NIPES

Journal of Materials Engineering, Structures and Computation

Journal homepage: www.nipesjournals.org.ng

Comparative Analysis of Composite Box Girder Bridges Using Beam on Elastic Foundation (BEF) Method, Space Frame Method, Finite Element Method and Computer Programming

Umeonyiagu I.E.* [a], *Nnebe F.O.*[b], *Ogbonna N.P.*[c]

[a,b] Department of Civil Engineering, Chukwuemeka Odumegwu Ojukwu University, Uli, Nigeria.
[c] Federal polytechnic Nekede Owerri, Nigeria.

ARTICLE INFORMATION

Article history:
Received 12 February 2023
Revised 01 March 2023
Accepted 05 March 2023
Available online 23 March 2023

Keywords:
Composite Box Girder Bridges, Beam on Elastic Foundation (BEF) Method, Space Frame Method, Finite Element Method and Computer Programming

https://doi.org/10.5281/zenodo.7759908

https://nipesjournals.org.ng

ABSTRACT

Composite steel box girder superstructures are designed for almost any span length and configuration but are particularly economic and efficient for medium and long-span highway bridges. The primary reason for the efficiency of steel boxes is their torsional stiffness. However, the analysis of such sections is more complicated due to the combination of flexure, shear, torsion, and distortion. In this study, comparisons were made using a beam on elastic foundation, space frame, finite element and java programming methods for analysing these complications. The results obtained from this study include deflections, transverse bending moments, longitudinal stresses and shear flow with and without bracing under direct loading. The numerical results of distortion at BEF and space frame were found to be the same. Divergence was observed on comparison with the space frame method, as the maximum percentage variation was 3.57% and 17.82% by falling into the range of 0.002m and 0.009m in distortion, and 3.51% and 11.76% by falling into the range of 0.001MNm/m and 0.002MNm/m in transverse bending moments for computer programming and finite element method respectively. Whereas there was a full convergence (below 2%) with the space frame method as the maximum percentage variation was 0.25% and 1.12% by falling into the range of 0.33Mpa and 1.10Mpa in longitudinal warping stresses for computer programming and finite element method respectively. The results obtained also showed that the load distribution behaviour of the structure was greatly improved when transverse bracing or framing was introduced at positions along the box. The numerical results were compared and validated to be in good agreement by falling within close range of the different methods used. Thereby proving the viability and workability of the methods developed and used.

1. Introduction

Composite box girder construction delivers an attractive and economic form of construction for medium-span highway bridges. According to [1] Composite steel box girder superstructures, can be designed for almost any span length and configuration, but are particularly efficient for medium- and long-span highway bridges, both tangent and curved, in spans of over 45.7m and up to 152.4m. The torsional properties of the closed section are often advantageous in reducing and simplifying the support arrangements and are particularly useful when curvature in the plan is required [2]. Steel boxes may either be tub sections or closed box sections, with either inclined webs or vertical webs. Many composite box girders built in the U.S. are tub girders having a solid bottom flange, two solid webs, and an open top with two separate top flanges on each web connected with top lateral bracing to form a pseudo box to resist the torsion before hardening of the concrete deck. Narrow non-composite closed steel boxes are often employed as straddle beams to provide support and are necessary under clearance. The vital reason for the efficiency of steel boxes, particularly for horizontally curved superstructures, is their torsional stiffness. The lateral bending stiffness of the deck is greatly enhanced by the fixity of the support at the girder lines provided by the torsional stiffness of the box. Which in turn distributes live loads over a much great tributary area engaging adjacent girders and correspondingly increasing the proportion of the superstructure cross-section resisting the vertical loads.

However, despite being an efficient cross-section, the analyses of such sections are more complicated due to the combination of flexure, shear, torsion, and distortion. Therefore, there is a great need to analyze the box girder bridge to know the effect of eccentric loading on the distortion of its cross-section. [3] gave three models of slab-on beam-type of bridges with a varying number of girders and varying span lengths which were loaded with Load Model 1 (LM1) according to Euro code 1 Part 2 (EN 1991-2:2003) and analyzed using Finite Element Analyses, Grillage Analogy, and Courbon's method. They proposed a calibration factor for the results from Courbon's method as a function of the bridge span length, which will enable Courbon's method to be used as a fast check for verification of results from computer methods. [4] carried out a detailed study of box girders of different cross sections namely rectangular, trapezoidal and circular using the finite element method. SAP2000 was utilized to carry out the linear analyses of these box girders. To analyze the complex behaviour of the different box girders, they employed three-dimensional 4-noded shell elements for the discretization of the domain. [5] carried out a comparative study of T-beam girder and Box girder superstructures. The aim of this study was to determine comparative results obtained from manual and computer methods (Java program and commercial finite element programs like STAAD-PRO). [6] assessed the behaviour of the box beam girder under pure torsion. He described various methods for the torsional strengthening of concrete box beams. In their assessment research, the box beam was strengthened practically with an external prestressing technique which used two different directions horizontally and vertically. [7] developed a computer program for the design of Balanced Cantilever Bridges. They compared the results from the developed program with those of the Manual method and also with those from STAAD PRO, a commercial finite element program.

