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Derivation of an Approximate Method for the Analyses of Ribbed Slabs (A Finite Element Approach)

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ABSTRACT

This study introduces a new approximate method for analyzing ribbed slabs, named Calibrated Eurocode Results (CER) or Ogbonna-Umeonyiagu's method, based on finite element method (FEM) results. The method was developed by calibrating existing approximate techniques, specifically the ACI coefficient and Eurocode methods, through comparison with FEM results obtained using SAFE 2016. Manual analyses were conducted on ribbed slabs with varying boundary conditions, revealing significant discrepancies between existing approximate methods and FEM results for ribbed slabs. Notably, the Eurocode method showed considerable convergence with FEM results. A statistical regression analysis was performed, yielding calibration factors that improved the accuracy of the Eurocode method, bringing its results within 6% of FEM values, compared to a previous discrepancy of 65%. The newly developed method offers enhanced bending moment coefficients for ribbed slabs, performing better than existing methods and facilitating easier analysis of two-way slabs. A Java program was also created to implement this method, demonstrating its effectiveness and adding to the repertoire of tools available for slab analysis.

1. **Introduction**

The slab is a key structural component in buildings and also serves as deck in bridges. The floor system of a structure can take many forms such as in situ solid slabs, ribbed slabs or precast units. Slabs may span in one direction or in two directions and they may be supported on monolithic concrete beams, steel beams, walls or directly by the structure's columns [1]. The shape, arrangement, and stiffness of the supporting beams all play a significant role in the slab's performance. Slabs are used almost in every building to enclose the space along with some other structural elements such as walls, columns etc. In comparison to other structural elements, a slab is highly indeterminate due to the integrity of the internal stress resultants and it offers several load paths to the applied loading. Due to the integrity of internal force-resultants, the structural behaviour of the slabs is highly susceptible to the type, layout of the supporting system and stiffness of the supporting structural element. Change in any parameters of the supporting systems causes a significant change in the moment-field induced in the slab under applied loading. Removal of even a single support will lead to a considerable change in the behaviour of the slab. In addition, even small changes in the depth of the supporting beams/system will cause a significant change in the moment field in the slab panel. Over the past few decades, several approximate methods for analyses of two-way slabs were developed. They include equivalent frame method (EFM) and direct design method (DDM), which have restrictions on their use or may be time-consuming, so many other approximate methods such as Rankine-Grashoff, Marcus, coefficient method in ACI 318-63 code and Euro code (EC2) are widely used due to their simplicity. The dilemma for the designer is to decide which of these methods is the most appropriate for a given two-way slab system. Several computer analysis programs are available using the finite element method with different element types and capabilities. Most past research has proved that the results from finite element programs are compatible with the elastic plate theory results. In 1998 [2] compared the results obtained from finite element analyses of two-way solid slab using STAAD-III with those obtained using approximate methods adopted by ACI codes, including coefficient method, direct design method and equivalent frame method. The results obtained from this research show that the finite element solution complies with the elastic plate theory for aspect ratios ranging from 1.0 to 5.0, where the maximum error in moments is equal to 2.60% and in deflection is equal to 2.04%, which is considered acceptable. [3] analyzed the waffle slab by an approximate method using Rankine-Grashoff method and compared it with the results from Timoshenko's plate theory and ETABS 2015. The results showed that finite element results and plate theory are in good agreement, However, Rankine-Grashoff method overestimates critical bending moment and shear force because they neglect negative moments at the supports. [4] made a parametric study of two-way slab analyses using various methods including Rankine-Grashoff method, Plate Theory, Stiffness Matrix Method and STAAD program. The study shows that Plate theory underestimates the bending moment shear force values when compared to other methods, but the most important result is that Rankine- Grashoff method and plate theory can be only used when the slab is simply supported on all sides since each of the methods does not take into consideration the deflection of boundary and their stiffness. The first approximate method of two-way slab analyses was published in 1910 by the National Association of Cement Users (NACU), the distribution of load in this method was based on the equality of the deflection in the middle of two perpendicular slab strips, and this principle was the main factor in development of other approximate methods. [5] worked on the comparative study of Analyses of Two-Way Solid R.C. Slab Using Approximate and Finite Element Methods. This research aims to derive an approximate method for the analyses of ribbed slabs, based on the finite element of solution. This was achieved by reviewing available approximate methods for analyses of two-way slab systems, comparison and verification of the numerical results generated from the methods with those of the finite element method, and derivation of a new approximate method based on FEM to analyse ribbed slabs.

1.1 Ribbed and Waffle Slabs

Two-way spanning ribbed slabs, known as waffle slabs, offer a material-efficient alternative to rectangular solid slabs, which tend to be wasteful, especially in cement usage. By modifying the slab's composition, its weight can be reduced without compromising strength or performance. Ribbed and waffle slabs exemplify such efficient designs. Flat slabs are generally more economical than ribbed slabs. However, the intricate formwork required for ribbed slabs can be cost-prohibitive for small domestic projects.

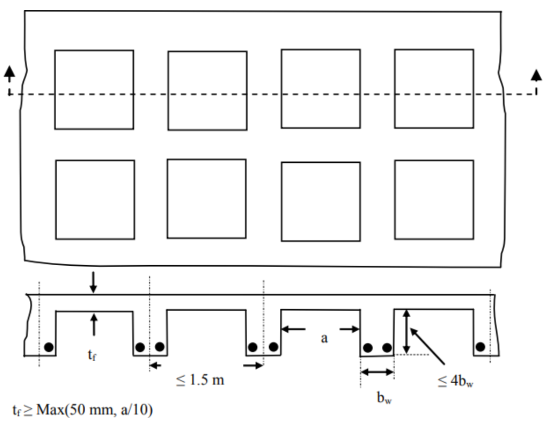

Figure 1. Ribbed slab [6]

Using hollow clay blocks can further reduce weight and simplify the formwork process. Grid or coffered floor systems, made up of beams placed at regular perpendicular intervals and integrated with a monolithic slab, are frequently utilized in large architectural spaces such as auditoriums and theaters to provide column-free areas. The rectangular or square voids formed in the ceiling are effectively used for concealed architectural lighting. According to [6], connecting a series of ribs (beams) with structural topping significantly reduced the weight of the slab between the ribs, as shown in Figure 2.5. Ribbed slabs can be constructed in various ways:

i) Ribbed slabs without permanent blocks. The space between the beams is created using square or rectangular plastic formers during casting. Reinforcement is laid between the former.
ii) Ribbed slabs with permanent hollow or solid blocks to obtain a flat ceiling.

1.2 Methods of Analyses for Ribbed and Waffle Slabs

A grid is a highly redundant and statically indeterminate structural system. Various methods are used for analyzing grid floor frames:

i.) Rankine-Grashoff Method:
This is an approximate method equating deflection at rib junctions and suitable for small grids with rib spacing ≤ 1.5m, but not for larger spans.

ii.) Plate Theory:
[7] analyzed moments and shears based on deflection surfaces. Maximum bending moments occur at span centers, torsional moments at corners, and shear forces at midpoints of longer sides.

iii.) Computerized Analysis and Design:
Necessary for rigorous elastic solutions due to slab sensitivity to support conditions. Computerization helps achieve economical designs through repeated analyses.

