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Evaluation of Asset Allocation Models

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Abstract

The main goal of investors is to minimize risk at any point of a given returns or/and maximize returns at any given risk. Asset allocation involves allotting investments among different assets. Optimal asset allocation minimizes risk of portfolio to the barest level and maximizes returns better. The aim of this paper is to investigate the two asset allocations; Black Litterman model (BLM) and Mean Variance model (MVM) and examine the model that minimizes risk better and maximizes return optimally. The data used are monthly data of groundnut oil, palm oil and palm kernel. The study shows that the BLM minimizes risk of portfolio better and maximizes return optimally than MVM.

1. Introduction

Portfolio management is the art of decision-making using all available information to date in order to formulate a most likely scenario for the future while also balancing risk against performance. The early theoretical development of portfolio theory heeds back to Harry Markowitz's article "Portfolio Selection" where he outlined the foundation for what is today known as the Mean-Variance (MV) theory. In his article, Markowitz postulated that investors are risk averse and that there is a tradeoff between risk and return. Markowitz's framework has since been further developed by scholars and one of the most influential contributions is the work by Fischer Black and Robert Litterman in 1991. The model proposed is known as the Black-Litterman model (BLM), which has in recent times come to achieve great recognition among portfolio managers worldwide.

MV portfolio optimization uses a complete set of expected returns while the BLM allows the investor to use any number of market views on future returns and combines them with an equilibrium leading to optimal portfolio weights. In other words, the BLM tilts the portfolio weights towards the assets where the investor has specified views, thereby avoiding extreme allocations given by the Mean Variance model (MVM), [1]. The concept of the model is that investors should take risk where they have views, and consequently take more risk where investors have stronger views [2]. Despite the extensive literature in this field, few empirical studies have tested the applicability of the BLM on equity markets. This inadequacy can partly be explained by the obscurity of the model's qualitative side, meaning there is no clear framework on how to incorporate and quantify market views in a consistent manner.

The two models BLM and MVM are used to evaluate both returns and risk of the estimated data; groundnut oil, palm oil and palm kernel. The remaining parts of this paper is organized as follow: section two reviews literature, section three explains the methodology, results and discussions are considered in section four while section five concludes the paper.

1.1. Review of Related Literature

Mean-Variance optimization (MVO) was developed by [3], it has become the foundation of modern finance theory. The technique considers the performance of the investor as well as the return, risk and diversification effects, which help to minimize the overall risk of the portfolio. It has become the foundation of modern finance theory. MVO model has thus become a key financial instrument for choosing asset allocation, but several difficulties arise.

Markowitz portfolio theory (MPT) asserts that a portfolio is diversified if its variance could not be reduced any further at the same level of expected return. It implies that a portfolio's variance may be used as a proxy for the fund's diversification level. Maximum diversification was introduced by [4] along with the concept of a Diversification Ratio (DR). [5], [6] established some basic concepts of modern portfolio theory, namely the efficient frontier and the capital market line.

The modern Markowitz theory on portfolio is indeed the mainstay of portfolio management. Diversification has been an enormous issue since MPT has been approved as a tool in managing asset portfolio. Many researchers have tried to model the rewards of developing diversification strategies for portfolio investments. The risk of well-diversified portfolio of an asset class is much higher than the volatility of its components. Second is the well-diversified portfolios within an asset class which are highly correlated; however, well-diversified portfolios of different asset classes are less correlated. All investors want to maximize the expected return, given implicitly; investors are risk averse and assume the mean-variance theory for selection criterion that is, the mean and the standard deviation of the return [7].

Investor can reduce risks in their portfolio simply by holding assets that are not positively correlated, thus diversifying the investments. This allows them to obtain the same return potential by reducing their portfolio volatility. The MVO model has thus become a key financial instrument for choosing asset allocations, but several difficulties arise. According to [8], it was established that problems incurred with mean variance optimization include creation of concentrated (or non-diversified) portfolio and unstable model causing significant changes in portfolio during small variations in initial data.

