NIPES

Journal of Materials Engineering, Structures and Computation

Journal homepage: www.nipesjournals.org.ng

Scheffe’s Polynomial Optimisation of Split Tensile Strength of Palm Kernel Shells Aggregate Modified Concrete

Abah Johnson Candy[a*], Emmanuel E. Ndububa[b], Ben Ughuoke[c] Ocholuje S. Ogbo[d],

.PhD. Student, Department of Civil Engineering, University of Abuja, Abuja, Nigeria
[b]Senior Lecturer, Department of Civil Engineering, University of Abuja, Abuja, Nigeria
[c]Senior Lecturer, Department of Mechanical Engineering, University of Abuja, Abuja, Nigeria
[d]Lecturer, Department of Civil Engineering, University of Abuja, Abuja, Nigeria
[*]Corresponding Author: ajonye2006@gmail.com

ARTICLE INFORMATION

Article history:
Received 24 October 2023
Revised 28 November 2023
Accepted 30 November 2023
Available online November 2023

Keywords:
Palm Kernel Shell; Optimisation; Scheffe’s Polynomial; Split Tensile Strength; wolfram mathematical

https://doi.org/10.5281/zenodo.10228303

ABSTRACT

The environmental consequence of natural aggregate exploitation and Agro-base waste management especially open-air incineration is immense. In this research a mathematical model was developed to optimize the mixture design of concrete using Palm Kernel Shells PKS as partial (60%) replacement in composite coarse aggregate of palm kernel shell and crushed granite (PKSCG) to forester environmental sustainability. The research work is to optimize the split tensile strength of Palm Kernel Shell aggregate modified concrete based on improved Scheffe`s second degree polynomial, where the strength was obtained for various ratios using the Scheffe`s simplex lattice method. To test for the adequacy of the model, student`s t-test and Fishers (ANOVA) were used where the results of test indicates that the model strengths are generally in good conformity with the corresponding observed experimental results. The optimised tensile strength by the model at 28 days of curing within the factor space was 1.06481Mpa, while the experimental results yield a maximum and minimum strength of 1.0963 MPa and 0.5658N MPa at same curing period with corresponding to mix ratio of [0.625:1.0:1.925:1.09:1.635]and [0.675:1.0:2.15:1.22:1.83] respectively for water, cement, fine aggregate, granite and PKS respectively. The coefficients of determination (R^2) was 99.99% for the split tensile response models, while the p-value obtained for the response coefficient fit parameters $\beta_i,\ \beta_{ij},\ \beta_{ijk}$ for $(i = 1, 2, 3, 4, 5)$ was 99.98% for the modelled strength. Wolfram Mathematica application was used for data fitting and optimization computation. The Scheffe`s optimization theory can be used to analyse and optimise concrete strength and possible mix proportions of concrete ingredient as the comparison of the split tensile strengths obtained between the laboratory test results and the model results show a good pact, consequently indicating that the optimisation equations can sufficiently predict the mechanical properties of the concrete at 28 days of curing. The palm kernel shells modified concrete is appropriate for applications necessitating Low- strength Concrete (LSC), and non-structural light weight concrete.