1.3 Reinforced Concrete Slab Failure Modes

Reinforced concrete (RC) slabs without shear reinforcement subjected to concentrated loads can fail due to bending, shear, and anchorage failure (Figure 2). Bending failure is desirable as it allows ductile deformation and redistribution of internal forces before collapse. However, shear failure is undesirable due to its brittle nature, causing sudden collapse and posing significant risk to life and property. Shear failures are categorized as one-way shear failure, often occurring in slab-wall systems, and punching shear failure, occurring in slab-column systems. Although anchorage failure due to bond-slip is common, it is not the primary focus of this study. The study of RC slab failure modes dates to [8], with extensive research conducted through laboratory tests, field tests, and numerical simulations to develop empirical and mechanical models [9].

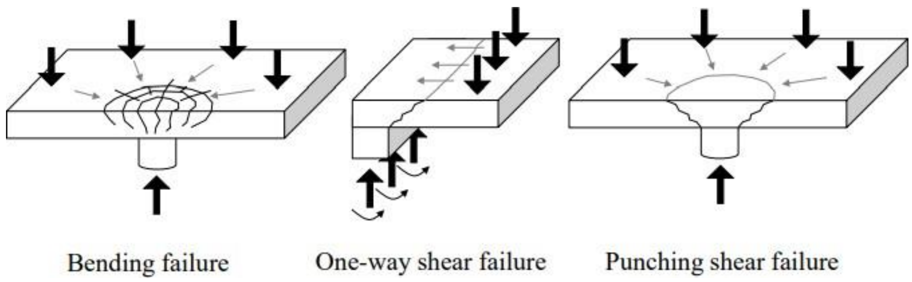

Figure 2. Three different failure modes for Reinforced Concrete Slabs [9]

1.4 Methods of Two-Way Slabs Analyses

The various methods that are used for analyses of two-way slabs include:

i) Elastic Plate Theory
ii) Finite Element Method (FEM)
iii) Approximate Methods (Rankine -Grashoff, Marcus, ACI, BS8110, EC2 etc).

1.4.1 ACI code coefficient methods

Determining exact moments in two-way slabs with various support conditions is complex and impractical for design practice. Hence, simplified methods are adopted. One popular method uses 'Moment Coefficients' from the 1963 ACI Code for two-way slabs supported on four sides by stiff beams. This method employs tables of moment coefficients for different support conditions. These coefficients are based on elastic analyses and account for inelastic redistribution, providing a practical approach to calculating moments, shears, and reactions in such slabs. The moment values in the middle strips are computed from

$$M_{a,max} = C_a w l_a^2 \tag{1}$$

$$M_{b,max} = C_b w l_b^2 \tag{2}$$

Where:

$M_{a,max}, M_{b,max} =$ *moments in short and long directions, respectively.*
$w =$ *total load per unit area.*
$C_a, C_b =$ *Tabulated moment coefficients*
$l_a, l_b =$ *length of clear span in short and long directions, respectively.*

This method provides the values of $M_{a.max}$ and $M_{b,max}$ along the central strip of the slab, as demonstrated in Figure 3 for a slab simply supported on all sides. As shown, the maximum moments are less elsewhere. Therefore, other design values can be reduced according to the variation shown. These variations in maximum moment across the width and length of a rectangular slab are accounted for approximately by designing the outer quarters of the slab span in each direction for a reduced moment.

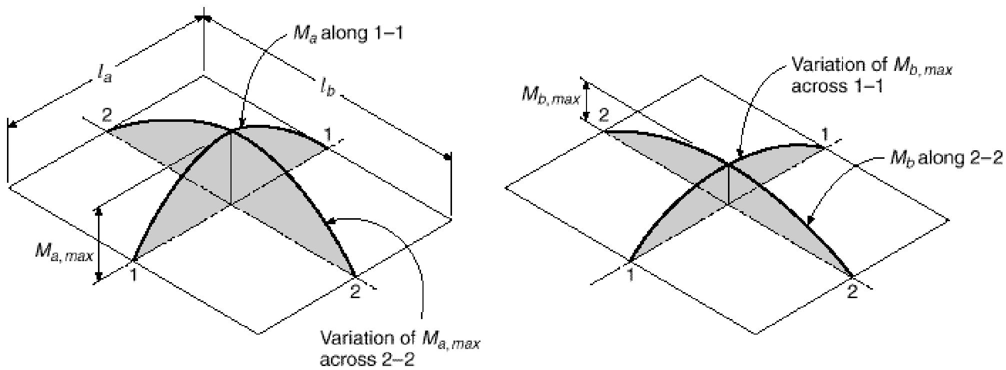

Figure 3 Variation of moments in a uniformly loaded slab simply supported on all sides

Compared to the idealized 'simply supported' slab, Figure 3 shows a more 'realistic' scenario where a system of beams supports a two-way slab.

1.4.2 Euro Code (EC2)

When a slab is supported on all four sides, it spans in both directions, often making it more economical to design it this way. The bending in each direction depends on the span ratio and support restraints [10]. For a square slab with similar restraints, the load spans equally in both directions. For a rectangular slab, more load is carried in the stiffer, shorter direction. If one span is much longer, the load is primarily carried in the shorter direction, potentially allowing for a one-way design. Moments are calculated using tabulated coefficients [1]

1.5 Simply supported slab spanning in two directions

A slab simply supported on its four sides will deflect about both axes under load and the corners will tend to lift and curl up from the supports, causing torsional moments. When no provisions have been made to prevent this lifting or to resist the torsion, then the moment coefficients of Table may be used and the maximum moments are given by: [10].

$$M_{sx} = a_{sx} n l_x^2 \quad (3)$$

and

$$M_{sy} = a_{sy} n l_y^2 \quad (4)$$

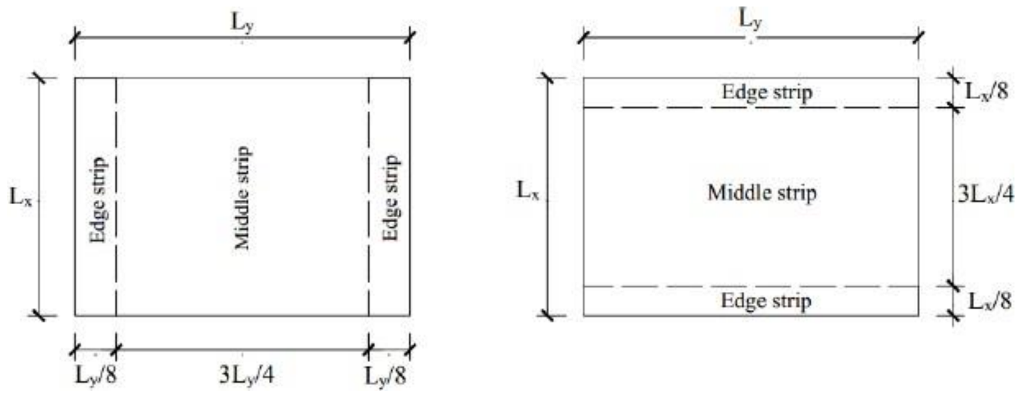

Figure 4. Division of slab into middle and edge strips [10].