1.2 Governing Equations of Composite box girder

The set of governing equations typically used in the combination of flexure, shear, torsion, and distortion for the composite box girder model is derived in Equations (1) to (12).

1.2.1 Beam ON Elastic Foundation method (BEF)

[8] provides the section properties attributed to the application of the BEF method which includes design charts with some Equations.

$$\beta = \left\{\frac{1}{4EI_b\delta_1}\right\}^{0.25} = \left\{\frac{1}{EI_c\delta_1}\right\}^{0.25} \tag{1}$$

$$w = 8EI_b\beta^3\frac{W}{P} \tag{2}$$

$$v = \frac{\frac{1}{D_c}[(2a+b)abc] + \frac{1}{D_a}[ba^3]}{(a+b)\left[\frac{a^3}{D_a} + \frac{2c}{D_c}(a^2+ab+b^2) + \frac{b^3}{D_b}\right]} \tag{3}$$

$$\delta_1 = \frac{ab}{24(a+b)}\left\{\frac{c}{D_C}\left[\frac{2ab}{a+b} - v(2a+b)\right] + \frac{a^2}{D_a}\left[\frac{b}{a+b} - v\right]\right\} \tag{4}$$

$$\delta_b = \frac{2\left[1+\frac{a}{b}\right] \times \delta_1}{\left[1 + \left\{\frac{(a+b)}{2h}\right\}^2\right]^{0.5}} \tag{5}$$

1.2.2 Space frame analyses of box-girder

[9] provides the section properties attributed to the cruciform members depending on whether shear flexibility is considered by the space frame, or not. Equations are derived below, first for shear-rigid members and then for shear-flexible members.

$$A_x = bd \tag{6}$$

$$C = \frac{bd^3}{6} \tag{7}$$

$$I_y = \frac{bd^2}{12} \tag{8}$$

$$I_z = \frac{ba^2d}{15} = \left(\frac{a}{b}\right)^2 \frac{b^3d}{15} \tag{9}$$

1.2.3 Grillage analyses of box-girder

[9] provides the distortion behaviour of the box-girder can be simulated by giving the spine beam members softened torsion constants.

$$\mathrm{C} = \frac{\mathrm{a}^2 l}{8\mathrm{G}} \frac{\mathrm{P}}{\mathrm{W}} \tag{10}$$

$$C_t = \frac{4A^2}{\oint \frac{ds}{t}} \tag{11}$$

$$\frac{1}{C_{dt}} = \frac{1}{C_d} + \frac{1}{C_t} \tag{12}$$

2. Methodology

2.1 Model Formulation

A single-span composite steel box–girder bridge supporting a vehicle of weight 2P over one web near midspan is considered. The following calculation examines the distortion caused by the antisymmetric component of the load, shown in Figure 1, which consists of the up and down loads of magnitude P at the two webs.

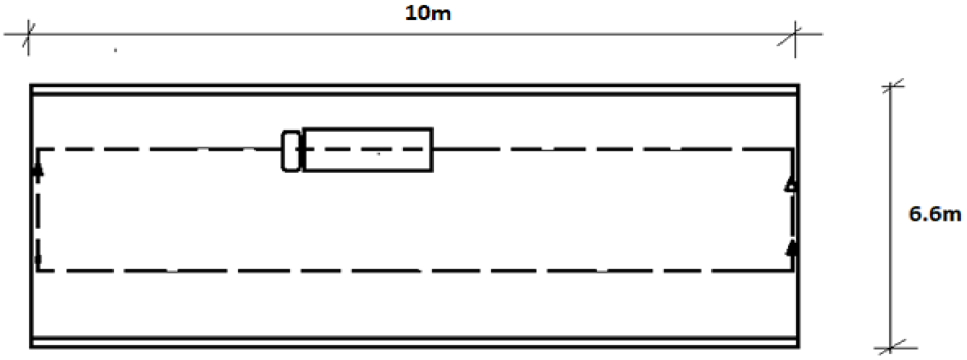

Figure 1. Plan of 10 metres Single span composite steel box-girder bridge deck

The basic raw section was derived using some simple rules of thumb for proportions, the angle of the web is such that the web plates can simply be cut square and a single-sided weld with partial penetration can be used. For example, using a trial section or rule of thumb, for bottom flange thickness to be fully effective