1. Introduction

By continuously exploiting our natural resources to find building materials like aggregates, among other things, the construction sector greatly contributes to environmental degradation. This is a well-known polluting activity. Because aggregate mining continuously degrades and destroys the beauty of our natural surroundings, it poses a major threat to the ecosystem. By continuously exploiting our natural resources to find building materials like aggregates, among other things, the construction sector greatly contributes to environmental degradation. This is a well-known polluting activity. Because aggregate mining continuously degrades and destroys the beauty of our natural surroundings, it poses a major threat to the ecosystem. The old-fashioned method of looking for standard building supplies, like hollow, solid, and hollow blocks as well as concrete, which are all obtained from naturally occurring resources, is bad for the environment because the country's natural resources are constantly being depleted [1]. Due to material loss and the lack of standards in the trial mix percentage procedure, construction projects are needlessly costly. In the building industry, errors resulting from trial-and-error methods or the so-called traditional mix procedure, which involve incorrect computation and mixing, frequently cause structural failure and waste material and time [2]. One amount is often changed at a time in the traditional method of mixture design, which makes it too expensive for most businesses to use in terms of both time and money [3]. In response to this, professionals working in the built environment, including architects and engineers, have long worked to minimize the negative effects of construction on the environment by using alternative building materials, such as recycled materials made primarily of palm kernel shell and Agro-based and industrial waste. Cement, fine and coarse aggregate, and water acting as a hydration agent are the elements that make up concrete by nature. Concrete has been deemed the most widely used building material worldwide because of its inexpensive cost, abundant supply of raw materials, ease of production, ease of casting into different shapes, and strong resistance to corrosion and fire [4]. The proportions of cement, aggregates, and water in the mixture when it is both pliable and hardened determine the characteristics of the concrete. Additionally, adding additional cementitious material or a chemical admixture might enhance these qualities [5]. Nigeria produces more than 2.5 million tons of PKS annually, and improper disposal or management could have a negative impact on the environment [6]. The oil palm belt of Nigeria extends over twenty-four states in the southern part of the country, which includes all of the Niger Delta states. Eighty percent of Nigeria's oil palm production comes from dispersed smallholders who collect semi-wild trees and process them by hand. Over an estimated range of 1.65 million hectares to 2.4 million hectares and up to 3 million hectares, several million smallholders are dispersed. The approximate area of oil palm plantations in Nigeria varies from 360,000 hectares to 169,000 hectares, with 72,000 hectares consisting of estate plantations and 97,000 hectares of smallholder plantings [7]. Following Indonesia, Malaysia, and Thailand, which account for more than 85% of the world's total palm oil production, Nigeria and Colombia are currently in fourth place in terms of palm plantations and oil production [8]. Waste material known as palm kernel shells (PKS) is produced in palm oil mills when palm nuts are crushed to extract palm oil. It is among the waste products that are produced in South East Asia and Africa in the greatest quantities. Since 1984, PKS has been tested in research as lightweight aggregates (LWAs) to create lightweight concrete (LWC), and a large number of researchers are currently engaged in this field [9]. The solid wastes from processing fresh fruit bunches (FFB) include empty fruit bunches (EFB), mesocarp fiber (MF), and palm kernel shell (PKS), which are significant biomass in the oil palm sector [10]. These wastes are referred to as FFB wastes. The agricultural waste known as palm kernel shell (PKS) is acquired from the threshing, crushing, and extraction processes carried out in the palm oil processing mill. It is most abundant in the tropical regions of the world, especially Asia and Africa. More than 1.5 million tons of palm kernel trash are produced as a result of Nigeria's high demand for palm kernel oil. Due to the large amount of PKS waste, disposal and environmental issues arise. In addition to being utilized in building, PKS is a fuel in the community. PKS particles vary in shape and have edges that are slightly rough and spiky, depending on the extraction technique used. The particles have a thickness ranging from 1.5 mm to 4 mm and are indicative of a smooth surface on the concave or convex face. Because PKS is an organic substance, its skin pores allow for a high absorption of water [11]. Large amounts of palm kernel shell (PKS) trash are produced during the processing of palm fruits to make palm oil. These wastes must be used for a variety of purposes, as the shells are typically burned in the open or left as waste [12]. Over time, palm kernel shells have been utilized as a lightweight aggregate in the manufacturing of concrete, showing promise in practically every lightweight concrete property [13]. For heavy traffic roads, PKS replacement of aggregates is determined to be appropriate at 10% and 50%, respectively, in place of coarse aggregates in highway pavements [14]. After 28 days, the PKS replacement of 10% and 25% of the coarse aggregate resulted in 4.78 MPa and 4.44 MPa of compressive strength, respectively. Based on these strengths, the lightweight structural concrete requirements were not met [15].

When 60% of the PKS is substituted for 40% of the coarse aggregate (granite) after 28 days of curing, Scheffe's Theory is applied to maximize the split tensile strength of the five component, two-degree (5, 2) polynomial model. Post-harvest waste management and disposal, particularly of industrial and Agro-based wastes, of which palm kernel shells are one, has long presented a number of challenges to all countries, regardless of their degree of development. Since this will leave us with secured and sustainable environment, and allow industrial waste products to be used profitably, it is applaudable for the elimination of the environmental hazard caused by aggregate mining and open incineration of PKS as solid fuel or disposal technique.

One crucial metric used to characterize the mechanical properties of concrete is splitting tensile strength. Airfield runways and pavement slabs are vulnerable to tensile pressures since their design is focused on bending strength, but plain concrete structures like dams require tensile strength in the event of an earthquake. Due to the conventional test procedure's known size effect and boundary condition constraints, the splitting tensile test is a frequently used method to ascertain the tensile strength of concrete. [6, 13].

2.0 Materials and method

2.1 Materials

Cement, water, fine, crushed granite, and palm kernel shells were the materials employed in this investigation. The palm kernel shell was acquired from Utonkon in the Nigerian state of Benue, in the Ado local government. The palm kernel shells were sun-dried for 72 hours after being cleaned with tap water. The crushed granite came from a quarry near Maru in the state of Zamfara, while the fine aggregate came from the riverbank in Kaura Namoda. The cement was a Dangote product that was purchased at the Kaura Namoda cement open market. Concrete was cast and cured using water from a borehole.

Figure1a. PKS

Figure 1b. Crushed granite

2.1 Mix design

The Concrete Mix Design Manual [16] was followed when creating the mix design. By measuring the computed amounts from the mix design shown in Table 1, Ordinary Portland Cement (OPC) Grade-42.5, fine-aggregate, coarse-aggregate, palm kernel shells, and water were proportioned.

2.2 Making of samples

To achieve a consistent consistency in the aggregate mixture, measured amounts of water, cement, fine-aggregate, palm kernel shells, and coarse-aggregate (as listed in Table 1) were added to the concrete mixer and mixed for approximately two minutes. After that, the wet mix was put into cylindrical molds measuring 150 mm in diameter by 300 mm in length, and a tamping rod was used to compact it. With a hand trowel, more of the wet mix was applied and levelled out. After a day, the samples were demoulded and put into a room-temperature curing tank filled with water, where they cured for a total of 28 day

2.4 Tests Method

At 28 days, the split tensile strengths of two samples per mix were evaluated. For the modelling and analysis, the mean value of the measured tensile strengths was employed. The Federal Polytechnic Kaura Namoda Department of Civil Engineering's Magnus Compression testing apparatus was used to perform the split tensile strength test.

2.4.1. Split Tensile strength test

Using cylindrical concrete measuring 150 mm in diameter and 300 mm in length, the splitting tensile test was carried out in compliance with [17, 18, 19]. Using the following equation, the specimen's measured splitting tensile strength (FT) was computed to the closest 0.05N/mm2. For the purpose of the research, sixty (60) cylindrical concrete pieces measuring 150 mm in diameter by 300 mm in length were created; thirty of them were used in the experimental procedure, and the remaining thirty served as the control for the fifteen points.