According to [11], for purposes of computerization, the values of α_{sx} and α_{sy} can be obtained from

$$\alpha_{sx} = \frac{k^4}{8(1+k^4)} \tag{5a}$$

And

$$\alpha_{sy} = \frac{k^2}{8(1+k^4)} \tag{5b}$$

$$Where \quad k = ly/lx \tag{5c}$$

1.6 Restrained slab spanning in two directions

When the slabs have fixity at the supports and reinforcement is added to resist torsion and to prevent corners of the slab from lifting, the maximum moments per unit width are given by: [10]

Where: β_{sx} , β_{sy} = moment coefficients

$$M_{sx} = \beta\, a_{sx} n l_x^2 \tag{6}$$

And

$$M_{sy} = \beta\,_{sy} n l_x^2 \tag{7}$$

Where: β_{sx} , β_{sy} = moment coefficients

1.7 Introduction to Mathematical Modelling

According to [12], models represent our beliefs about how the world functions, and mathematical modeling expresses these beliefs using the language of mathematics. This approach offers several advantages like clear formulation and identification of assumptions, provision of well-defined rules which facilitate efficient expression and analysis, utilization of extensive proven mathematical results and high computational ability.

1.7.1 Regression and Correlation

Regression and correlation are statistical methods for analyzing relationships between variables, but they serve different purposes. Regression models the relationship between a dependent variable and one or more independent variables, aiming to create a predictive model for estimating the dependent variable based on the independents. It focuses on understanding the nature and strength of these relationships, aiding in prediction and identifying how changes in independent variables affect the dependent variable. Correlation, on the other hand, measures the strength and direction of the relationship between two variables without establishing causation.

1.8 Computer Analyses and Design Program

Computer programming packages are available for analyses of thermal effects on bridges. Most of the software like STAADPRO, SAFE are sold as commercial packages and will require adequate training to use. Computer programs can also be written to validate the results of manual analyses and to reduce the time spent as well as the error made during manual calculations. Several object-oriented programming languages have been developed in recent years. These include C++, C#, and Java. Java is one of the more popular object-oriented programming languages because it has several unique features. During the last fifteen years, finite element development has gradually shifted from procedural approach (Fortan, C) towards an object-oriented approach. Mostly, object-oriented finite element algorithms have been implemented in C++ programming language. It was shown that an object-oriented approach with the C++ programming language could be used without sacrificing computational efficiency as compared to Fortran. The Java language, introduced by Sun Microsystems in over two decades ago, possesses features that make it attractive for use in computational modeling. Java is a simple language (simpler than C++). It has a rich collection of libraries implementing various APIs. Java makes it is easy to create GUIs and to communicate with other computers over a network. With Java memory leaks is prevented with built-in garbage collection mechanism. Another advantage of Java is its portability. Java virtual machines (JVM) are developed for all major computer systems. JVM is embedded in most popular Web browsers in form of applets. Applets can be downloaded through the Internet and executed within a web browser. Useful for object-oriented design Java features are packages for organizing classes and prohibition of class multiple inheritance. This allows cleaner object-oriented design in comparison to C++.

2. Materials and Methods

2.1 Specifications for Design of Interior Panel of Ribbed (Waffle) Slabs using Approximate Methods

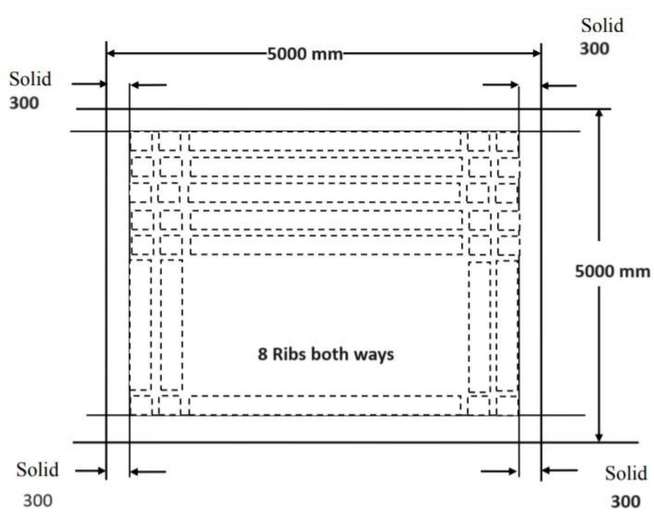

Figure 5a. Plan of waffle slab.

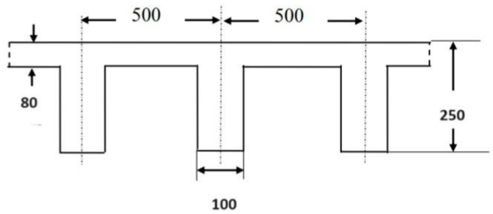

Figure 5b. Section through the slab

a. Slab Identification

The Interior panel of a waffle R.C. Slab with different aspect ratios will be analyzed by finite element using SAFE 2016 program. The slab has the following geometrical and material properties:

$Slab\ Thickness = 80mm$

$Modulus\ of\ Elasticity\ = 26.667\ kN/mm^2$

$Poisson\ Ratio = 0.2$

Loads:

The loads on floor slab are calculated on the basis of density of reinforced concrete and floor finish considered as 10 kN/m^2.

$Live\ load\ = 2\ kN/m^2$

$Dead\ load = 10kN/m^2$

$Ultimate\ load\ = 1.4G_k + 1.6Q = 1.4 \times 10 + 1.6 \times 2 = 17.2\ kN/m^2$

5m x 5m Slab (aspect ratio=1.0)

b. Model dimensions

The model has dimensions in short direction = 5 m and long direction varies with variable aspect ratios (1.0, 1.25, 1.5, 1.75, 2.0).

c. Sample slab analyses using approximate methods:

This section aims to analyze interior ribbed slab with dimensions 5 x 5 (aspect ratio = 1.0) and is restrained on all four sides. using the approximate methods such as Eurocode (EC2) and ACI 318-63 coefficient method.

