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On the Class of Exponential-Type Imputation Estimators of Population Mean with Known Population Mean of Auxiliary Variable

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Abstract

This paper considered a new class of exponential-type imputation scheme and the point estimator of the proposed scheme for estimating population mean was derived. Members of the derived estimator for different pairs of constants κ_1 and K2 ((k_1=1, k_2=1), (k_1=1, k_2=-1), (k_1=-1, k_2=1), (k1=-1, k_2=-1)) were obtained. The properties (bias and MSE) of the proposed estimator were derived up to second degree approximation using Taylor's series procedure. Efficiency conditions for which the new class of estimators outperformed their counterparts were also derived by theoretically compared their MSEs. Empirical study was conducted through simulations using four probability distributions. The numerical results revealed that the members of the proposed estimator have less biases, smaller MSEs and larger PREs compared to other estimators in the study.

1. Introduction

Non-response is one of the challenges statisticians or surveyors do encountered during the course of data or information collection due to absence, refusal or inaccessibility of some respondents. This poses some problems during data compilation, computations and estimations stages. The normal procedure is to go back to the field to collect the missing information (that is, call-back). However, this approach incurred additional cost, time and logistics. To solve this problem, several imputation schemes for estimating the responses of non-respondents have been developed by several researchers. These researchers include [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11].
In this paper, we have proposed some new imputation schemes under the assumption that the population mean of auxiliary variable is known, derived their estimators for population mean and compared their efficiency with that of some similar existing estimators.

1.2 Some Existing Imputation Estimators

Under the Mean Method of imputation, values found missing are to be replaced by the mean of the rest of observed values. The study variable thereafter, takes the form given as,

$$y_{.i} = \begin{cases} y_i & \text{if} \quad i \in R \\ \bar{y}_r & \text{if} \quad i \in R^c \end{cases} \tag{1}$$

Under the method of imputation, sample mean denoted by t_0, bias and MSE of t_0 are given by (2), (3) and (4) respectively as

$$t_0 = \frac{1}{r}\sum_{i\in R} y_i = \bar{y}_r \tag{2}$$

$$Bias(t_0) = 0 \tag{3}$$

$$MSE(t_0) = \lambda_{r,N}\bar{Y}^2 C_Y^2 \tag{4}$$

where $\lambda_{r,N} = \frac{1}{r} - \frac{1}{N}$, $C_Y = \frac{S_Y}{\bar{Y}}$, $S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2$, $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$

Ratio imputation scheme for population mean estimators with known population mean of auxiliary variable is given as

$$y_{.i} = \begin{cases} y_i & i \in R \\ \frac{\bar{y}_r}{n-r}\left(n\left(\frac{\bar{X}}{\bar{x}_r}\right) - r\right) & i \in R^C \end{cases} \tag{5}$$

Under the above method, the resultant estimator of the population mean $\bar{Y}$ given as

$$t_1 = \bar{y}_r\left(\frac{\bar{X}}{\bar{x}_r}\right) \tag{6}$$

The bias, mean square error and minimum mean square error of t_1 respectively are given by:

$$Bias\left(t_1\right) = \bar{Y}\lambda_{r,N}\left(C_X^2 - \rho C_Y C_X\right) \tag{7}$$

$$MSE\left(t_1\right) = \bar{Y}^2\lambda_{r,N}\left(C_Y^2 + C_X^2 - 2\rho_{XY} C_X C_Y\right) \tag{8}$$

[7] proposed Exponential-Type Compromised Imputation method as

$$y_{.i} = \begin{cases} \nu\frac{n}{r} y_i + (1-\nu)\bar{y}_r \exp\left(\frac{\bar{X} - \bar{x}_r}{\bar{X} + \bar{x}_r}\right) & \text{if} \ i \in R \\ (1-\nu)\bar{y}_r \exp\left(\frac{\bar{X} - \bar{x}_r}{\bar{X} + \bar{x}_r}\right) & \text{if} \ i \in R^c \end{cases} \tag{9}$$