At the 28-day curing period, two specimens of each combination were crushed, and the average strength was noted.

$$F_T = \frac{2P}{\pi L D} \qquad (1)$$

where;
F_T = splitting tensile strength in N/mm^2
P = maximum load applied to the specimen in kN
L = length of the specimen in mm
D = diameter of specimen in mm

Figure2. Split tensile Testing

2.5 Theory/Calculations

2.5.1Scheffe's factor space

The creation of a simplex lattice ordered line arrangement connecting experimental places is necessary to implement Scheffe's hypothesis. This is made up of m+1 vector point that are uniformly spaced within values ranging from 0 to 1 for design mixtures with m degree [20, 21]. Equation (2) provides the minimum number N of experimental trials needed for the sample responses to be robustly modelled. Considering the sensitivity of the use of non-conventional aggregates like periwinkle shell in concrete **[22],** a robust objective function is required for optimisation computations, therefore, the second-degree Scheffe's polynomial is considered. Given the 5-component mixture considered for this work, the computation of the variation of $N(q,m)$ with m where, $q = 5$ and $m = 1,2,3,4,5,$ yields N-values as shown in Figure 5.

$$N(q,m) = \frac{(q+m-1)!}{m!\,(q-1)!} \qquad (2)$$

It was determined that N(5,2)=15 was the representative required number of responses needed for the split tensile stresses test of the hardened concrete, and that this objective function would be used in the optimization simulations. It can be inferred that a minimum of fifteen simplex lattice mixture points are necessary. Pseudo-component factions X1, X2, X3, X4, and X5 represent the components in the following order: water, cement, fine aggregate, coarse aggregate, and palm kernel shell.

2.5.2. Split tensile strength response modeling

A combination of the linear and quadratic form of the response model is required for which $0 \leq i \leq j \leq k \leq 5$, with the general presented in Eqn. (3).

$$n(X) = \sum_{i=1}^{n} \beta_i X_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \beta_{i_j} X_i X_j + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \beta_{i_{i_j}} X_i X_i X_j \tag{3}$$

The five components that varied in the mix proportions and were identified by Eqn. (2) comprised the general response models that relate the split tensile strength responses of the hardened concrete. The model equations, which served as objective functions in conjunction with the maximum and lowest values of compressive and flexural strengths discovered by laboratory testing, were fed the results of the laboratory experiments.

$$\begin{aligned} n(X_i) \; = \; & X_1\beta_1 + X_2\beta_2 + X_3\beta_3 + X_4\beta_4 + X_5\beta_5 + X_1X_2\beta_{1_2} + X_1X_3\beta_{1_3} \\ & + X_1X_4\beta_{1_4} + X_1X_5\beta_{1_5} + X_2X_3\beta_{2_3} + X_2X_4\beta_{2_4} + X_2X_5\beta_{2_5} \\ & + X_3X_4\beta_{3_4} + X_3X_5\beta_{3_5} + X_4X_5\beta_{4_5} \end{aligned} \tag{4}$$

Equations (5) and (6) describe the restraint parameters, which are utilized in conjunction with the goal function for the optimization computations. The Wolfram language's "Maximize" and "Minimize" functions were implemented with the necessary constraint conditions, and the optimized values of each pseudo-mixture component, (X i), were found. Definition of Equation (5):

$$X_1 + X_2 + X_3 + X_4 + X_5 = 1 \tag{5}$$

Where characteristic values of X_i are positive real numbers such that:

$$0 \le X_i \le 1, \, i = 1, 2, 3, 4, 5 \tag{6}$$

A further boundary was constructed using the laboratory lowest and maximum strength values, designated as f_{min} and f_{max}, respectively, and the goal function of Eqn. (4) inside the bounds defined in Eqns. (5) and (6) (see Eqn. (7) [23]). Since the control mixture's mechanical properties were returned by the optimization using Equations (5) and (6), the introduction of this boundary was required. Therefore, adding Eqn. (7) improved the solution's robustness.

$$f_{min} \le n(X) \le f_{max} \tag{7}$$

The proportioning of the constituent water, cement, fine-aggregate, coarse aggregate, and palm kernel shell was carried out by trials in line with the requirements of **[24,25].**

2.6 Mixture component modelling

The 5-component mixture pseudo components for a N (5, 2) = 15 simplex lattice combination that needs 15 trial mixtures were produced using the information displayed in Table 1.

2.6.1 Pseudo components

Scheffe's method of mixes was used to construct a total of fifteen trial mixtures for a simplex lattice structure of N(5,2). Points on the (5, 2) simplex tetrahedron whose vertex corresponds to a pseudo component, X1, X2, X3, X4, where used to determine the distribution of the mixed points. These components represent water, cement, fine aggregate, and composite coarse aggregate made of crushed granite and palm kernel shell, as shown in Table 1. Five of the fifteen required mixture components are located at the vertices of the simplex tetrahedron, as can be seen in Figure 6, which illustrates how the pseudo components were constructed.