It was observed that the volatility facing by an investors was portfolio risk which leads to a basic and essential point that the volatility of a stock should be estimated not only by variance also by covariance. Notably, correlations are useful for constructing portfolio allocation strategies, but do not offer a complete and accurate measure of overall market integration. Furthermore, one cannot fully account for the structure of risk since simple correlations simplify the factor structure. One would need to include the full covariance matrix. Investors use MVO choice models because they are well understood; most investors use them because of their simplicity and transparency.

[9] suggested BLM as an alternative to Markowitz optimization. Black and Litterman introduced an intuitive optimization method to resolve the Mean-Variance optimization difficulties. This method makes it possible to combine allocations resulting from market equilibrium according to CAPM with portfolio managers' views.

The most essential aspect of Markowitz model was his elucidation of the effect on portfolio diversification by the number of securities (risky and riskless) within a portfolio and their covariance relationships. [10] established that modern portfolio theory provides a rigorous understanding of what diversification is and how it works to improve investment opportunities. MVT has been used to formulate an ex-post frame work of international portfolio diversification but a flaw in this approach is that investment is on intuition which makes investor's decision to be uncertain and vulnerable to huge risk. They observed few parameters uncertainty, owing to the lack of historical data and low data frequency. [11] provided an extension to the BLM for an additional factor which is uncorrelated with the market. They showed how it intuitively impacts the expected returns computed from the model.

[12] provided a detailed transformation between the two specifications of the BLM formula for the estimated asset returns. BLM is relatively flexible when it comes to the method used to choose the portfolio as declared by [13]. [14] declared that, under the economic theory of choice, an investor chooses among the opportunities by specifying the indifference curves or utility function. These curves are constructed so that the investor is equally happy along the same curve which leads to an analysis of the assumed investor's profile. The extreme sensitivity of portfolio weights to expected returns which investors focus on is itself not sensitive to how investors make his choice; there is a trade-off between portfolio risk and portfolio return, the more risk an investor is willing to accept, the higher the expected return of the investment. Therefore, for a given amount of risk, there is an "optimal" portfolio that produces the highest possible return, as long as it reflects a reasonably smooth trade-off between risk and expected return. Black and Litterman leave quite a lot of freedom to the investor in terms of their portfolio choice model. However, [15] believes the BLM combines views of the investor and the market equilibrium on the expected return of the assets in one formula. This formula should be a better approximation of the expected returns. These expected returns, or more precisely the estimator of the expected return, makes BLM gives better result than MVM, analysis in this paper proves this.

2. Methodology

As discussed earlier that two asset allocation models are involved in this study therefore two methodologies are equally considered; BLM and MVM.

2.1. Black Litterman Model

A portfolio of *n* assets is denoted by a vector $x \in R^n$ with $\sum_{i=1}^{n} x_i = 1$. Let the returns of an asset be denoted by $\Re_i$ and expected return of asset *i* be $E(\Re_i)$. Then the expected return vector is $E(\Re) = col\{E(\Re_i)\} \in R^n$, *(i=1, 2,...,n)*. The covariance matrix is denoted by $\sum \in R^{nxn}$. The covariance of assets *i* and *j* is given as σ_{ij} [16] the return $\Re_p$ of portfolio is estimated by

$$\Re_p = \sum_{i=1}^{n} x_i \Re_i$$

$$E(\Re_p) = E\left(\sum_{i=1}^{n} x_i \Re_i\right)$$

$$\sum_{i=1}^{n} E(x_i \Re_i) = \sum_{i=1}^{n} x_i E(\Re_i)$$
$$= x' E(\gamma) \quad (1)$$

The variance of return of the portfolio can be computed as:

$$\sigma_p^2 = \sigma_i^2 (\sum_{i=1}^{n} x_i \Re_i)$$
$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} (x_i \Re_i, x_j \Re_j)$$
$$= \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} (\Re_i \Re_j)$$
$$= \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij}$$
$$= x' \Sigma x \quad (2)$$

