi > 2.8067e-5$	$\varpi > 2.8612e-4$
t_{ZB2}	$\varpi > 1.6535e-4$	$\varpi > 1.3194e-4$	$\varpi > 4.4535e-5$	$\varpi > 3.1989e-4$
t_{ZB3}	$\varpi > 1.4848e-4$	$\varpi > 1.2648e-4$	$\varpi > 3.3578e-5$	$\varpi > 2.9760e-4$
t_{ZB4}	$\varpi > 1.5346e-4$	$\varpi > 1.2104e-4$	$\varpi > 3.6789e-5$	$\varpi > 3.0421e-4$
t_{ZB5}	$\varpi > 1.4047e-4$	$\varpi > 1.1917e-4$	$\varpi > 2.8468e-4$	$\varpi > 2.8695e-4$
t_Z	$\varpi > 4.7015e-5$	$\varpi > 4.7015e-5$	$\varpi > 4.7015e-5$	$\varpi > 4.7015e-5$
t_p	$\varpi = 1.1778e-4$	$\varpi = 1.1778e-4$	$\varpi = 1.1778e-4$	$\varpi = 1.1778e-4$

Tables 1 and 2 showed MSE and PRE proposed, [2] and [3] estimators for sample sizes 20, 50 and 100 respectively. The results of the table revealed that the proposed estimator has minimum MSE and higher PRE compared to all methods of robust estimators considered in the study. The results of efficiency conditions are presented in Table 3 and the results revealed that all the conditions for which the proposed estimator superseded others are satisfied.

4. Conclusion

From the empirical results, it is obtained that the proposed estimator is more efficient than estimators suggested by [2] and [3].

Nomenclature

N	Population size
n	Sample size
Y	Variable of study or interest
X	Auxiliary variable
$\bar{Y}, \bar{X}$	Population means of Y, X
R	Respondents group
$\bar{y},\ \bar{x}$	Sample means of Y, X
S_Y^2, S_X^2	Population variances of Y, X
C_Y, C_X	Population coefficients of variation of Y, X
ρ_{YX}	Population correlation coefficient of Y, X
$\beta_2(x)$	Population kurtosis of X.

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