The point estimator for the population mean in the case of Exponential-Type Compromised Imputation method is given as:

$$t_2 = \nu\bar{y}_r + (1-\nu)\bar{y}_r \exp\left(\frac{\bar{X} - \bar{x}_r}{\bar{X} + \bar{x}_r}\right) \tag{10}$$

The bias and MSE of t_2 up first order approximation is given by (11) and (12) respectively as;

$$Bias\left(t_2\right) = (1-\nu)\lambda_{r,N}\bar{Y}\left(\frac{3}{8}C_X^2 - \frac{1}{2}C_{YX}\right) \tag{11}$$

$$MSE\left(\lambda_2\right) = \varphi_{r,N}\bar{Y}^2\left(C_Y^2 + \frac{(1-\nu)^2}{4}C_X^2 - (1-\nu)C_{YX}\right) \tag{12}$$

where $\nu = 1 - 2C_{YX} / C_X^2$.

[10] suggested imputation scheme defined in (13)

$$y_{.i}=\begin{cases} z\frac{n}{r}y_i\frac{\bar{X}}{\bar{x}_r}+(1-z)\bar{y}_r\frac{\bar{x}_r}{\bar{X}} & i\in R \\ (1-z)\bar{y}_r\frac{\bar{x}_r}{\bar{X}} & i\in R^C \end{cases} \tag{13}$$

The point estimator of population mean $\bar{Y}$ under proposed method of imputation is:

$$t_3=\bar{y}_r\left(z\frac{\bar{X}}{\bar{x}_r}+(1-z)\frac{\bar{x}_r}{\bar{X}}\right) \tag{14}$$

The bias and MSE of t_3 respectively are given as

$$Bias(t_3)=\bar{Y}\lambda_{n,N}\left(zC_X^2+(1-2z)\rho_{XY}C_XC_Y\right) \tag{15}$$

$$MSE(t_4)=\bar{Y}^2\lambda_{r,N}\left(C_Y^2+(1-2z)^2C_X^2+2(1-2z)\rho_{XY}C_XC_Y\right) \tag{16}$$

Where $z=2^{-1}(1-\rho_{XY}C_Y/C_X)$.

[11] proposed an exponential type imputation method using regression approach defined as

$$y_{.i}=\begin{cases} y_i & \text{if } i\in R \\ \bar{y}_r\left(w_1+w_2\left(\bar{X}-\bar{x}_r\right)\right)\exp\left(\frac{\lambda\left(\bar{X}-\bar{x}_r\right)}{\lambda\left(\bar{X}+\bar{x}_r\right)+2\eta}\right) & \text{if } i\in R^c \end{cases} \tag{17}$$

The point estimator of the population mean, bias and MSE under this method are given as

$$t_4=\frac{r}{n}\bar{y}_r+\left(1-\frac{r}{n}\right)\bar{y}_r\left(w_1+w_2\left(\bar{X}-\bar{x}_r\right)\right)\exp\left(\frac{\lambda\left(\bar{X}-\bar{x}_r\right)}{\lambda\left(\bar{X}+\bar{x}_r\right)+2\eta}\right) \tag{18}$$

$$Bias(t_4)=\bar{Y}\left(\left(\frac{r}{n}-1\right)+\left(1-\frac{r}{n}\right)\lambda_{r,N}\left(\begin{array}{l}\left(\frac{3}{2}\mu^2w_1+\mu w_2\bar{X}\right)C_X^2\\ +\left(\mu w_1+w_2\bar{X}\right)C_{YX}\end{array}\right)\right) \tag{19}$$

$$MSE(t_8)_{\min}=\bar{Y}^2\left(1-\frac{2r}{n}+\frac{r^2}{n^2}\left(1+\lambda_{r,N}C_Y^2\right)-\frac{BC^2+AD^2-2CDE}{AB-E^2}\right) \tag{20}$$