$$\frac{t_b}{t_c} < 24$$

Where
t_b is thickness of bottom flange
t_c is thickness of web

The structure will be analyzed for spans of 20m, 25m, 30m, 35m and 45m respectively. The cross-section has a concrete top flange 6.6 m wide and 0.2m thick and the box dimensions are a=3.2, b= 2.8m, c=1.5m, d =3.4m, h=1.5m. The bottom flange is 0.02m thick and the webs are 0.010m thick. the structure is first considered without any cross-bracing or diaphragms, except at supports. It is assumed that Young’s Modulus, E is 200 000 MPa for steel and 30 000 MPa for concrete. Poison’s ratio v is assumed to be 0.25 for steel and concrete. All section properties for concrete are converted below to their equivalent for steel using a modular ratio m = 0.15.

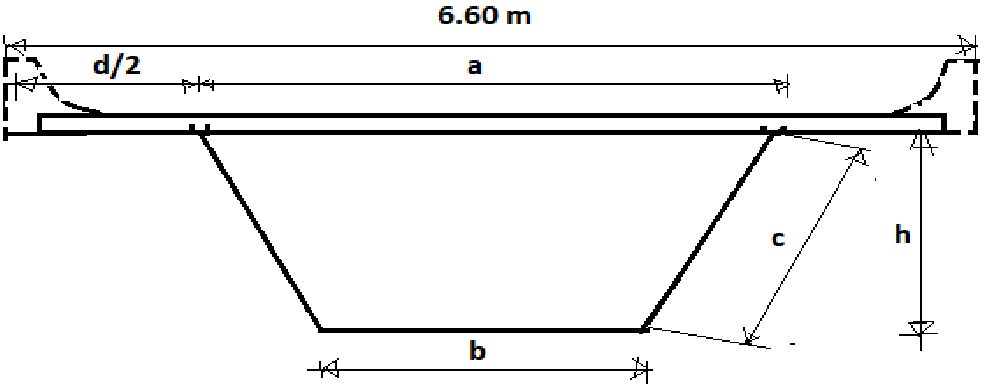

Figure 2. Cross sectional of the Box Girder Bridge

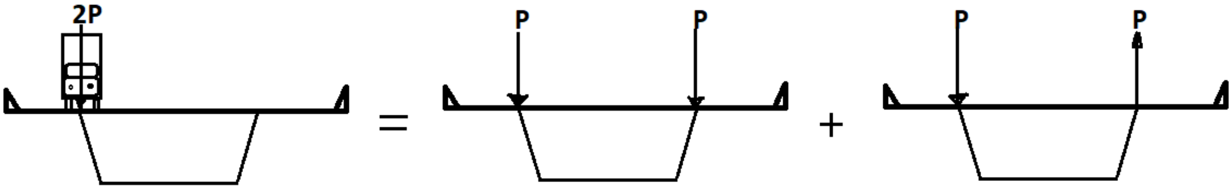

Figure 3. Components of loads on the composite Box Girder Bridge

2.2 Creation of a computer program for the analyses of box girder bridge

BOX Bridge is a computer program written in Java Programming language for the comparative analyses of composite box girder bridges. It is written to reduce the time used in the analyses of composite box girder bridges and has rich a graphical interface to aid the user to visualize the result of the analyses. Design of bridge substructures using BOX Bridge is organized into several classes. Using a Unified Modeling Language (UML) diagram, the various packages classes are presented in the next section:

2.2.1 Package BOX Bridge

This package shall contain the main classes which include BOX Main,BOXDetails, BOX Space, BOX Graphs, BOXBEF, BOX Frame and BOX Finite