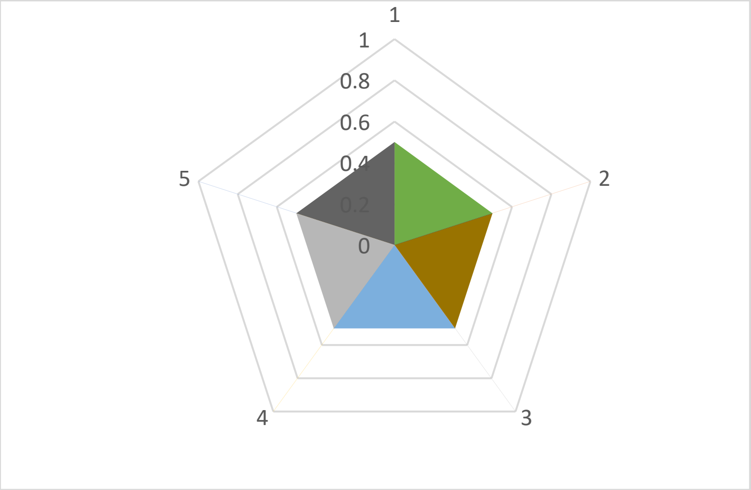


Figure 6. A multidimensional plot of the 5-pseudo-components factor space

2.6.2 Actual components (Z_i)

The relationship between the constant actual components A, the pseudo components X and the real component Z is given by Eqn. (8).

$$Z = X\,A \tag{8}$$

Equation (9) presents the values of the components of the A- matrix, which are made up of five trial mix ratios Zi for the values of i = 1, 2, 3,... 5, derived from trials mixes and experiments. Equations (8) and (9) are used to calculate the corresponding values of the real components (Zi), which are then displayed in Table 1.

$$A = \begin{bmatrix} 065 & 0.60 & 0.55 & 0.70 & 0.68 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 2.0 & 1.75 & 1.55 & 2.3 & 2.15 \\ 1.14 & 1.0 & 0.88 & 1.3 & 1.22 \\ 1.71 & 1.5 & 1.32 & 1.95 & 1.83 \end{bmatrix} \tag{9}$$

2.6.3 Transformation of optimized values of pseudo–components to actual mix components

The usage of component mix ratio values greater than unity is restricted by the criteria of Eqn. (5) for $\sum I = Xi$ (Obam, 2006). It becomes necessary to convert the optimal values of the real mixture values Zi for (i=1-5) once the optimization algorithm has been applied to get the optimal values of Xi for (I =1–5). For the transformation, a variant of Eqn. (8), i.e., Eqn. (10), is consequently necessary.

$$Z_i = A_i^T \cdot X_i \tag{10}$$

Where (Zi) are the converted values, Ai^ T is the transpose of Ai, and Xi is the vector of optimized values of the pseudo components. (Zi) stands for sets of matrices that match the serial numbers 1–5, 6–10, and 11–15 of the corresponding groups.

Table 1. Matrix table for Scheffe's $N(5,\ 2)$- water, cement, fine-aggregate, granite and PKS

S/No.	Points	Pseudo Components					Response	Real Components				
		X1	X2	X3	X4	X5		Z1	Z2	Z3	Z4	Z5
1	Z1	1	0	0	0	0	0.813	0.650	1.000	2.000	1.140	1.710
2	Z2	0	1	0	0	0	0.743	0.600	1.000	1.750	1.000	1.500
3	Z3	0	0	1	0	0	0.743	0.550	1.000	1.550	0.880	1.320
4	Z4	0	0	0	1	0	0.920	0.700	1.000	2.300	1.300	1.950
5	Z5	0	0	0	0	1	0.920	0.680	1.000	2.150	1.220	1.830
6	Z12	0.5	0.5	0	0	0	0.743	0.625	1.000	1.875	1.070	1.605
7	Z13	0.5	0	0.5	0	0	0.849	0.600	1.000	1.775	1.010	1.515
8	Z14	0.5	0	0	0.5	0	0.778	0.675	1.000	2.150	1.220	1.830
9	Z15	0.5	0	0	0	0.5	0.955	0.665	1.000	2.075	1.180	1.770
10	Z23	0	0.5	0.5	0	0	1.026	0.575	1.000	1.650	0.940	1.410
11	Z24	0	0.5	0	0.5	0	0.743	0.650	1.000	2.025	1.150	1.725
12	Z25	0	0.5	0	0	0.5	0.884	0.640	1.000	1.950	1.110	1.665
13	Z34	0	0	0.5	0.5	0	1.096	0.625	1.000	1.925	1.090	1.635
14	Z35	0	0	0.5	0	0.5	0.566	0.615	1.000	1.850	1.050	1.575
15	Z45	0	0	0	0.5	0.5	1.061	0.690	1.000	2.225	1.260	1.890

3. Results and Discussions

3.1 Split tensile strengths

The split tensile F_T strengths gotten from the experiments specify the range of response values achievable from the factor space and the results are as presented in Table 2.