The result of the analyses are as follows:

i. Eurocode Method

Moment in each direction

Short direction moments:

$Negative\ moment\ coefficient\ at\ continuous\ edge\ = -0.031$

$M_{sx} = \beta_{sx} w l_x^2$

$= 0.031 \times 17.2 \times 5^2 = 13.33\ kN.m$

$Slab\ width\ supported\ by\ one\ rib\ =\ 500\ mm.$

$The\ moment\ per\ rib\ =\ 13.33\ \times\ 0.5 =\ 6.67\ kNm$

$Positive\ moment\ at\ mid-span = 0.024$

$M_{sx} = \beta_{sx} w l_x^2$

$= 0.024 \times 17.2 \times 5^2 = 10.32\ kN.m$

$The\ moment\ per\ rib\ =\ 10.32\ \times\ 0.5 =\ 5.16kNm$

Long direction Moments:

$Negative\ moment\ coefficient\ at\ continuous\ edge\ =\ -0.032$

$M_{sy} = -\beta_{sy} w l_y^2$

$$= 0.032 \times 17.2 \times 5^2 = 13.33 kN.m$$

$The\ moment\ per\ rib\ =\ 13.33\ \times\ 0.5 =\ 6.67\ kNm$

$Positive\ moment\ at\ mid - span = 0.024$

$M_{sy} = \beta_{sy} w l_y^2$

$$= 0.024 \times 17.2 \times 5^2 = 10.32\ kN.m$$

$The\ moment\ per\ rib\ =\ 10.32\ \times\ 0.5 =\ 5.16 kNm$

ii. ACI 318-63 Coefficient Method

$M_a\ =\ C_a w l_a^2$

$M_b\ =\ C_b w l_b^2$

$la\ =\ 5.00\ -\ 0.30\ =\ 4.70\ m,$

$l_b =\ 5.00\ -\ 0.30\ = 4.70\ m$

$\frac{la}{lb}\ = \frac{4.70}{4.70}\ = 1.0$

$C_a\ =\ 0.045,\ C_y\ =\ 0.036$

Short direction moment

$Coefficient\ for\ negative\ moment\ = -0.045$

$M_a\ =\ C_a w l_a^2\ =\ 0.045\ (17.2)\ (4.7)^2\ \ = 17.1 kNm$

$The\ moment\ per\ rib\ =\ 17.1\ \times\ 0.5 =\ 8.55 kNm$

$Coefficient\ for\ positive\ moment\ = 0.018$

$$M_a\ =\ C_a w l_a^2\ =\ 0.018(17.2)(4.7)^2\ \ = 6.84\ kNm$$

$$The\ moment\ per\ rib\ =\ 6.84 \times\ 0.5 =\ 3.42 kNm$$

Long direction moment

$Coefficient\ for\ negative\ moment\ = -0.045$

$M_b\ =\ C_b w l_b^2\ =\ 0.045\ (17.2)(4.7)^2\ \ = 17.1 kNm$

$The\ moment\ per\ rib\ =\ 17.1\ \times\ 0.5 =\ 8.55\ kNm$

$Coefficient\ for\ positive\ moment\ = 0.036$

$M_b\ =\ C_b w l_b^2\ =\ 0.036(17.2)(4.7)^2\ \ = 13.68\ kNm$

$The\ moment\ per\ rib\ =\ 13.68 \times\ 0.5 =\ 6.84 kNm$

To obtain α and, β for simply supported slab:

$$\alpha = \frac{8M_a}{wl_x^2} \tag{10}$$

$$\beta = \frac{8M_b}{wl_y^2} \tag{11}$$

Apply M_a and M_b in equations (10) and (11):

$$\alpha = \frac{8C_a l_a^2}{l_x^2} \tag{12}$$

$$= \frac{8C_b l_b^2}{l_y^2} \tag{13}$$

$$-\alpha = \frac{8(0.045)(4.7)^2}{5^2} = 0.32, \qquad \alpha = \frac{8(0.036)(4.7)^2}{5^2} = 0.25,$$

$$-\ \beta = \quad \frac{8(0.045)(4.7)^2}{5^2} \quad = 0.32\,, \quad \beta = \quad \frac{8(0.036)(4.7)^2}{5^2} \quad = 0.25$$

iii. Finite Element Analyses of Ribbed Slab (Interior Panel)

Finite element Modelling of 5m x 5m Slab

The following are results were obtained from SAFE 2016 finite element program for the analyses of ribbed slab. The slab is of 5 m X 5 m dimension, with boundary beams of 30 cm X 50 cm and slab thickness is equal to 80 mm

Results of finite element analyses:

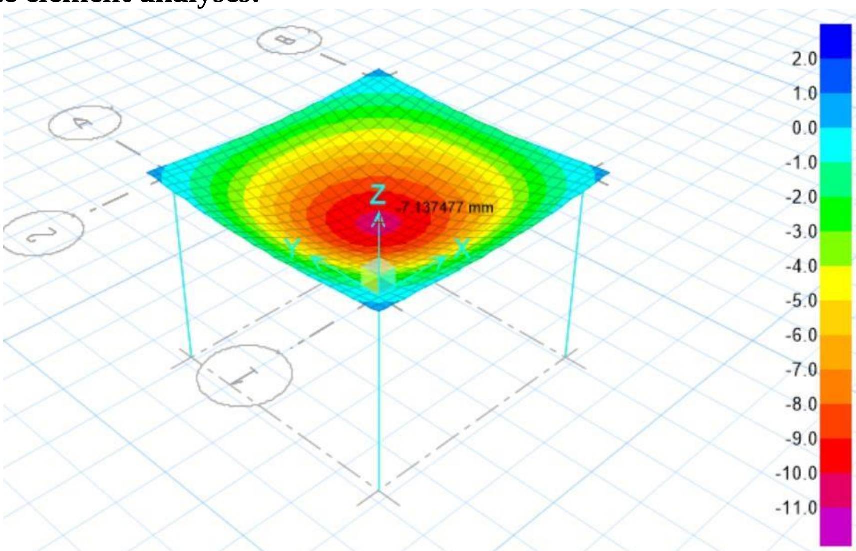

Figure 6. Deflection shape of model

Figure 6 shows deflection shape of slab and boundary beams with maximum deflection equal to 7.14 mm

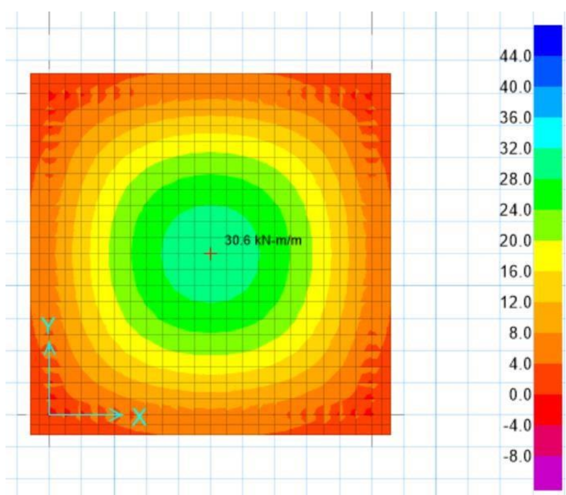

Figure 7. Bending moment diagram in X direction (short)

Bending moment map for slab in direction X and Y axis is shown in Figures 7 and 8, respectively. From both figures, it is evident that the maximum moment in the slab occurs at midspan of the slab.