Where $w_1=\frac{DE-BC}{AB-E^2}$, $w_2=\frac{CE-AD}{AB-E^2}$, $\mu=\frac{\lambda\bar{X}}{2\left(\lambda\bar{X}+\eta\right)}$

$$A=\left(1-\frac{r}{n}\right)^2\left(1+\lambda_{r,N}\left(C_Y^2+4\mu^2C_X^2-4\mu C_{YX}\right)\right),\ B=\left(1-\frac{r}{n}\right)^2\lambda_{r,N}\bar{X}C_X^2$$

$$C=\left(1-\frac{r}{n}\right)\left(\frac{r}{n}\left\{1+\lambda_{r,N}\left(\frac{3}{2}\mu^2C_X^2-2\mu C_{YX}\right)\right\}-\left\{1+\lambda_{r,N}\left(\frac{3}{2}\mu^2C_X^2-\mu C_{YX}\right)\right\}\right)$$

$$D=\left(1-\frac{r}{n}\right)\left(\frac{r}{n}\lambda_{r,N}\bar{X}\left(\mu C_X^2-2C_{YX}\right)+\lambda_{r,N}\bar{X}\left(\mu C_X^2-C_{YX}\right)\right),\ E=2\left(1-\frac{r}{n}\right)^2\lambda_{r,N}\bar{X}\left(\mu C_X^2-C_{YX}\right)$$

2. Methodology

2.1 Proposed Imputation Scheme and their Estimators

In this study, we proposed the following class of imputation schemes defined in (21).

$$y_{.i}=\begin{cases}\theta_1\dfrac{n}{r}y_i & \text{i}\in R\\ \dfrac{n}{n-r}\bar{y}_r\left(\theta_2\left(\dfrac{\bar{X}}{\bar{x}_r}\right)^{\kappa_1}+\theta_3\exp\left(\dfrac{\kappa_2\left(\bar{X}-\bar{x}_r\right)}{\bar{X}+\bar{x}_r}\right)\right) & \text{i}\in R^{\text{c}}\end{cases}\tag{21}$$

where $\kappa_1,\kappa_2\in(1,-1)$

Table 1: Members of Proposed Imputation Schemes

SN	κ_1	κ_2	Imputation Schemes
1	1	1	$y_{.i}=\begin{cases}\theta_1\frac{n}{r}y_i & \text{i}\in R\\ \frac{n}{n-r}\bar{y}_r\left(\theta_2\frac{\bar{X}}{\bar{x}_r}+\theta_3\exp\left(\frac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r}\right)\right) & \text{i}\in R^{\text{c}}\end{cases}$
2	1	-1	$y_{.i}=\begin{cases}\theta_1\frac{n}{r}y_i & \text{i}\in R\\ \frac{n}{n-r}\bar{y}_r\left(\theta_2\frac{\bar{X}}{\bar{x}_r}+\theta_3\exp\left(\frac{\bar{x}_r-\bar{X}}{\bar{x}_r+\bar{X}}\right)\right) & \text{i}\in R^{\text{c}}\end{cases}$
3	-1	1	$y_{.i}=\begin{cases}\theta_1\frac{n}{r}y_i & \text{i}\in R\\ \frac{n}{n-r}\bar{y}_r\left(\theta_2\frac{\bar{x}_r}{\bar{X}}+\theta_3\exp\left(\frac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r}\right)\right) & \text{i}\in R^{\text{c}}\end{cases}$
4	-1	-1	$y_{.i}=\begin{cases}\theta_1\frac{n}{r}y_i & \text{i}\in R\\ \frac{n}{n-r}\bar{y}_r\left(\theta_2\frac{\bar{x}_r}{\bar{X}}+\theta_3\exp\left(\frac{\bar{x}_r-\bar{X}}{\bar{x}_r+\bar{X}}\right)\right) & \text{i}\in R^{\text{c}}\end{cases}$

The point estimators of population mean from the proposed schemes are obtained as

$$t_p=\frac{1}{n}\left(\sum_{i=1}^{r}\theta_1\frac{n}{r}y_i+\sum_{i=1}^{n-r}\frac{n}{n-r}\bar{y}\left(\theta_2\left(\frac{\bar{X}}{\bar{x}_r}\right)^{\kappa_1}+\theta_3\exp\left(\frac{\kappa_2\left(\bar{X}-\bar{x}_r\right)}{\bar{X}+\bar{x}_r}\right)\right)\right)\tag{22}$$