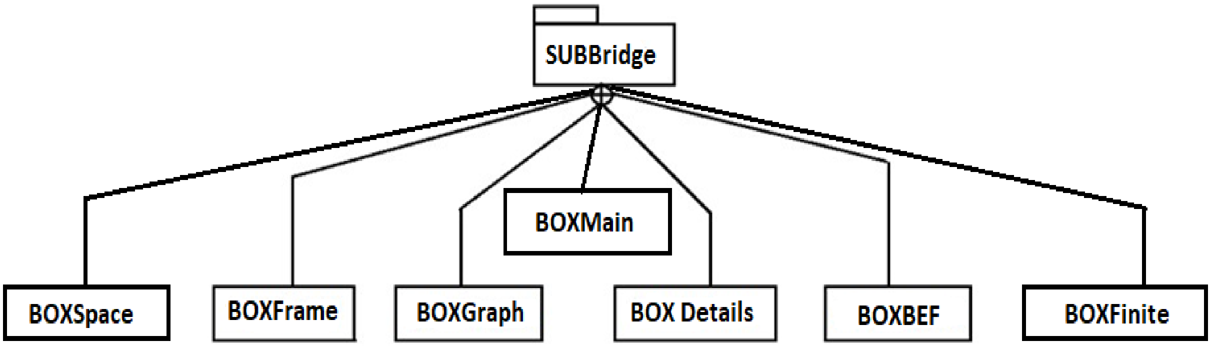

Figure 4. Package BOX bridge showing its member classes

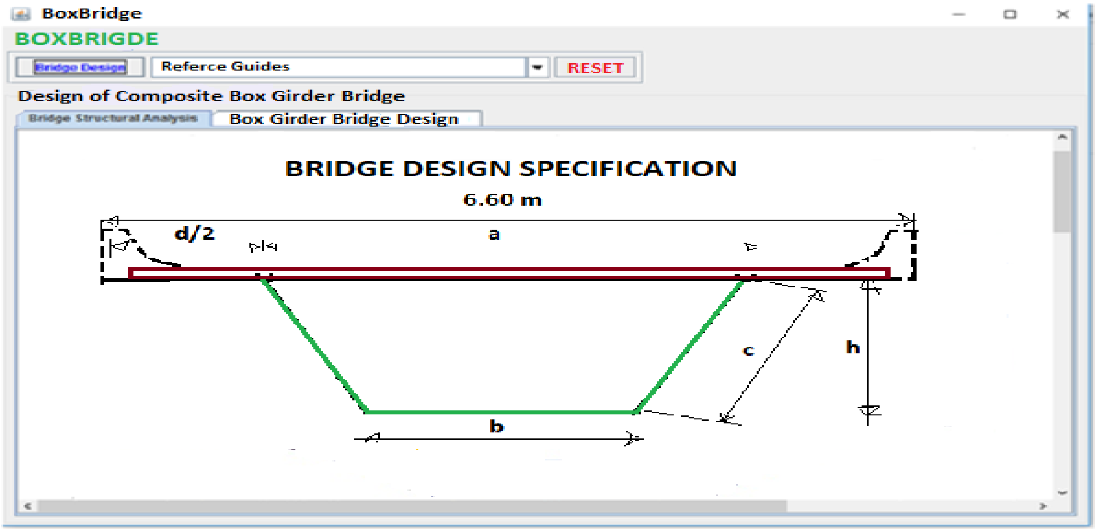

Figure 5. The Main Application Window for the developed Computer Program

3. Results and Discussion

3.1 Analyses of composite box girder bridge using BEF, space frame and finite element method

The results of the analyses performed using Beam on Elastic Foundation, Space frame method, finite element method and the developed computer program (Box Bridge) are presented here. Table 1, shows the results of the structural analyses performed on the steel composite box girder bridge.

Table 1: Distortion at midspan of the composite box-girder bridge

Main span length	BEF method	Space frame method	Finite element method	Computer program (box bridge)
20	0.015	0.015	0.013	0.015
25	0.020	0.020	0.017	0.019
30	0.03	0.03	0.025	0.03
35	0.045	0.045	0.038	0.047
40	0.050	0.050	0.042	0.050
45	0.055	0.055	0.046	0.057

A graph of the results in Table 1 is presented which shows the comparison of the Distortion at the Mid-span of the steel composite box girder bridge for 20m to 45m spans when subjected to antisymmetric loading of 1MN. The results from the BEF, Space Frame methods and the developed computer program and similar but differs slightly from the results obtained from the finite element method. The deflection from the space frame the method is 0.03m, which is mainly due to torsion with a small contribution from torsion.

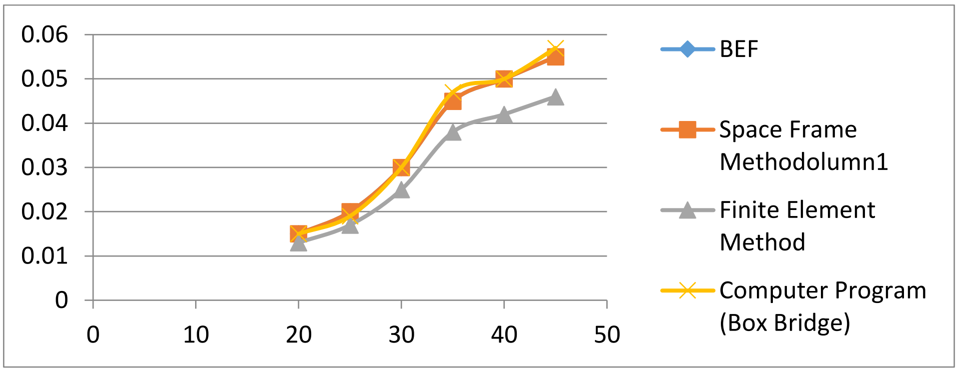

Figure 6. Graph of distortion at midspan of the composite box-girder bridge

Table 2: Transverse bending moment

Main span length	Space frame method (MNm/m)	Finite element method (MNm/m)	Computer program (box bridge) (MNm/m)
20	0.014	0.013	0.013
25	0.018	0.016	0.018
30	0.022	0.021	0.022
35	0.026	0.025	0.026
40	0.029	0.028	0.028
45	0.033	0.031	0.033