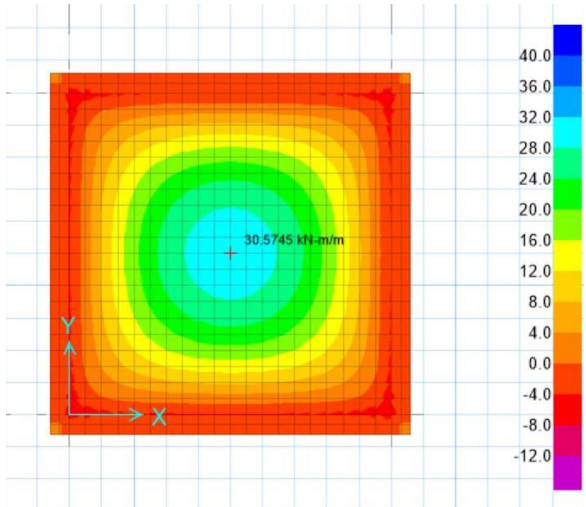

Figure 8. Bending moment diagram in Y direction (long)

To compare finite element results with other approximate methods, the average moment in a strip with width equal to 1.0 m in both directions, as shown in Figure 9 and Figure 10, respectively, will be taken. Note that the results of strip moment are less than the maximum moment shown in Figure 8 and Figure 9, since strip moment shows the average value of moment at 1.0 m width.

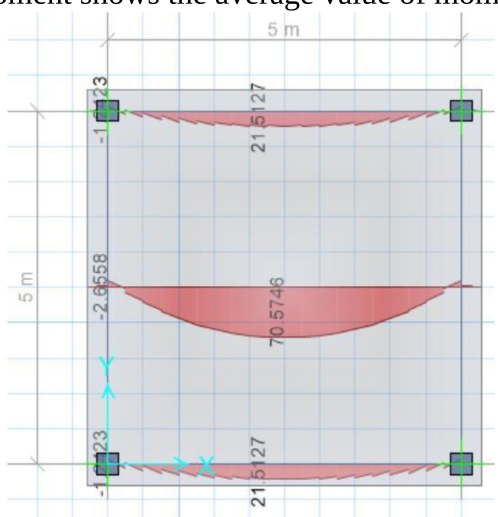

Figure 9. Bending moment for 1.0m strip in X direction (short), kN.m

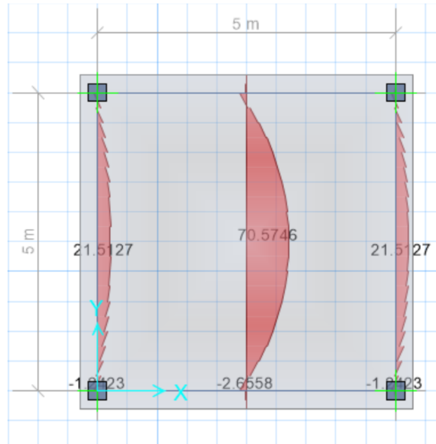

Figure 10. Bending moment for 1.0m strip in Y direction (long), kN.m

Determination of α and β for the Finite Element Method

$$\alpha = \frac{8M_x}{wl_x^2}, \quad \beta = \frac{8M_b}{wl_y^2}$$

$When\ ly/lx = 1.0,$

$$\alpha = 8 \times 30.6/17.2 \times 5^2 = 0.569$$
$$\beta = 8 \times 11.95/17.2 \times 5^2 = 0.222$$

2.2 Development of Java Program for the Application of the derived Approximate methods

The study will develop a Java based computer program for the application of the newly derived approximate Method. This method is called Calibrated Eurocode Results (CER) or Ogbonna-Umeonyiagu's coefficient method.

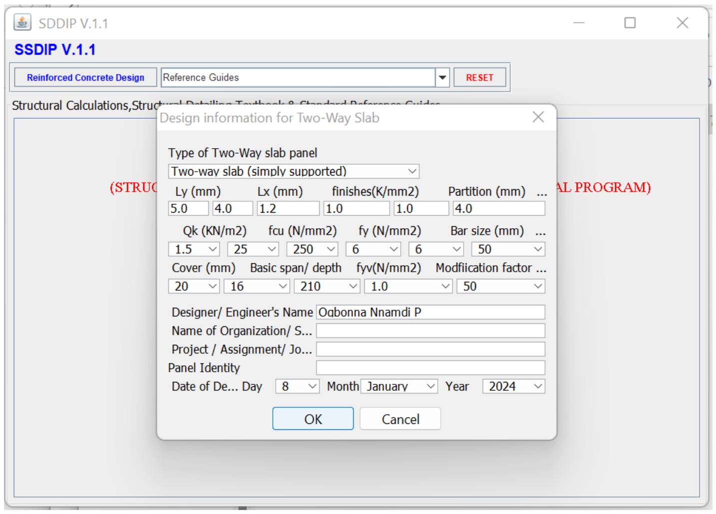

Figure 11. The Java based program SDDIP

The developed computer program is called SDDIP v.1.1, it has graphical user interface (GUI) for the analyses of rectangular solid slabs and ribbed slabs. It shows the SDDIP V.1.1 program with a design dialog box which allow the user to enter the required design information which the program uses to analyze the slab based on the coefficients derived using the CER method. The design of the slab is in accordance to [10] and Eurocode 2 provisions.

3 Results and Discussion

3.1 Finite element versus approximate methods for Ribbed Slabs (Interior Panels)

In this section, we will compare the results of finite element analyses with other approximate methods mentioned for interior panel of ribbed slabs.

Table 1. Bending Moment and Load Distribution in Short direction for Interior Panel of Ribbed Slabs)

Method	Finite Element		Eurocode (EC2)		ACI 318-63	
Ly/lx	α	Moment (kN.m)	α	Moment (kN.m)	α	Moment (kN.m)
1.00 (5x5)	0.66	30.6	0.192	10.32	0.250	6.84
1.25 (5x6.25)	0.79	42.48	0.269	14.62	0.184	10.25
1.50 (5x7.5)	0.903	48.56	0.319	17.2	0.230	12.16
1.75 (5x8.75)	0.970	52.17	0.352	18.92	0.243	13.30
2.0 (5x10)	1.01	54.11	0.118	20.64	0.254	14.10

Table 2. Bending Moment and Load Distribution in Long Direction for Interior Panel of Ribbed Slab

Method	Finite Element		Eurocode (EC2)		ACI 318-63	
Ly/lx	β	Moment (kN.m)	β	Moment (kN.m)	β	Moment (kN.m)
1.00 (4x4)	0.66	30.6	0.192	10.32	0.25	6.84
1.25 (4x5)	0.51	40.46	0.192	16.13	0.080	6.70
1.50 (4x6)	0.388	46.90	0.192	23.22	0.044	5.35
1.75 (4x7)	0.327	53.99	0.113	31.61	0.023	3.68
2.0 (4x8)	0.291	62.47	0.118	41.28	0.016	3.24

Tables 1 and 2 represent the values of moment load distribution coefficients in short and long direction which were used to perform a statistical regression on each of the approximate methods and the finite element method.