By simplifying (22), we obtained the proposed estimators as

$$t_p=\bar{y}_r\left(\theta_1+\theta_2\left(\frac{\bar{X}}{\bar{x}_r}\right)^{\kappa_1}+\theta_3\exp\left(\frac{\kappa_2\left(\bar{X}-\bar{x}_r\right)}{\bar{X}+\bar{x}_r}\right)\right)\tag{23}$$

Table 2: Members of the Estimator for Proposed Imputation Schemes

SN	κ_1	κ_2	Imputation Schemes
1	1	1	$t_{p1}=\bar{y}_r\left(\theta_1+\theta_2\left(\frac{\bar{X}}{\bar{x}_r}\right)+\theta_3\exp\left(\frac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r}\right)\right)$
2	1	-1	$t_{p2}=\bar{y}_r\left(\theta_1+\theta_2\left(\frac{\bar{X}}{\bar{x}_r}\right)+\theta_3\exp\left(\frac{\bar{x}_r-\bar{X}}{\bar{x}_r+\bar{X}}\right)\right)$
3	-1	1	$t_{p3}=\bar{y}_r\left(\theta_1+\theta_2\left(\frac{\bar{x}_r}{\bar{X}}\right)+\theta_3\exp\left(\frac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r}\right)\right)$
4	-1	-1	$t_{p4}=\bar{y}_r\left(\theta_1+\theta_2\left(\frac{\bar{x}_r}{\bar{X}}\right)+\theta_3\exp\left(\frac{\bar{x}_r-\bar{X}}{\bar{x}_r+\bar{X}}\right)\right)$

2.2 Biases and MSEs of the Proposed Estimators

To obtain bias and mean square error (MSE) of the proposed estimator up to the first order approximation, we use following transformations:
$\Delta_o=\left(\bar{y}_r-\bar{Y}\right)/\bar{Y},\ \Delta_1=\left(\bar{x}_r-\bar{X}\right)/\bar{X},$ such that $\left|\Delta_i\right|\approx 0,\ \forall\ \ \mathrm{i}=(0,1).$ This implies that $\bar{y}_r=\bar{Y}\left(1+\Delta_0\right),\ \bar{x}_r=\bar{X}\left(1+\Delta_1\right)$

The expected values of ε_i up to first order approximation are given by (24) as

$$\left.\begin{array}{l}E\left(\Delta_0\right)=E\left(\Delta_1\right)=0, E\left(\Delta_0^2\right)=\lambda_{r,N}C_Y^2,\\ E\left(\Delta_1^2\right)=\lambda_{r,N}C_X^2, \mathrm{E}\left(\Delta_0\Delta_1\right)=\lambda_{r,N}\rho C_X C_Y\end{array}\right\}\tag{24}$$

Using the defined error terms, (23) can be expressed as in (25)

$$t_p=\bar{Y}\left(1+\Delta_0\right)\left(\theta_1+\theta_2\left(1+\Delta_1\right)^{-\kappa_1}+\theta_3\exp\left(-\kappa_2\Delta_1\left(2+\Delta_1\right)^{-1}\right)\right)\tag{25}$$

Simplify the inverse terms up to first order approximation in (25), we obtained (26) as

$$t_p=\bar{Y}\left(1+\Delta_0\right)\left(\theta_1+\theta_2\left(1-\kappa_1\Delta_1+\frac{\kappa_1\left(\kappa_1+1\right)}{2}\Delta_1^2\right)+\theta_3\exp\left(\kappa_2\left(-\frac{\Delta_1}{2}+\frac{\Delta_1^2}{4}\right)\right)\right)\tag{26}$$

Simplify the exponential terms up to first order approximation in (26), we obtained (27) as

$$t_p=\bar{Y}\left(1+\Delta_0\right)\left(\theta_1+\theta_2\left(1-\kappa_1\Delta_1+\frac{\kappa_1\left(\kappa_1+1\right)}{2}\Delta_1^2\right)+\theta_3\left(1-\frac{\kappa_2}{2}\Delta_1+\frac{\kappa_2}{4}\Delta_1^2+\frac{\kappa_2^2}{8}\Delta_1^2\right)\right)\tag{27}$$