Table 2. Results of sample response to Scheffe's $N(5, 2)$water, cement, fine-aggregate, coarse aggregate,
palm kernel shell - Lattice composition

S/No.	Points	Tensile Strength Test (MPa)		Standard Deviation	Mean
		Test 1	Test 2		
1	Z_1	0.7780	0.8487	0.05	0.8134
2	Z_2	0.6365	0.8487	0.15	0.7426
3	Z_3	0.7073	0.7780	0.05	0.7427
4	Z_4	0.9902	0.8487	0.1	0.9195
5	Z_5	0.9195	0.9195	0.0	0.9195
~~6~~	~~Z_{12}~~	~~0.7073~~	~~0.7780~~	~~0.05~~	~~0.7427~~
7	Z_{13}	0.7780	0.9195	0.1	0.8488

8	Z_{14}	0.7073	0.8487	0.1	0.7780
9	Z_5	0.9902	0.9195	0.05	0.9549
10	Z_{23}	1.1316	0.9195	0.15	1.0256
11	Z_{24}	0.6365	0.8487	0.15	0.7426
12	Z_{25}	0.6365	1.1316	0.35	0.8841
13	Z_{34}	1.0609	1.1316	0.05	1.0963
14	Z_{35}	0.5658	0.5658	0.0	0.5658
15	Z_{45}	1.2024	0.9195	0.2	1.0610

3.2 Response models

The coefficients βi (for i = 1,2...,5) needed to fit the data from the Split tensile strength laboratory test were determined by using the Wolfram language's "Non-Linear Model Fit" function. The models given in Equations (11) show values for n(Xt) of 99.98%. Table 3 contains a list of the coefficient values. After that, the generated model served as the computation's objective function for optimization.

$$\begin{aligned} n(X_t) = {} & 0.81279X_1 + 0.742021X_2 + 0.13717X_1X_2 + 0.741728X_3 + 0.287768X_1X_3 \\ & + 1.14132X_2X_3 + 0.918719X_4 + 0.350138X_1X_4 + 0.344208X_2X_4 \\ & + 1.07248X_3X_4 + 0.919021X_5 + 0.357684X_1X_5 + 0.221748X_2X_5 \\ & + 1.04357X_3X_4 + 0.58375X_4X_5 \end{aligned} \tag{11}$$

The coefficient of determinations R^2 of the models were *99.99%* $n(X\quad)$. The details of the coefficients are listed in Table 3.

Table 3. Coefficients of Scheffe's second-degree polynomial for Split tensile strengths

S/No.	β_i	β_1 $(estimate)$ Split tens. strength	Standard error Split tens. strength	t-Statistic Split tens. Strength	p-Value Split tens. strength
1	β_1	0.81279	0.0122302	66.4573	0.00957865
2	β_2	0.742021	0.0121928	60.8572	0.0104599
3	β_3	0.741728	0.0121657	60.9686	0.0104408
4	β_4	0.918719	0.0121648	75.523	0.00842899
5	β_5	0.919021	0.0121928	75.374	0.00844565
6	β_{12}	-0.13717	0.0598577	-2.2916	0.261948
7	β_{13}	0.287768	0.059806	4.81168	0.13045
8	β_{14}	-0.350138	0.0598	-5.85515	0.107689
9	β_{15}	0.357684	0.0598112	5.98021	0.105478
10	β_{23}	1.14132	0.0591951	19.2807	0.032989
11	β_{24}	-0.344208	0.0591191	-5.82228	0.108286
12	β_{25}	0.221748	0.0587617	3.77369	0.164909
13	β_{34}	1.07248	0.0581493	18.4435	0.0344835
14	β_{35}	-1.04357	0.0570215	-18.3014	0.0347507
15	β_{45}	0.58375	0.0565765	10.3179	0.0615085

3.3 Validation of Scheffe's third-degree polynomial models

The mixed sample data's Scheffe's 2nd -degree polynomial model response shows a R^2 value of 99.99% for the split tensile strength responses; Table 4 displays further statistical parameters. Furthermore, 99.98% was the average p-value for the split tensile response model. For the split tensile strength model, the response coefficient fit parameters βi had an average value of 0.0145 for the t-statistics. The degree to which the t-statistics values are close to nullity represents the model's accuracy and fitness, calculated at a maximum of 100,000 iterations. As a result, the derived model is appropriate for determining mixture components for concrete's split tensile strength that integrate PKS.

Table 4. Parameter ANOVA of Scheffe’s simplex lattice third-degree polynomial for the compressive and flexural strength models

Source of Variance	SS	df	MS
Between Groups	0.0129	1	0.0129
Within Groups	0.4505	28	0.0161
Total	**0.4634**	**29**	

3.4 Material composition optimization computation

Thus, the values of X1–X5 are calculated using the objective functions, i.e., Eqns. (10) and (11). The models' suitability for forecasting the split tensile strength of concrete using non-conventional aggregates is indicated by the R^2value of 99.99% from Equations (11). The "Minimize" and "Maximize" routines in the linear programming method of the Wolfram programming language repository were used to optimize the computation of the laboratory test results for the five-component mixture. The laboratory tests yielded a maximum and minimum value for split tensile strength of 1.0963 MPa and 0.5658 MPa. The optimisation problem's boundary limitations were derived from these findings. The constraint conditions specified in Eqns. (5), (6), and (7) were applied, together with the "Minimize" and "Maximize" functions in Wolfram Mathematica, to the resulting goal functions in Eqns. (11) for split tensile strength response. The pseudo components for water, cement, fine-aggregate, granite, and PKS were found to be 0.000, 0.000, 0.000, 0.499661, and 0.500339, respectively. The region of adequacy for the "Maximise" and "Minimise" operations, along with corresponding values of Zi by means of Eqn. (10), was also determined. To produce optimal values of the real components of Zi for use in the construction of palm kernel shell concrete, the Ai matrix of mixture proportions was transformed using the optimised values of Xi in Table 5. After that, Eqn. (11) is used to calculate the split tensile strength that results from employing the optimized Zi parameters.