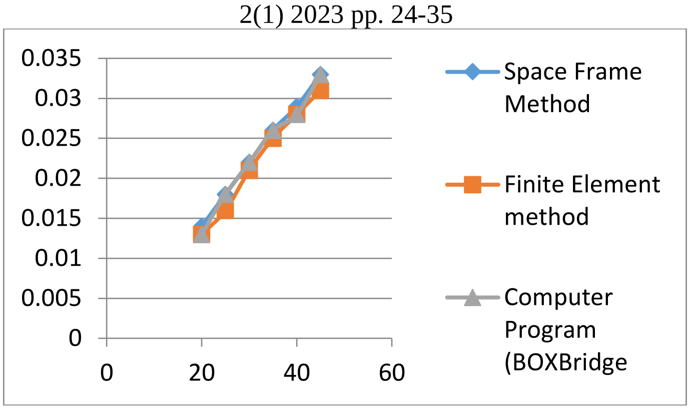

Figure 7. Graph of the transverse bending moment against the bridge span

Table 4 and Figure 7, illustrates the transverse bending moments calculated by the transverse space frame members, the developed computer program and the finite element method.

Table 3: Longitudinal warping stresses

Main span length	Space frame method (MPa)	Finite element method (MPa)	Computer program (box bridge) (MPa)
20	64.67	65.67	64.63
25	80.83	81.43	80.81
30	97.0	98.1	97.2
35	113.17	114.20	113.15
40	129.33	130.34	129.0
45	145.6	146.3	145.3

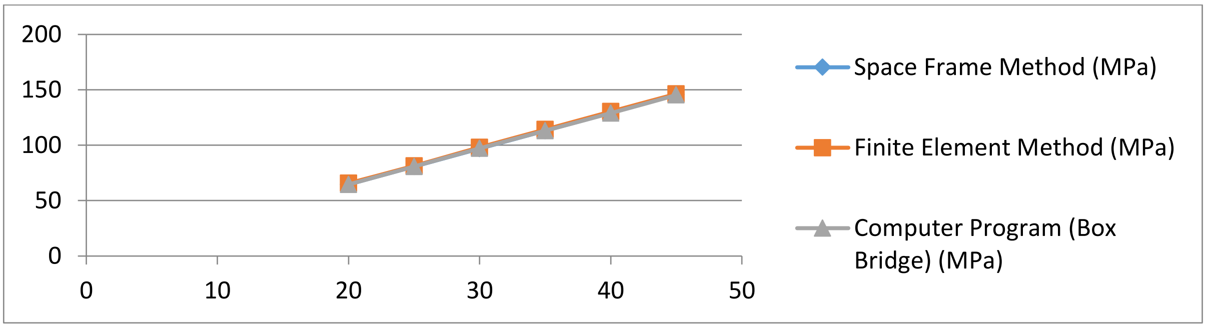

Figure 8. Graph of longitudinal warping stresses against bridge spans

Table 3. and Figure 8 shows the longitudinal in-plane stresses at midspan derived from the axial forces and the in-plane bending moments in the longitudinal members of the space frame. The transverse bending stresses and longitudinal in-plane stresses from the finite elements are close to the space frame results. The deflection from the finite elements are slightly smaller than those from the space frame and the developed computer program respectively.

Table 4: Deflections, transverse bending moments, longitudinal stresses and shear flow of composite steel box girder bridge without bracing under direct loading

Main span length	Dead loading	Live loading	Deflection	Transverse bending moments	Longitudinal stresses	Shear flow
20	0.08	0.04	0.059	-0.019	133	0.8
25	0.08	0.04	0.074	-0.023	167	1.0
30	0.08	0.04	0.089	-0.028	200	1.2
35	0.08	0.04	0.104	-0.033	233	1.4

40	0.08	0.04	0.138	-0.037	266	1.6
45	0.08	0.04	0.155	-0.042	300	1.8

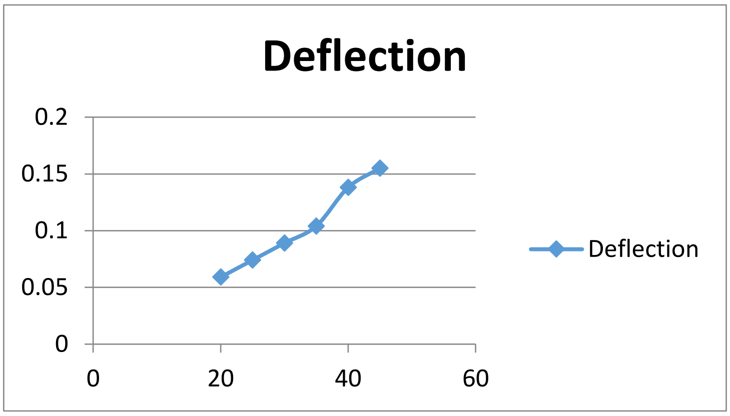

Figure 9. Graph of deflections against bridge spans of composite steel box girder bridge without bracing under direct loading