Subtract $\bar{Y}$ from both sides of (27) and simplify the expressions up to first order approximation, we obtained

$$t_p-\bar{Y}=\bar{Y}\left(\Delta_0-\left(\theta_2\kappa_1+\frac{\theta_3}{2}\kappa_2\right)\Delta_1+\left(\theta_2\frac{\kappa_1\left(\kappa_1+1\right)}{2}+\theta_3\frac{\kappa_2\left(\kappa_2+2\right)}{8}\right)\Delta_1^2-\left(\theta_2\kappa_1+\frac{\theta_3}{2}\kappa_2\right)\Delta_0\Delta_1\right)\tag{28}$$

By taking expectation of (28) and apply the results of (24), we get the biases of proposed estimators as

$$Bias\left(t_p\right)=\bar{Y}\lambda_{r,N}\left(\left(\theta_2\frac{\kappa_1\left(\kappa_1+1\right)}{2}+\theta_3\frac{\kappa_2\left(\kappa_2+2\right)}{8}\right)C_X^2-\left(\theta_2\kappa_1+\theta_3\frac{\kappa_2}{2}\right)\rho_{XY}C_X C_Y\right)\tag{29}$$

Also, square (28), take expectation of the results and apply the results of (24), we get the MSEs of proposed estimators as

$$MSE\left(t_p\right)=\bar{Y}^2\lambda_{r,N}\left(C_Y^2+\psi^2C_X^2-2\psi\rho_{XY}C_XC_Y\right) \tag{30}$$

where $\psi=\theta_2\kappa_1+\theta_3\dfrac{\kappa_2}{2}$.

Differentiate (30) partially with respect to ψ, equate the results to zero and solve for ψ, we obtained

$$\psi=\rho_{XY}C_Y/C_X \tag{31}$$

To obtain expressions for $\theta_i, i=1,2,3$, the following system of linear equations are used

$$\left.\begin{array}{l}\theta_1+\theta_2+\theta_3=1\\ \kappa_1\theta_2+\kappa_2\theta_3/2=\rho_{XY}C_XC_Y\\ \bar{Y}\lambda_{r,N}\left(\left(\dfrac{\kappa_1(\kappa_1+1)}{2}C_X^2-\kappa_1\rho_{XY}C_XC_Y\right)\theta_2+\left(\dfrac{\kappa_2}{2}\left(\dfrac{(\kappa_2+2)}{4}C_X^2-\rho_{XY}C_XC_Y\right)\right)\theta_3\right)=0\end{array}\right\} \tag{32}$$

Solving (32), we obtained (33) as

$$\left.\begin{array}{l}\theta_3=4\left(2^{-1}(\kappa_1+1)C_X-\rho_{XY}C_Y\right)\rho_{XY}C_Y/\kappa_2\left(\kappa_1-\kappa_2/2\right)C_X^2\\ \theta_2=-\left(2^{-1}(\kappa_2+2)C_X-2\rho_{XY}C_Y\right)\rho_{XY}C_Y/\kappa_1\left(\kappa_1-\kappa_2/2\right)C_X^2\\ \theta_1=1+\left(2\left(4^{-1}\kappa_2(\kappa_2+2)-\kappa_1(\kappa_1+1)\right)C_X-(\kappa_2-2\kappa_1)\rho_{XY}C_Y\right)\rho_{XY}C_Y/\kappa_1\kappa_2\left(\kappa_1-\kappa_2/2\right)C_X^2\end{array}\right\} \tag{33}$$

Setting $\kappa_1=\kappa_2=1$ in (33) and substitute the results in (23), the proposed scheme becomes

$$y_{.i}=\begin{cases}\dfrac{C_X^2-5\rho_{XY}C_XC_Y+4\rho_{XY}^2C_Y^2}{C_X^2}\dfrac{n}{r}y_i & \text{i}\in R\\ \dfrac{n}{n-r}\bar{y}_r\left(\dfrac{-(3C_X-4\rho_{XY}C_Y)\rho_{XY}C_Y}{C_X^2}\dfrac{\bar{X}}{\bar{x}_r}+\dfrac{8(C_X-\rho_{XY}C_Y)\rho_{XY}C_Y}{C_X^2}\exp\left(\dfrac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r}\right)\right), & \text{i}\in R^{\text{c}}\end{cases} \tag{34}$$