Table 5. Optimised mix-proportions

Mixture proportion	X_1	X_2	X_3	X_4	X_5
Optimised value	0.00	0.00	0.00	0.499661	0.500339

The computation results are displayed in Table 6. The model predicts a maximum split tensile strength value of 1.06481 MPa. This grade of concrete is suitable for building sidewalks and roadways with less foot traffic. It is also suggested for use as a filler where concrete is essential. Eqns. (11) disclose the split tensile strength concept.

$$\begin{aligned} n(X_t) = {} & 0.81279X_1 + 0.742021X_2 + 0.13717X_1X_2 + 0.741728X_3 + 0.287768X_1X_3 \\ & + 1.14132X_2X_3 + 0.918719X_4 + 0.350138X_1X_4 + 0.344208X_2X_4 \\ & + 1.07248X_3X_4 + 0.919021X_5 + 0.357684X_1X_5 + 0.221748X_2X_5 \\ & + 1.04357X_3X_4 + 0.58375X_4X_5 \end{aligned} \tag{11}$$

Table 6. Recommended mix proportions for tensile strength for the use of PKS in concrete

S/No.	Z1	Z2	Z3	Z4	Z5	Tensile Strength. MPa
1	0.6900	1.0000	2.2249	1.2600	1.8900	0.5658
2	0.6200	1.0000	1.8624	1.0599	1.5899	0.9195
3	0.6525	1.0000	2.0376	1.1551	1.7326	0.6167
Average	0.654	1.000	2.042	1.158	1.737	0.701

The presence of eccentric particles inside the concrete matrix may be the cause of the modest drop in the split tensile strength's optimal value. Due to the minuscule pore spaces found in palm kernel shells, the concrete mix may contain micro voids. The link between the core shell and concrete is weakened by the alkaline concrete matrix, which also embrittles the surface of the core shell [26]. Therefore, it is recommended that additional research be conducted to determine the degree to which concrete's mechanical qualities are impacted by embrittlement of components caused by alkali. On the other hand, the model indicates that concrete with a maximum split tensile strength value of 1.06481MPa may include precisely determined amounts of PKS. In any event, there is a good degree of agreement between the split tensile strengths obtained from the model results and the laboratory test results, suggesting that the optimisation equations are able to adequately predict the mechanical properties of the concrete after 28 days of curing (Table 7).

Practical considerations

The palm kernel shells modified concrete is suitable for applications requiring Low- strength Concrete (LSC), such as sidewalk slabs, kerbs, flooring for residential buildings, squat walls, landscaping projects, garden beds, sound barrier panels, decorative features, and partitions, according to the split tensile strength (indirect compressive strength) obtained from the optimum model [26].

Table 7. Experimental and model results for Split tensile strengths.

S/No.	Symbol of Response	Laboratory Result	Model	Difference
		Split Tensile	Split Tensile	Split Tensile
1	Z1	0.8134	0.8135	-1E-04
2	Z2	0.7426	0.743	-0.0004
3	Z3	0.7427	0.7425	0.0002
4	Z4	0.9195	0.9195	0
5	Z5	0.9195	0.920	-0.0005
6	Z12	0.7427	0.7425	0.0002
7	Z13	0.8488	0.849	-0.0002
8	Z14	0.778	0.778	0
9	Z15	0.9549	0.955	-1E-04
10	Z23	1.0256	1.026	-0.0004
11	Z24	0.7426	0.743	-0.0004
12	Z25	0.8841	0.8845	-0.0004
13	Z34	1.0963	0.0965	0.9998
14	Z35	0.5658	0.8840	-0.3182
15	Z45	1.061	1.061	0

Discussion

In formulating the predictive model for the split tensile strength of composite water, cement, fine-aggregate, granite, and palm kernel shells hardened concrete at 28 days, Scheffe's second-degree polynomial N (5,2) was rummage-sale.

The values of the split tensile strength for the concrete were obtained using Scheffe`s simplex model, and the model obtained the highest strength of 1.0963N/mm^2 corresponding to mix ratio of 0.6: 1.0: 1.775:1.01:1.515 for water, cement, fine and coarse aggregates and PKS respectively. This is an indication that the improved value of the tensile strength was achieved by the addition of about 25.68% by weight of the PKS as a fifth component in the mix with a water-cement ratio of 0.60. The minimum strength value from the model was found to be 0.5658N/mm^2 corresponding to the mix ratio of 0.675:1.0:2.15:1.22:1.83 for water, cement, fine and coarse aggregates and PKS respectively. This further indicates that the minimum value of the tensile strength was achieved by the addition of about 26.62% by weight of the PKS as a fifth component in the mix with a water-cement ratio of 0.675 (Table 1 and 7). Split tensile strength obtained from the experiment is between 1.0963 N/mm^2 - of 0.565796 N/mm^2 which is lower than the 2.0Mpa minimum specified by [27] for structural lightweight aggregate concrete. The low split tensile strength in this research may be ascribed to the lightweight, shapes and semi-porous nature of PKS aggregate or breakdown of bonds between the aggregates and the paste, failure of shell aggregate and aggregate-paste interface as obtainable in [28]. The split tensile strength of all points in the simplex can be derived using this model, and the suitability of the model for forecasting the split tensile strength of concrete using non-conventional aggregates is indicated by the R^2value of 99.99%.

A computer program "***WOLFRAM MATHEMAICAL***" was used to select the optimized tensile strength of the palm kernel shells concrete as in Appendix I and vice - versa.

4. Conclusion

In order to formulate the predictive model for the split tensile strength of composite water, cement, fine-aggregate, granite, and palm kernel shells hardened concrete at 28 days, Scheffe's second-degree polynomial N (5,2) was rummage-sale.