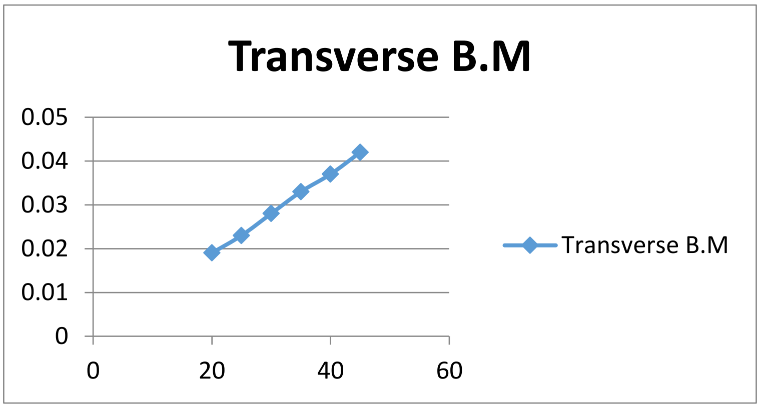

Figure 10. Graph of transverse bending moments against bridge spans of composite steel box girder bridge without bracing under direct loading

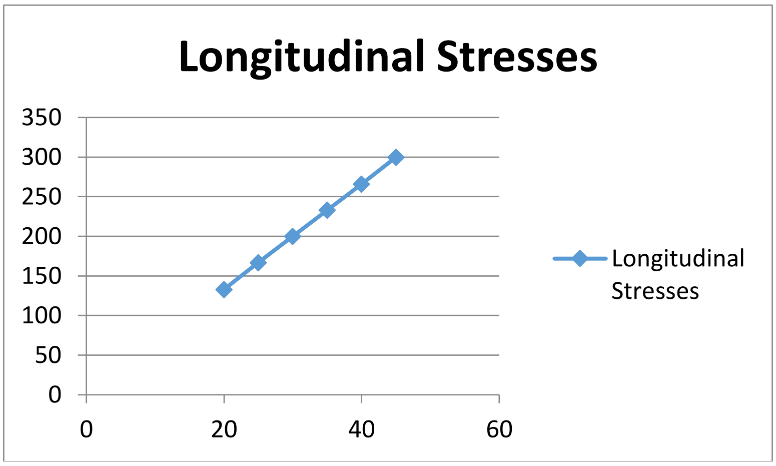

Figure 11. Graph of longitudinal stresses against bridge spans of composite steel box girder bridge without bracing under direct loading

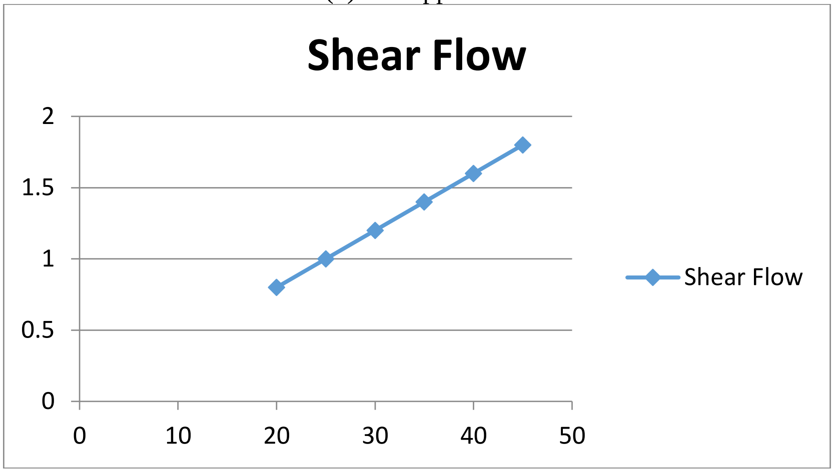

Figure 12. Graph of shear flow against bridge spans of composite steel box girder bridge without bracing under direct loading

Table 5: Deflections, transverse bending moments, longitudinal stresses and shear flow of composite steel box girder bridge with bracing under direct loading