Also, setting $\kappa_1=1,\ \kappa_2=-1$ in (33) and substitute the results in (23), the proposed scheme becomes

$$y_{.i}=\begin{cases}\dfrac{3C_X^2+7\rho_{XY}C_XC_Y-4\rho_{XY}^2C_Y^2}{3C_X^2}\dfrac{n}{r}y_i & \text{i}\in R\\ \dfrac{n}{n-r}\bar{y}_r\left(\dfrac{-(C_X-4\rho_{XY}C_Y)\rho_{XY}C_Y}{3C_X^2}\dfrac{\bar{X}}{\bar{x}_r}-\dfrac{8(C_X-\rho_{XY}C_Y)\rho_{XY}C_Y}{3C_X^2}\exp\left(\dfrac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r}\right)\right), & \text{i}\in R^{\text{c}}\end{cases} \tag{35}$$

Similarly, setting $\kappa_1=-1,\ \kappa_2=1$ in (33) and substitute the results in (23), the proposed scheme becomes

$$y_{.i}=\begin{cases}\dfrac{3C_X^2+3\rho_{XY}C_XC_Y-12\rho_{XY}^2C_Y^2}{3C_X^2}\dfrac{n}{r}y_i & \text{i}\in R\\ \dfrac{n}{n-r}\bar{y}_r\left(\dfrac{(3C_X-4\rho_{XY}C_X)\rho_{XY}C_X}{3C_X^2}\dfrac{\bar{X}}{\bar{x}_r}+\dfrac{8\rho_{XY}^2C_Y^2}{3C_X^2}\exp\left(\dfrac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r}\right)\right), & \text{i}\in R^{\text{c}}\end{cases} \tag{36}$$

Similarly, setting $\kappa_1=-1,\ \kappa_2=-1$ in (33) and substitute the results in (23), the proposed scheme becomes

$$y_{.i} = \begin{cases} \dfrac{C_X^2 + \rho_{XY} C_X C_Y + 4\rho_{XY}^2 C_Y^2}{C_X^2} \dfrac{n}{r} y_i & \text{i} \in R \\ \dfrac{n}{n-r} \bar{y}_r \left(\dfrac{-\left(C_X - 4\rho_{XY} C_X\right) \rho_{XY} C_X}{C_X^2} \dfrac{\bar{X}}{\bar{x}_r} - \dfrac{8\rho_{XY}^2 C_Y^2}{C_X^2} \exp\left(\dfrac{\bar{X} - \bar{x}_r}{\bar{X} + \bar{x}_r} \right) \right), & \text{i} \in R^c \end{cases} \quad (37)$$

Remark: *For practical purposes, unknown parameters C_Y (population coefficient of variation for the study variable) and ρ_{XY} (population coefficient of correlation for the study and auxiliary variables) are to be estimated using their corresponding statistics.*

2.3. Efficiency Comparisons

In this section, efficiency of the estimators obtained from the new proposed schemes was be compared to other estimators considered in the study theoretically.

i. $MSE\left(t_0\right) - MSE\left(t_p\right) > 0$ if $\psi < 2\rho_{XY} C_Y / C_X$

ii. $MSE\left(t_1\right) - MSE\left(t_p\right) > 0$ if $\psi > 2\rho_{XY} C_Y / C_X - 1$

iii. $MSE\left(t_2\right) - MSE\left(t_p\right) > 0$ if $2\psi - v > 4\rho_{XY} C_Y / C_X - 1$

iv. $MSE\left(t_3\right) - MSE\left(t_p\right) > 0$ if $\psi - 2z > 2\rho_{XY} C_Y / C_X - 1$

v. $MSE\left(t_4\right) - MSE\left(t_p\right) > 0$ if

$$\left(1 - \frac{2r}{n} + \frac{r^2}{n^2}\left(1 + \lambda_{r,N} C_Y^2\right) - \frac{BC^2 + AD^2 - 2CDE}{AB - E^2}\right) - \lambda_{r,N}\left(C_Y^2 + \psi^2 C_X^2 - 2\rho_{XY} C_Y C_X\right) > 0$$