The following are the research's conclusions:

With a maximum model strength of 1.06481 MPa, the highest and minimum tensile strength values from the laboratory test result are 1.0963 MPa and 0.5658 MPa, respectively. These results correspond to mix ratios of [0.625:1.0:1.925:1.09:1.635] and [0.675:1.0:2.15:1.22:1.83] of the constituent parts.

The split tensile strength response models yielded coefficients of determination (R^2) of 99.99% and average p-values of 99.98%, indicating a satisfactory fit of the models. The reduced mechanical properties of the PKS concrete are due to the air voids trapped by the shells in the concrete as such generating bonding problems.

Low-strength concrete, also known as non-structural concrete, is a type of optimum mix that is appropriate for light traffic roads, pedestrian walkways, kerbs, courtyard slabs, residential construction flooring, etc.

Utilizing these substitute elements in concrete, such as waste from industries and agriculture, would improve environmental sustainability.

The comparison of the split tensile strengths obtained between the laboratory test results and the model results shows a good fit, indicating that the optimisation equations can sufficiently predict the mechanical properties of the concrete at 28 days of curing. This suggests that Scheffe's optimization theory can be used to analyse and optimise concrete strength and possible mix proportions of concrete ingredient.

References

[1] M. Safiuddin, M. Z. Jumaat, M. Salam, M. Islam, and R. Hashim, (2010), "Utilization of solid wastes in construction materials," International journal of physical sciences, vol. 5, pp. 1952-1963.

[2] V. Gollu, D. Allam, and S. Erla, (2016) "Causes of concrete failure," International Journal of Advanced Technology in Engineering and Science, vol. 4, pp. 198-206.

[3] P. Domone and J. Illston,(2010) Construction materials: their nature and behaviour: CRC press.

[4] L. Anyaogu and J. Ezeh,(2013) "Optimization of compressive strength of fly ash blended cement concrete using Scheffe's Simplex Theory," Academic Research International, vol. 4, p. 177.

[5] M. Shahbandeh, (2022) "Leading producers of palm oil worldwide from 2021/2022," On line:< https://www. statista. com/statistics/856231/palm-oil-top-global-produ c ers/>. Accessed, vol. 25, p. 2022.

[6] A. PIND, (2011) "A report on palm oil value chain analysis in the Niger Delta," Foundation for Partnership Initiatives in the Niger Delta, pp. 2-4.

[7] N. F. Jalani, A. A. Aziz, N. A. Wahab, W. H. W. Hassan, and N. H. Zainal, (2016) "Application of palm kernel shell activated carbon for the removal of pollutant and color in palm oil mill effluent treatment," Journal of Earth, Environment and Health Sciences, vol. 2, p. 15.

[8] M. Sundalian, D. Larissa, and O. Suprijana, (2021) "Contents and utilization of palm oil fruit waste," Biointerface Research in Applied Chemistry, vol. 11, pp. 10148-10160.

[9] Z. Itam, S. Beddu, N. L. M. Kamal, M. A. Alam, and U. I. Ayash, (2016) "The feasibility of palm kernel shell as a replacement for coarse aggregate in lightweight concrete," in IOP Conference Series: Earth and Environmental Science, p. 012040.

[10] G. Kabir, A. M. Din, and B. Hameed, (2017)"Pyrolysis of oil palm mesocarp fiber and palm frond in a slow-heating fixed-bed reactor: A comparative study," Bioresource technology, vol. 241, pp. 563-572.

[11] I. Uchegbulam, E. O. Momoh, and S. A. Agan, (2022) "Potentials of palm kernel shell derivatives: A critical review on waste recovery for environmental sustainability," Cleaner Materials, p. 100154.

[12] U. J. Alengaram, B. A. Al Muhit, and M. Z. bin Jumaat, (2013) "Utilization of oil palm kernel shell as lightweight aggregate in concrete–A review," Construction and Building Materials, vol. 38, pp. 161-172.

[13] U. J. Alengaram, H. Mahmud, and M. Z. Jumaat, (2011) "Enhancement and prediction of modulus of elasticity of palm kernel shell concrete," Materials & Design, vol. 32, pp. 2143-2148.

[14] D. O. Oyejobi, M. Jameel, N. Sulong, S. A. Raji, and H. A. Ibrahim, (2020) "Prediction of optimum compressive strength of light-weight concrete containing Nigerian palm kernel shells," Journal of King Saud University-Engineering Sciences, vol. 32, pp. 303-309

[15] COREN (2017) Concrete Mix Design Manual. Abuja: Council for the Regulation of Engineering in Nigeria.

[16] C. ASTM, "Standard test method for splitting tensile strength of cylindrical concrete specimens," ed, 2011.

[17] B. S. Institution, "BS 1881-117 (1983) Testing concrete–Method for determination of tensile splitting strength," ed: British Standards Institution.

[18] B. En, "12390-6:(2009). Testing Hardened Concrete. Tensile Splitting Strength of Test Specimens," British Standard Institution, London.

[19] Simon, M J (2003) “Concrete mixture optimization using statistical methods: final report”. Gaithersburg: Federal Highway Authority.

[20] Ndububa E. E and Osadebe N. N. (2007) “An Optimization of the Flexural Strength of Fibre Cement Mixtures using Scheffe’s Simplex Lattice” NSE Technical Transactions, Vol.42, No.1. Pp. 1 – 17. Lagos.