3.2 Regression Analyses for Interior Panel of Ribbed Slabs (Short Direction)

The results from the analyses of Interior Panel of Ribbed Slab are summarized in Table 3

The results are used to derive new models for the analyses of interior panels of ribbed slabs through statistical regression methods. The application of linear regression method was performed using Microsoft Excel's Data Analyses Tool and the results are present in the sub-sections as follows:

Table 3. Summary of Bending Moment in Short Direction for Interior Panel of Ribbed Slabs

Aspect Ratio	Finite Element	Eurocode	ACI
1	30.6	10.32	6.84
1.25	42.48	14.62	10.25
1.5	48.56	17.2	12.16
1.75	52.17	18.92	13.30
2	54.11	20.64	14.10

Table 4. Regression Statistics for Interior Panel of Ribbed Slabs (Short Direction)

Regression Statistics			
	FEM	Eurocode	ACI
Multiple R	0.946377	0.97759	0.958962
R Square	0.89563	0.955682	0.919608
Adjusted R Square	0.860839	0.940909	0.89281
Standard Error	3.534464	0.980551	0.948457
Observations	5	5	5

From Table 4, the Pearson correlation coefficient value (Multiple R) is equal to 0.946377 for the finite element regression results, which is very strong. Values of 0.97759, and 0.958962 were also reported for the Eurocode and the ACI methods respectively. From the ANOVA table, the significance F or P -Value of the regression models for the moments obtained from the various approximate analyses methods and the finite element method are shown in Table 5

Table 5. ANOVA Results for Interior Panel of Ribbed Slabs (Short Direction)

Analyses Method	significance F- Value
Finite Element Method	0.014785
Eurocode	0.004014
ACI-316, 63	0.009918

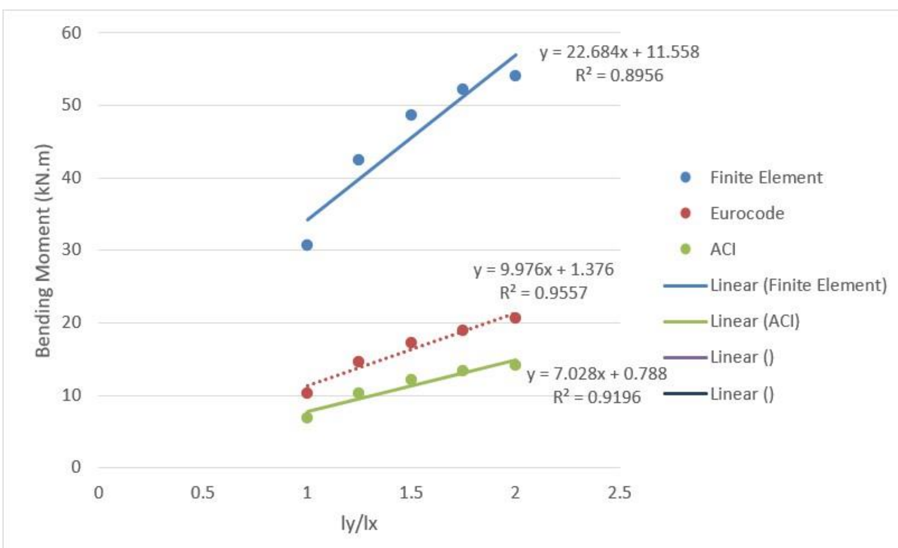

Figure 12. Variation of bending moment for Interior Panel of Ribbed Slabs using different methods (Short Direction)

3.3 Determination of the Calibration Factor (CF) for the Eurocode method using the Finite Element Method Results of Interior panel of Ribbed Slabs (Short Direction)

$$The\ Regression\ Calibration\ equation\ = y\ = 0.423x\ - 2.925 \qquad (14)$$

$$R^2\ =\ 1$$

$Where$

$y =\ the\ Eurocode\ Results\ and$

$x =\ the\ finite\ element\ results$

From Table 4.154,

$The\ slope\ of\ the\ regression\ equation\ = 0.423$

$The\ Intercept\ of\ the\ regression\ equation\ = -2.925$

To adjust the Eurocode results to match the Finite Element results, we need to transform the regression equation by inverting Equation 4.10 (Montgomery, Peck, & Vining, 2012).

$$Calibrated\ Eurocode\ result\ CER = x = \frac{y + 2.925}{0.423} \qquad (15)$$

From Equation 4.13,

$$when\ ly/lx = 1, Calibrated\ Eurocode\ Result\ CER = \frac{10.32 + 2.925}{0.423} = 31.31 kN.m$$

$$when\ ly/lx = 1.25, Calibrated\ Eurocode\ Result\ CER = \frac{14.62 + 2.925}{0.423} = 41.48 kN.m$$

$$when\ ly/lx = 1.5, Calibrated\ Eurocode\ Result\ CER = \frac{17.2 + 2.925}{0.423} = 47.58\ kN.m$$

$$when\ ly/lx = 1.75, Calibrated\ Eurocode\ Result\ CER = \frac{18.92 + 2.925}{0.423} = 51.64\ kN.m$$

$$when\ ly/lx = 2, Calibrated\ Eurocode\ Result\ CER = \frac{20.64 + 2.925}{0.423} = 55.71\ kN.m$$

Table 6. Comparison of the Bending Moment results from the Finite Element, The Eurocode and the CER methods for Interior Panels of Ribbed Slabs (Short Direction)

Aspect Ratio	Finite Element (kN.m)	Eurocode (kN.m)	CER (kN.m)	Percentage Difference (%) Between FEM & CER	Percentage Difference (%) Between Eurocode & CER
1	30.6	10.32	31.31	2.27	67.01
1.25	42.48	14.62	41.48	2.35	64.75
1.5	48.56	17.2	47.58	2.02	63.85
1.75	52.17	18.92	51.64	1.02	60.22
2	54.11	20.64	55.71	2.87	62.95

Table 6, compares the results of design momnts from the Finite Element, Eurocode and the CER methods for Interior Panel of Ribbed Slabs. The percentage difference between the bending moments obtained from Finite Element method and the Calibrated Eurocode Results (CER) method ranges 1.08% to 2.87% for aspect ratios between 1.0 to 2.0 (Table 4.6). The percentage difference between the design moments obtained from Eurocode method and the calibrated Eurocode results (CER) method is approximately 63.75% for aspect ratios between 1.0 to 2.0.