3. Results and Discussion

In this section, simulation studies were conducted to assess the performance of the suggested estimators with respect to some existing estimators. The steps for stimulation are as follows;

Step1: Populations of size $N = 500$ with values (X, Y) are generated from the distributions in Table 3

Table 3: Populations used for Simulation Study

Populations	Auxiliary variable x	Study variable y
1	$X \sim \exp(0.7)$	$Y = 3 + 2X + 5X^2 + e,$ $e \sim N(0,1)$
2	$X \sim chisq(2)$	
3	$X \sim uniform(50,100)$	
4	$X \sim gamma(3,2)$	

Step 3: calculate biases, MSEs and PREs of the proposed and other considered estimators 10,000 times and take the average as given below;

$$Bias\left(\hat{\theta}\right)=\frac{1}{10000}\sum_{j=1}^{10000}\left(\hat{\theta}_j-\bar{Y}\right) \tag{38}$$

$$MSE\left(\hat{\theta}\right)=\frac{1}{10000}\sum_{j=1}^{10000}\left(\hat{\theta}_j-\bar{Y}\right)^2 \tag{39}$$

$$PRE\left(\hat{\theta}\right)=\left(MSE\left(\hat{\theta}\right)/MSE\left(\bar{y}_r\right)\right)\times 100 \tag{40}$$

where $\hat{\theta}_j$ is the estimate of the estimators based on j^{th} sample of sizes $n=100$ with $r=80$ from populations defined in step 1.

Table 4: Biases, MSEs and PREs of t_0,t_1,t_2,t_3,t_4 and t_p using Population 1

Estimators	Bias	MSE	PRE
Sample mean t_0	0.07129936	20.56328	100
Ratio estimator t_1	-0.1183695	5.101494	403.0834
Singh et al. (2014) t_2	-0.3392347	2.897702	709.6408
Sing and Gogoi (2018) t_3	0.7354008	78.01035	26.35967
Audu et al. (2020) $t_4(\lambda=1,\eta=1)$	0.584759	3.760751	546.7865
$t_4(\lambda=1,\eta=-1)$	0.658953	4.610823	445.9784
$t_4(\lambda=1,\eta=0)$	0.5780256	3.523319	583.6337
$t_4(\lambda=1,\eta\in\Re)$	0.5912655	3.523319	484.9325
Proposed $t_p(\kappa_1=1,\kappa_2=1)$	0.02831592	2.754686	746.4836
$t_p(\kappa_1=1,\kappa_2=-1)$	0.02883345	2.757029	745.8492
$t_p(\kappa_1=-1,\kappa_2=1)$	0.03341576	2.782802	738.9414
$t_p(\kappa_1=-1,\kappa_2=-1)$	0.04858547	2.875489	715.1228

Table 5: Biases, MSEs and PREs of t_0,t_1,t_2,t_3,t_4 and t_p using Population 2

Estimators	Bias	MSE	PRE
Sample mean t_0	0.002526502	65.38015	100
Ahmed et al. (2006) t_1	-0.332948	17.95152	364.2039
Singh et al. (2014) t_2	-0.7541572	9.086104	719.562
Sing and Gogoi (2018) t_3	1.1703	248.2631	26.33502
Audu et al. (2020) $t_4(\lambda=1,\eta=1)$	0.9629443	11.38413	574.3099
$t_4(\lambda=1,\eta=-1)$	0.9347575	9.72647	672.1879
$t_4(\lambda=1,\eta=0)$	0.9535352	10.78415	606.2615

$t_4(\lambda=1,\eta\in\Re)$	0.9766825	10.78415	499.4336
Proposed $t_p(\kappa_1=1,\kappa_2=1)$	-0.0466231	8.340084	783.9268
$t_p(\kappa_1=1,\kappa_2=-1)$	-0.04509603	8.35497	782.5301
$t_p(\kappa_1=-1,\kappa_2=1)$	-0.03611758	8.450985	773.6395
$t_p(\kappa_1=-1,\kappa_2=-1)$	-0.009655374	8.756904	746.6127