The expected return of equilibrium portfolio as:

$$\Pi = \delta \sum x_{mkt} \quad (3)$$

where Π is the expected return of market equilibrium, δ is the risk aversion, x_{mkt} is the market weight. The improvement in the BLM allows the investors to combine their views directly in the model in an intuitive way. The views can be relative. The views have to be in the same format with constraints. The investors should be able to fix a level of confidence in his views. This requirement may be as follows:

$$P.E(\Re) = Q + \varepsilon \quad (4)$$

where P is the vector that describes the assets concerned by the views, Q is the vector of their performances and ε is the random normal vector of error terms, $\varepsilon \sim N(0, \Omega)$ with diagonal variance matrix Ω. It is assumed that the market is rotating around an equilibrium point and the same with investors' portfolio in respect to CAPM hypothesis [17].

Let the mean $E(\Re) = \Pi$, the covariance, assumed to be proportional to Σ, with factor of uncertainty τ, $E(\Re) \sim N(\Pi, \tau\Sigma)$.

The Equation (5) is known as the Black Litterman model and it represents the expected return vectors that is produced from a Bayesian mixing of the implied equilibrium excess return vector (Π) and the vector of investor views (Q).

$$E(\Re) = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q] \quad (5)$$

2.2. Mean Variance Model

Three assets (groundnut oil, palm oil and palm kernel oil) are used, consequently three assets model is considered:

Let groundnut oil, palm oil and palm kernel represent 1,2 and 3 respectively in the equation:

$$R_p = w_1 R_1 + w_2 R_2 + w_3 R_3 \quad (6)$$

$$\delta_p = \sqrt{\delta_p^2} \quad (7)$$

$$\delta_p^2 (gnut\, oil) = w_1^2 \delta_1^2 + w_2^2 \delta_2^2 + w_3^2 \delta_3^2 + 2 w_1 w_2 r_{1,2} \delta_1 \delta_2 + 2 w_1 w_3 r_{1,3} \delta_1 \delta_3 \quad (8)$$

$$\delta_p^2 (palm\, oil) = w_1^2 \delta_1^2 + w_2^2 \delta_2^2 + w_3^2 \delta_3^2 + 2 w_2 w_1 r_{2,1} \delta_2 \delta_1 + 2 w_2 w_3 r_{2,3} \delta_2 \delta_3 \quad (9)$$

$$\delta_p^2 (palm\, \mathrm{ker}\, nel) = w_1^2 \delta_1^2 + w_2^2 \delta_2^2 + w_3^2 \delta_3^2 + 2 w_3 w_1 r_{3,1} \delta_3 \delta_1 + 2 w_3 w_2 r_{3,2} \delta_3 \delta_2 \quad (10)$$

where R_p is return of portfolio, R_1, R_2, R_3 are returns of groundnut oil, palm oil and palm kernel respectively, δ_p is standard deviation of portfolio, $\delta_p^2(gnut\,oil)$ is variance of groundnut oil, $\delta_p^2(palm\,oil)$ is variance of palm oil, $\delta_p^2(palm\,\ker nel)$ is variance of palm kernel, w_1, w_2, w_3 are weights of groundnut oil, palm oil and palm kernel respectively, $\delta_1, \delta_2, \delta_3$ are standard deviations of groundnut oil, palm oil and palm kernel respectively, $r_{1,2}$ is correlation coefficient of groundnut oil and palm oil, $r_{1,3}$ is correlation coefficient of groundnut oil and palm kernel, $r_{2,3}$ is correlation coefficient of palm oil and palm kernel.

3. Data Analysis

The sample data were explored from monthly data of Groundnut Oil, Palm Oil and Palm Kernel from yahoo finance DataStream. The data spans from 2010 to 2016. The actual data for this study are non-stationary. The non-stationary data were transformed to stationary by first differencing. Stationary data was used for the analysis of this study.