Main span length	**Dead loading**	**Live loading**	**Deflection**	**Transverse bending moments**	**Longitudinal stresses**	**Shear flow**
20	0.08	0.04	0.043	-0.023	92	0.733
25	0.08	0.04	0.054	-0.028	115	0.917
30	0.08	0.04	0.065	-0.034	138	1.10
35	0.08	0.04	0.076	-0.040	161	1.28
40	0.08	0.04	0.087	-0.045	184	1.47
45	0.08	0.04	0.098	-0.051	207	1.65

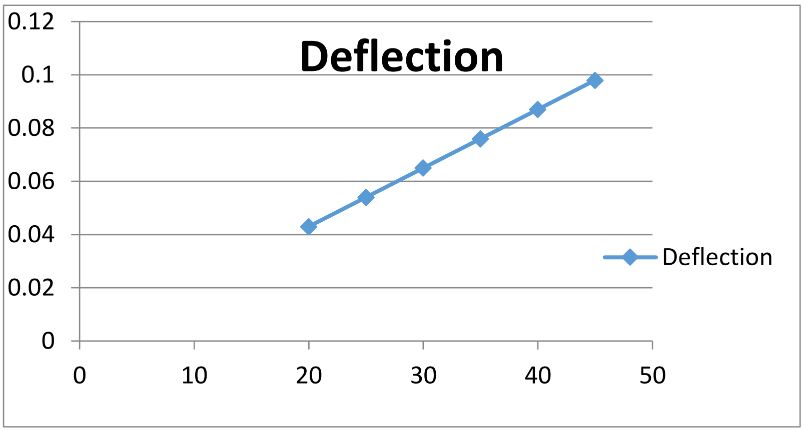

Figure 13. Graph of deflections against bridge spans of composite steel box girder bridge with bracing under direct loading

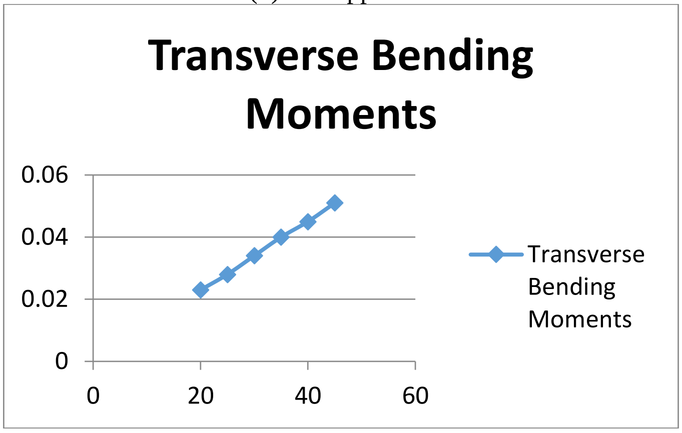

Figure 14. Graph of transverse bending moments against bridge spans of composite steel box girder bridge with bracing under direct loading

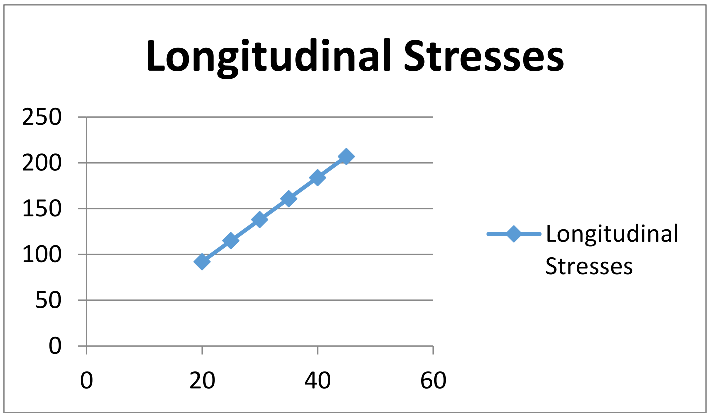

Figure 15. Graph of Longitudinal stresses against spans of composite Steel Box Girder Bridge with Bracing under Direct Loading

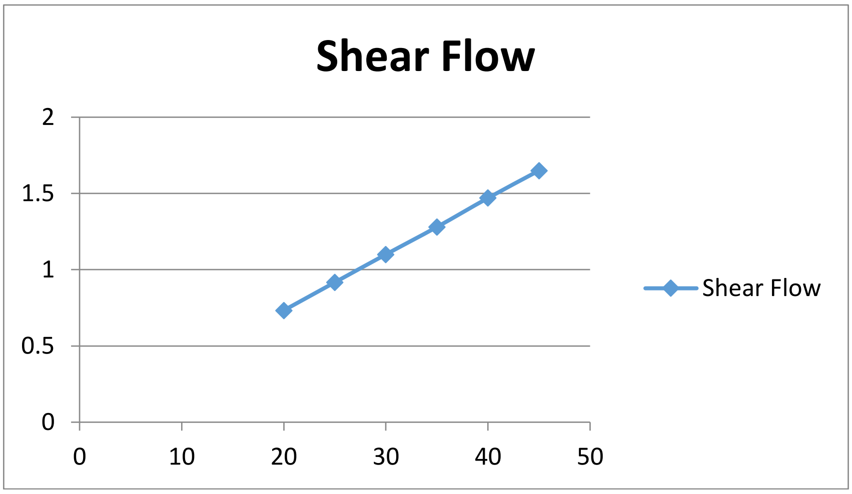

Figure 16. Graph of shear flow against bridge spans of composite steel box girder bridge with bracing under direct loading