Table 6: Biases, MSEs and PREs of t_0, t_1, t_2, t_3, t_4 and t_p using Population 3

Estimators	Bias	MSE	PRE
Sample mean t_0	-10.51727	1238432	100
Ratio t_1	-15.14461	296853	417.1868
Singh et al. (2014) t_2	-24.93687	10581.74	11703.48
Sing and Gogoi (2018) t_3	15.01676	4928759	25.12664
Audu et al. (2020) $t_4(\lambda=1,\eta=1)$	34.30684	11692.43	10591.74
$t_4(\lambda=1,\eta=-1)$	34.30788	11659.84	10621.35
$t_4(\lambda=1,\eta=0)$	34.30735	11676.31	10606.36
$t_4(\lambda=1,\eta\in\Re)$	34.27137	11676.31	9445.085
Proposed $t_p(\kappa_1=1,\kappa_2=1)$	0.9705278	9053.167	13679.54
$t_p(\kappa_1=1,\kappa_2=-1)$	0.9653535	9056.111	13675.09
$t_p(\kappa_1=-1,\kappa_2=1)$	0.966375	9063.558	13663.86
$t_p(\kappa_1=-1,\kappa_2=-1)$	0.9849391	9079.185	13640.34

Table 7: Biases, MSEs and PREs of t_0, t_1, t_2, t_3, t_4 and t_p using Population 4

Estimators	Bias	MSE	PRE
Sample mean t_0	-0.04221389	3.922419	100
Ahmed et al. (2006) t_1	-0.05397056	0.7460236	525.7768
Singh et al. (2014) t_2	-0.0884436	0.3221238	1217.674
Sing and Gogoi (2018) t_3	0.06888382	14.95297	26.2317
Audu et al. (2020) $t_4(\lambda=1,\eta=1)$	0.1367046	0.3576383	1096.756
$t_4(\lambda=1,\eta=-1)$	0.1380231	0.3426267	1144.808
$t_4(\lambda=1,\eta=0)$	0.1360855	0.3477281	1128.013
$t_4(\lambda=1,\eta\in\Re)$	0.1375578	0.3477281	1037.974
Proposed $t_p(\kappa_1=1,\kappa_2=1)$	-4.61776e-05	0.3140942	1248.803
$t_p(\kappa_1=1,\kappa_2=-1)$	-0.0001870142	0.3141481	1248.589
$t_p(\kappa_1=-1,\kappa_2=1)$	-0.000175067	0.314711	1246.356
$t_p(\kappa_1=-1,\kappa_2=-1)$	0.0005346201	0.3167301	1238.411

Tables 4-7 showed the results of the biases, MSEs and PREs of the proposed and other estimators considered in the study using populations 1-4 respectively. The following results were obtained;

i. All the estimators considered in the study including the proposed estimators have smaller MSEs and larger PREs compared to sample mean except [10] t_3.
ii. All the proposed estimators are less biased compared to other estimators.
iii. All the proposed estimators have smaller MSEs and larger PREs compared to all other estimators considered in the study.
iv. The proposed estimator t_1 was the most efficient among other estimators.

4. Conclusion

From the empirical study in section 3, the results revealed that the proposed estimators outperformed their counterparts for all the populations considered in the study, therefore it was concluded that the proposed estimators are more efficient and can produce better estimates of population means. Hence, the new proposed schemes are recommended for use for both practical and theoretical purposes.

Nomenclature

N	Population size
n	Sample size
Y	Variable of study or interest
X	Auxiliary variable
$\bar{Y}, \bar{X}$	Population means of Y, X
R	Respondents group
R^c	Non-respondents group
r	Number of respondents from sample units
$\bar{y}_r, \bar{x}_r$	Sample means of Y, X based on r respondents units
S_Y^2, S_X^2	Population variances of Y, X
C_Y, C_X	Population coefficients of variation of Y, X
ρ_{YX}	Population correlation coefficient of Y, X
Δ_0, Δ_1	Error terms of Y, X

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