4. Results and Discussion

As stated above that this research is carried out to investigate the two asset allocations; BLM and MVM and examine the model that minimizes risk better and maximizes return optimally. The results of our investigation are presented in Table1 and 2. Moreover, the risks divulged by the two models are given in Table1 while the returns revealed by the two models are presented in Table2.

Table 1: Risk of the models

Assets	**Black Litterman Risk**	**Mean Variance Risk**
Groundnut oil	0.0016	0.0021
Palm oil	0.0017	0.0025
Palm kernel	0.0018	0.0021

Table2: Return of the models

Asset	**Black Litterman Return**	**Mean Variance Return**
Groundnut oil	0.350	0.005
Palm oil	0.380	0.005
Palm kernel	0.400	0.005

Considering Table1, BLM generated risk 0.0016, 0.0017 and 0.0018 for groundnut oil, palm oil and palm kernel respectively while MVM produced risk 0.0021, 0.0025 and 0.0021 respectively. The risk of BLM is minimized better than MVM. Correspondingly, in Table2 had return 0.350, 0.380and 0.400 and MVM contained 0.005 all through for the three assets respectively which shows BLM maximized return optimally compared with MVM. The Figure 1 and 2 shows the degrees of minimization of risk and maximization return in BLM compared to MVM.

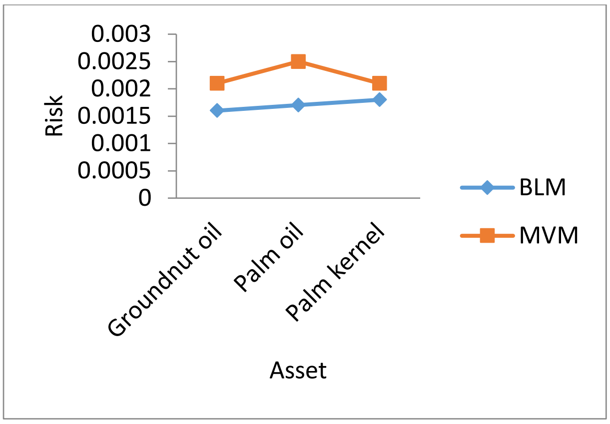

Figure 1: Graph of Risk

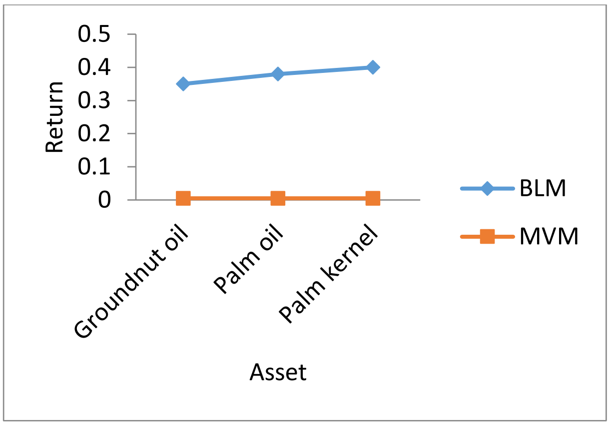

Figure 2: Graph of Return

5. Conclusion

This research is carried out to investigate the two asset allocations; BLM and MVM and examine the model that minimizes risk better and maximizes return optimally. As shown vividly in the result above that asset allocation BLM minimizes risk better and maximizes return optimally. From the study it was discovered that BLM minimized risk of groundnut oil by 31.25%, palm oil 47.06% and palm kernel 17.67% in compared with MVM. Equally, it maximized groundnut oil by 98.57%, palm oil 98.68% and palm kernel 98.75%. According to [9] and [15], the results in this paper validate the Literature which says Black and Litterman developed BLM to improve the weakness of MVM. As shown in the results, BLM is better both to minimize risk and maximize return. Therefore, it is recommended that investors should invest using asset allocation; Black Litterman model instead of Mean Variance model.