3.4 Regression Analyses for Interior Panel of Ribbed Slabs (Long Direction)

From Table 7, the Pearson correlation coefficient value (Multiple R) is equal to 0.997672 for the finite element regression results, which is very strong. Values of 0.995185 and 0.969739 were also reported for the Eurocode and the ACI methods respectively. From the ANOVA table, the significance F or P -Value of the regression models for the moments obtained from the various approximate analyses methods and the finite element method are shown in Table 8

Table 7. Regression Statistics for Interior Panel of Ribbed Slabs (Long Direction)

	Regression Statistics		
	FEM	**Eurocode**	**ACI**
Multiple R	0.997672	0.995185	0.969739
R Square	0.995349	0.990392	0.940394
Adjusted R Square	0.993798	0.98719	0.920525
Standard Error	0.96437	1.391819	0.469766
Observations	5	5	5

Table 8 ANOVA Results for Interior panel of Ribbed Slabs (Long Direction)

Analyses Method	significance F- Value
Finite Element Method	0.000135
Eurocode	0.000401
ACI-316, 63	0.00629

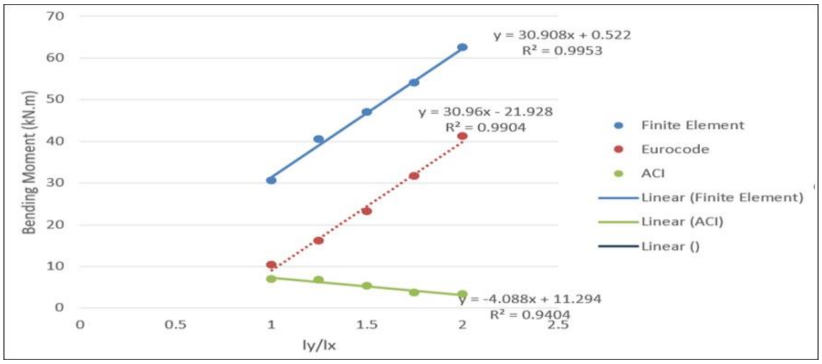

Figure 13. Variation of bending moment for Interior Panel of Ribbed Slabs using different methods (Long Direction)

The variation of bending moment for ribbed slabs using the different approximate methods and the finite element method for the longer direction of the slab is shown in Figure 13

3.5 Determination of the Calibration Factor (CF) for the Eurocode method using the Finite Element Method Results (Long Direction)

$$The\ Regression\ Calibration\ equation\ = y\ = 0.995x\ - 22.13 \quad (16)$$

$$R^2\ =\ 1$$

$Were$

$y =\ the\ Eurocode\ Results\ and$

$x =\ the\ finite\ element\ results$

$The\ slope\ of\ the\ regression\ equation\ = 0.995$

$The\ Intercept\ of\ the\ regression\ equation\ = -22.13$

To adjust the Eurocode results to match the Finite Element results, we need to transform the regression equation by inverting Equation 16 [13].

$$Calibrated\ Eurocode\ result\ CER = x = \frac{y + 22.13}{0.995} \quad (17)$$

From Equation 17,

$$when\ ly/lx = 1, Calibrated\ Eurocode\ Result\ CER = \frac{10.32 + 22.13}{0.995} = 32.61kN.m$$

$$when\ ly/lx = 1.25, Calibrated\ Eurocode\ Result\ CER = \frac{16.13 + 22.13}{0.995} = 38.45kN.m$$

$$when\ ly/lx = 1.5, Calibrated\ Eurocode\ Result\ CER = \frac{23.12 + 22.13}{0.995} = 45.48\ kN.m$$

$$when\ ly/lx = 1.75, Calibrated\ Eurocode\ Result\ CER = \frac{31.61 + 22.13}{0.995} = 54.01\ kN.m$$

$$when\ ly/lx = 2, Calibrated\ Eurocode\ Result\ CER = \frac{41.28 + 22.13}{0.995} = 63.72\ kN.m$$

The percentage difference between the design moments obtained from finite element method and the calibrated Eurocode results (CER) method ranges 2 % to 6.16 % for aspect ratios between 1.0 to 2.0 (Table 12). It can also be seen from Table 12 that the percentage difference between the results from the Eurocode method and the CER method for Short Direction of Interior Panel of Ribbed Slabs is approximately 50.4%.

The percentage difference between the design moments obtained from finite element method and the calibrated

Eurocode results (CER) method ranges 2 % to 6.16 % for aspect ratios between 1.0 to 2.0 (Table 9). It can also be seen from Table 12 that the percentage difference between the results from the Eurocode method and the CER method for Short Direction of Interior Panel of Ribbed Slabs is approximately 50.4%.

Table 9. Comparison of Bending Moment results from the Finite Element, The Eurocode and the CER methods for Interior Panel of Ribbed Slabs (Long Direction)

Aspect Ratio	Finite Element (kN.m)	Eurocode (kN.m)	CER (kN.m)	Percentage Difference Between FEM & CER(%)	Percentage Difference between Eurocode & CER (%)
1	30.6	10.32	32.61	6.16	68.35
1.25	40.46	16.13	38.45	4.97	58.04
1.5	46.9	23.22	45.48	3.03	48.94
1.75	53.99	31.61	54.01	2.00	41.47
2	62.47	41.28	63.72	2.00	35.22

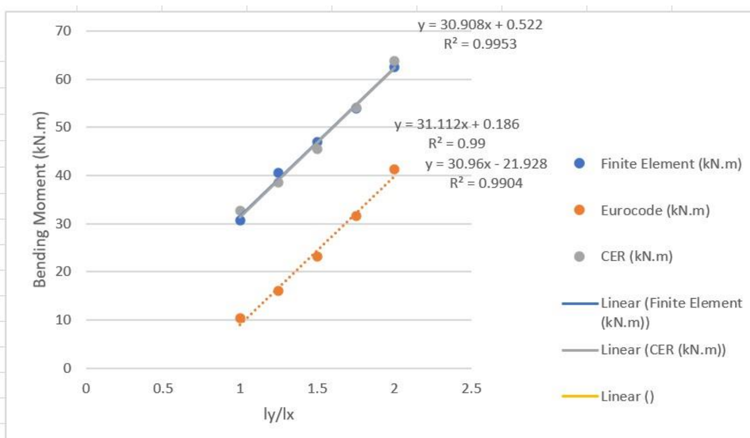

Figure 14. Comparison between the Eurocode, Finite Element and the Calibrated Eurocode (CER) methods.

3.7 Derivation of Approximate Method for the Analyses of Ribbed Slab Spanning in Two Directions (Interior Panels)

The bending moment coefficients for Interior Panels of Restrained slabs given in [10] used in the analysis of Interior Panel of Ribbed Slabs. Table 10 shows the ratios of Bending Moment results from Eurocode and CER methods for Interior panel of Ribbed Slab (Short Direction). The average ratio of the bending moments is 2.813.

Table 10. Ratios of the Bending Moment results from Eurocode and CER methods for Interior panel of Ribbed Slab (Short Direction)

Aspect Ratio	Eurocode (kN.m)	CER (kN.m)	Ratio
1	10.32	31.31	3.03
1.25	14.62	41.48	2.84
1.5	17.2	47.58	2.77
1.75	18.92	51.64	2.73
2	20.64	55.71	2.67

Table 10 shows the ratios of Bending Moment results from Eurocode and CER methods for Interior panel of Restrained Slab (Long Direction). The average ratio of the bending moments is 2.14 for the long direction and 2.48 for either direction. The value of 2.48 is used to multiply the original a_{sx} *and* a_{sy} ratios from Eurocode [10] to obtain the derived a_{sx} *and* a_{sy} coefficient for the CER method in Table 11

.