3.2 Comparison of results from the developed program

The significance of displacement on any bridge structure needs to be viewed within the context of the overall loading and behaviour of the structure. Table.4 and Figures 9 to 16, illustrate the results from the space frame analyses due to the total loading of dead load of 80 kN/m and distributed live load of 40 kN/m along one side. The load distribution behaviour of the structure is greatly improved if some transverse bracing or framing is introduced at predominant positions along the box. Cross bracing was introduced into the space frame model without difficulty.
Table 5 shows the results calculated by the space frame under the same loading as Table 4. when cross-bracing is included at 7.5m spacing with area A =0.002m2. This relatively small quantity of bracing has prevented most of the distortion so that there is little variation in deflection across the box in Figure 4 and a little variation in the longitudinal bending stresses in Figure 7.

4. Conclusion

The purpose of this study was to perform analyses of steel composite box girder bridges using methods such as beam on elastic foundation, space frame and finite elements. The results from these methods were compared with those obtained from a computer program in Java Programming Language called BOX Bridge. The results obtained from both manual and computer methods included deflections, transverse bending moments, longitudinal stresses and shear flow with and without bracing under direct loading. This thesis performed analyses of steel composite box girder bridges. It also developed a Java-based computer program for quick and accurate analysis and design of steel composite box girder bridges. The results obtained, showed that the load distribution behaviour of the structure was greatly improved when transverse bracing or framing was introduced at predominant positions along the box. It was evident that this relatively small quantity of bracing had prevented most of the distortion of the steel composite box girder bridge. The developed program will also serve as a useful interactive program for teaching structural engineering students and a valuable tool for practicing structural engineering. Further research on concrete box girders of similar shape, is recommended.

Nomenclature

β	Beam on Elastic foundation parameter
δ_1	Vertical deflection of one web per unit torsional load
δ_b	Diagonal brace elongation
A_x	Compression area
C	Torsion constant
C	Equivalent torsion constant
C_{dt}	Distortion with torsion constant
C_d	Equivalent torsion constant for distortion
C_t	Pure torsion constant
D_b	Flexural rigidity for bottom flange
D_c	Flexural rigidity for webs
E	Young’s modulus
L	Span length
I_c	Moment of inertia
I_y	Out-of-plane bending inertia
I_z	In-plane bending inertia
P	Concentrated torsional load
w	Dimensionless deflection term
W	Distortion deflection

References

[1] W.F Chen. and L Duan (2000): Bridge Engineering Handbook, CRC Press LLC.
[2] Iles DC. Design guide for composite box girder bridges. Steel Construction Institute; 2004.

[3] V.O Okonkwo, O.U. Ubani and J.N Udemba (2019). "Comparative Analyses of Live Load Distribution on External Right Bridge Deck Girders under Traffic Load Model 1 (EN 1991-2)

[4] P.K. Gupta, Singh K.K and A. Mishia . (2010). "Parametric study on behaviour of box girder bridge structures by using finite element method", Asian Journal of civil engineering, Vol.11. pp 135-148

[5] A. Saxena and S.Maru (2013). "Comparative study of the analysis with design of box girder and T shaped beam superstructure", International journal of research in engineering and advanced technology, Vol 1(2)pp 24-28

[6] A El-HakimKhalil ,E. Etman , A. Atta and S. Fayed.(2015) Torsional strengthening of RC box beams using external prestressing technique. IOSR Journal of Mechanical and Civil Engineering.12(2) pp30-41.

[7] I.E.Umeonyiagu, F.E. Onwuka and N.P. Ogbonna. (2021). "Computer program for the design of Balanced Cantilever Bridges", The International Journal of Engineering and Science (IJES), Vol. 10(3). pp. 15-33

[8] R.H. Wight., S.R. Abdel-Samuel and A.R. Robinson. (1968) BEF Analogy forAnalysis of Box-Girders. *Journal of Structural Division ASCE*, Vol 94(7), Proceeding Paper No 6025, pp. 1719-1743

[9] Hambly, E. C. (1991). Bridge Deck Behaviour, E FN Spon, London &New York.