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Appendix

Table A1: Sample of Non-Stationary Data

Date	Groundnut Oil	Palm Oil	Palm Kernel
2010M01	1316.00	793.00	878.00
2010M02	1380.00	798.00	894.00
2010M03	1380.00	832.00	995.00
2010M04	1360.00	830.00	1020.00
2010M05	1353.00	811.00	1030.00
2010M06	1342.00	798.00	1051.00
2010M07	1300.00	807.00	1059.00
2010M08	1334.00	905.00	1165.00
2010M09	1270.00	912.00	1260.00
2010M10	1331.00	987.00	1412.00
2010M11	1727.50	1109.00	1626.00
2010M12	1753.00	1228.00	1820.00
2011M01	1788.00	1281.00	2120.00
2011M02	1730.00	1292.00	2296.00
2011M03	1650.00	1180.00	1977.00
2011M04	1680.00	1149.00	1899.00
2011M05	1830.00	1159.00	1958.00
2011M06	1980.00	1133.00	1765.00
2011M07	2120.00	1089.00	1371.00
2011M08	2150.00	1083.00	1375.00
2011M09	2195.00	1065.00	1268.00
2011M10	2240.00	994.00	1085.00
2011M11	2225.00	1053.00	1298.00
2011M12	2270.00	1027.00	1367.00
2012M01	2345.00	1061.00	1366.00
2012M02	2420.00	1106.00	1362.00
2012M03	2495.00	1153.00	1370.00
2012M04	2570.00	1181.00	1395.00
2012M05	2555.00	1085.00	1239.00
2012M06	2520.00	999.00	1093.00
2012M07	2468.00	1015.00	1067.00
2012M08	2553.00	997.00	1008.00
2012M09	2408.00	967.00	984.00
2012M10	2375.00	839.00	862.00
2012M11	2303.00	813.00	815.00
2012M12	2216.00	776.00	762.00

Table A2: Sample of Stationary Data

Date	Groundnut Oil	Palm Oil	Palm Kernel
2010M01	-0.04863	-0.00631	-0.01822
2010M02	0	-0.04261	-0.11298
2010M03	0.014493	0.002404	-0.02513
2010M04	0.005147	0.022892	-0.0098
2010M05	0.00813	0.01603	-0.02039
2010M06	0.031297	-0.01128	-0.00761
2010M07	-0.02615	-0.12144	-0.10009
2010M08	0.047976	-0.00773	-0.08155
2010M09	-0.04803	-0.08224	-0.12063
2010M10	-0.2979	-0.12361	-0.15156
2010M11	-0.01476	-0.1073	-0.11931
2010M12	-0.01997	-0.04316	-0.16484
2011M01	0.032438	-0.00859	-0.08302
2011M02	0.046243	0.086687	0.138937
2011M03	-0.01818	0.026271	0.039454
2011M04	-0.08929	-0.0087	-0.03107
2011M05	-0.08197	0.022433	0.09857
2011M06	-0.07071	0.038835	0.223229
2011M07	-0.01415	0.00551	-0.00292
2011M08	-0.02093	0.01662	0.077818
2011M09	-0.0205	0.066667	0.144322
2011M10	0.006696	-0.05936	-0.19631
2011M11	-0.02022	0.024691	-0.05316
2011M12	-0.03304	-0.03311	0.000732
2012M01	-0.03198	-0.04241	0.002928
2012M02	-0.03099	-0.0425	-0.00587
2012M03	-0.03006	-0.02428	-0.01825
2012M04	0.005837	0.081287	0.111828
2012M05	0.013699	0.079263	0.117837
2012M06	0.020635	-0.01602	0.023788
2012M07	-0.03444	0.017734	0.055295
2012M08	0.056796	0.03009	0.02381
2012M09	0.013704	0.132368	0.123984
2012M10	0.030316	0.030989	0.054524
2012M11	0.037777	0.04551	0.065031
2012M12	0.052347	-0.08376	-0.04331