Table 11. Comparison of the Bending Moment results from Eurocode and CER methods for Interior panel of Restrained Slab (Long Direction)

Aspect Ratio	Eurocode (kN.m)	CER (kN.m)	Ratio
1	10.32	32.61	3.12
1.25	16.13	38.45	2.38
1.5	23.22	45.48	1.96
1.75	31.61	54.01	1.71
2	41.28	63.72	1.54

Table 12. Bending moment coefficients for the Analyses of Interior Panels of Restrained Slab Spanning in Two Directions (CER Method)

Types of panel and moments considered	Short Direction coefficients for values (βsx) ly/lx								Long Direction coefficients for all values of ly/lx (βsy)
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
					Interior Panels				
Negative moment a continuous edge	0.077	0.102	0.104	0.114	0.124	0.131	0.146	0.156	0.089
Positive moment at mid-span	0.060	0.069	0.079	0.087	0.091	0.099	0.109	0.119	0.060

3.8 Application of the Developed Computer Program for the Analyses of Ribbed Slabs using CER Method

The results from the analyses of Interior Panel of Ribbed Slab (Short Direction) are summarized in Table 13.

Table 13. Summary of Bending Moment in Interior Panel of Ribbed Slab (Short Direction)

Aspect Ratio	Finite Element (kN.m)	CER (Manual Analyses) (kN.m)	CER (SDDIP V.1.1) (kN.m)
1	30.6	31.31	31.94
1.25	42.48	41.48	42.14
1.5	48.56	47.58	47.17
1.75	52.17	51.64	52.26
2	54.11	55.71	55.10

Figure 15, shows a comparison between the results from the Finite Element Program, the Manual Analyses using CER Coefficients and the CER results from the Developed Program SDDIP V.1.1 for Interior Panel of Ribbed Slabs (Short Direction).

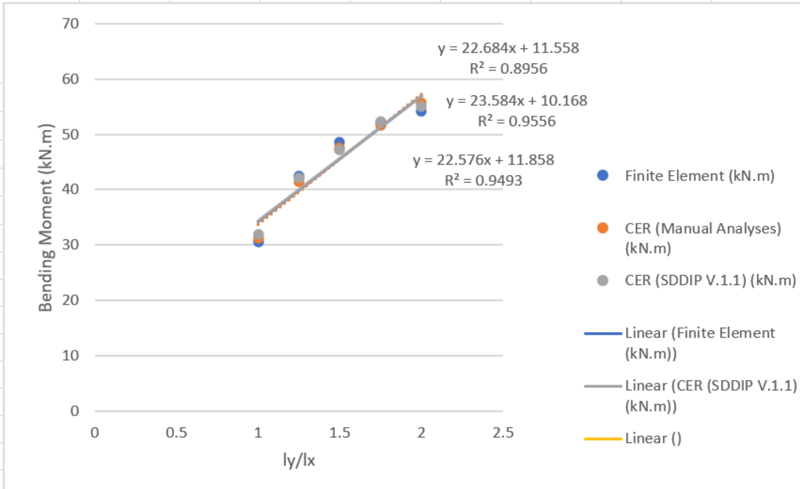

Figure 15. Comparison between the Finite Element method, the Manual analyses using CER Coefficients and the CER results from the Developed Program SDDIP V.1.1 for Interior Panel of Ribbed Slabs (Short Direction)

Table 14 and Figure 16, shows a comparison between the results from the Finite Element Program, the Manual Analyses using CER coefficients and the CER results from the Developed Program SDDIP V.1.1 for Interior Panel of Ribbed Slabs (Long Direction).

Table 14. Summary of Bending Moment in Interior Panel of Ribbed Slab (Long Direction)

Aspect Ratio	Finite Element (kN.m)	CER (Manual Analyses) (kN.m)	CER (SDDIP V.1.1) (kN.m)
1	30.6	32.61	32.55
1.25	40.46	38.45	38.14
1.5	46.9	45.48	46.14
1.75	53.99	54.01	54.72
2	62.47	63.72	63.46

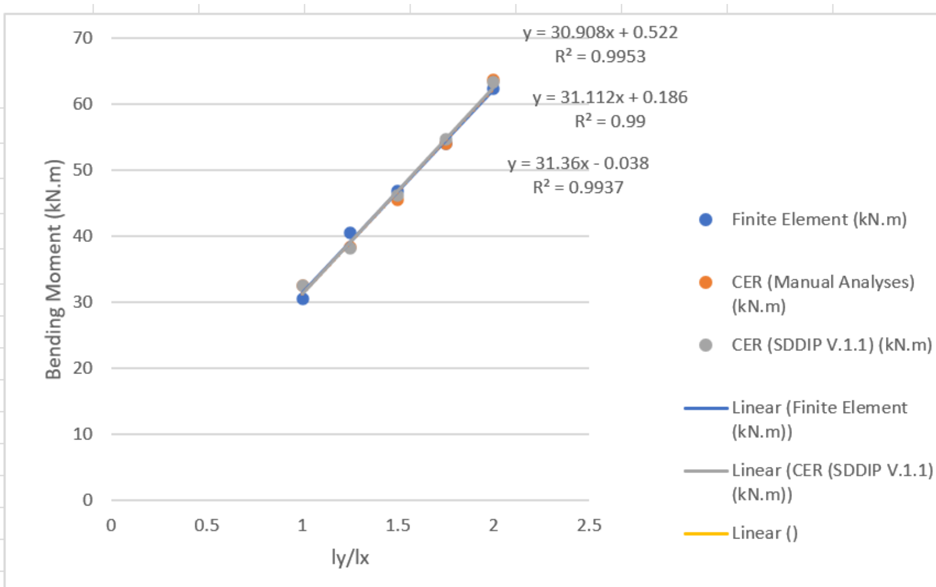

Figure 16. Comparison between the Finite Element Method, the manual analyses using CER Coefficients and the CER results from the Developed Program SDDIP V.1.1(Ribbed Slabs)

4. Conclusion

The findings of this research can be summarized as follows:

i.) The design moments derived from both approximate and finite element methods showed a high degree of similarity for two-way simply supported rectangular slabs.

ii.) In contrast, the design moments for restrained slabs and ribbed (waffle) slabs revealed significant discrepancies between the results of the approximate methods and the finite element methods, necessitating further statistical regression analyses.

iii.) The Rankine-Grashoff and Marcus methods were found to be inadequate for analyzing restrained and ribbed slabs.

iv.) Among the evaluated methods, the results obtained using the Eurocode method demonstrated superior alignment with the finite element method compared to other existing approximate methods.

v.) The statistical regression analysis, utilizing the finite element method as the independent variable and the Eurocode method as the dependent variable, produced a calibration factor. This factor serves to refine the accuracy of the Eurocode method's results.

vi.) By applying this calibration factor to the Eurocode method's outputs, the results closely approximate those derived from the finite element method, reducing the error margin from over 65% to about 6%.

vii.) The established regression calibration factor can facilitate the formulation of a new approximate coefficient, simplifying and enhancing the analysis of slabs.

viii.) Additionally, the developed Java software provides a practical tool for implementing the newly derived approximate method, known as Ogbonna-Umeonyiagu's method / CER method, in computational applications.

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