[21] Mohanta, N R, and M Murmu (2022) "Alternative coarse aggregate for sustainable and eco-friendly concrete - a review." Journal of Building Engineering 59: 105079.

[22] CEN (2007) European Committee for Standardization: BS EN 1997-2:2007 Eurocode 7: Geotechnical Design, Part 2. Eurocodes.

[23] O.S. Ogbo[a], E. O. Momoh[b*], E. E. Ndububa[c], O. O. Afolayan[d], S. Onuche[e], J. O. Agada[f] (2023) "Scheffe's Polynomial Optimisation of Laterite Concrete incorporating Periwinkle Shells and Coir" Journal of Civil Engineering (0000) 00(0) 1-19.

[24] BS EN 12390-5 (2009) Testing Hardened Concrete Part 5: Flexural Strength of Test Specimens. London: BSI.

[25] Momoh, E O, A I Osofero, and M Oleksandr (2021) "Bond behaviour of oil palm broom fibres in concrete for eco-friendly construction." Proceedings of the Institution of Civil Engineers – Construction Materials 174 (1): 47-64

[26] Eid, R., Dancygier, A. N. & Jaber, G., (2021). "Mechanical properties of low-performance concrete (LPC) and shear capacity of old unreinforced LPC squat walls." Materials, 14: 7310.

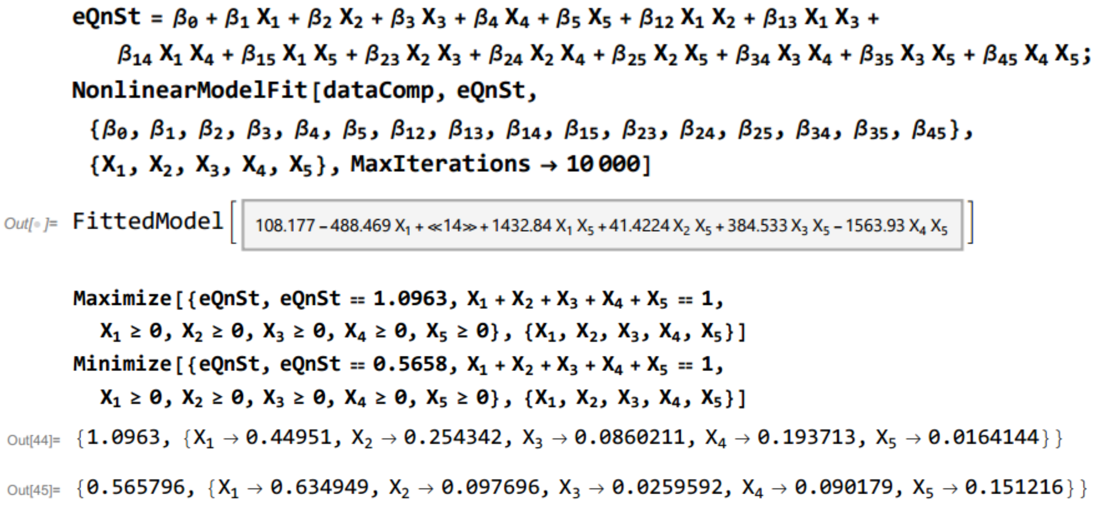
[27] Muhammad Aslam, Payam Shafigh, Mohammad Alizadeh Nomeli, Mohd Zamin Jumaat, (2017) " Manufacturing of high- strength lightweight aggregate concrete using blended coarse lightweight aggregates," Journal of Building Engineering, 13, 53-62.

[28] J. Oti, J. Kinuthia, R. Robinson, and P. Davies, (2015) "The use of palm kernel shell and ash for concrete production," International Journal of Civil and Environmental Engineering, vol. 9, pp. 263-270.

Appendix 1

Split Tensile Strength component composition optimization

In[□]:= **data Comp = Import["C:Split Tensile Strength Test.csv"];**



```
eQnSt = β0 + β1 X1 + β2 X2 + β3 X3 + β4 X4 + β5 X5 + β12 X1 X2 + β13 X1 X3 +
    β14 X1 X4 + β15 X1 X5 + β23 X2 X3 + β24 X2 X4 + β25 X2 X5 + β34 X3 X4 + β35 X3 X5 + β45 X4 X5;
NonlinearModelFit[dataComp, eQnSt,
 {β0, β1, β2, β3, β4, β5, β12, β13, β14, β15, β23, β24, β25, β34, β35, β45},
 {X1, X2, X3, X4, X5}, MaxIterations → 10000]

Out[∘ ]= FittedModel[ 108.177 - 488.469 X1 + «14» + 1432.84 X1 X5 + 41.4224 X2 X5 + 384.533 X3 X5 - 1563.93 X4 X5 ]

Maximize[{eQnSt, eQnSt == 1.0963, X1 + X2 + X3 + X4 + X5 == 1,
  X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0, X5 ≥ 0}, {X1, X2, X3, X4, X5}]
Minimize[{eQnSt, eQnSt == 0.5658, X1 + X2 + X3 + X4 + X5 == 1,
  X1 ≥ 0, X2 ≥ 0, X3 ≥ 0, X4 ≥ 0, X5 ≥ 0}, {X1, X2, X3, X4, X5}]
Out[44]= {1.0963, {X1 → 0.44951, X2 → 0.254342, X3 → 0.0860211, X4 → 0.193713, X5 → 0.0164144}}

Out[45]= {0.565796, {X1 → 0.634949, X2 → 0.097696, X3 → 0.0259592, X4 → 0.090179, X5 → 0.151216}}
```