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Evaluating Dissolved Oxygen Dynamics in Fresh Water Fish Tank Using the Modified Cobb Douglas Approach

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Abstract

A modified Cobb Douglas model was used to evaluate dissolved oxygen dynamics in a fibre glass fish tank with Chicken Manure as the treatment. Though the model was originally modeled to handle two input variable with their output elasticities, this study looked at seven input variables which includes pH, Temperature, Turbidity, Ammonia, Transparency, TDS (Total dissolved solid), EC (Electrical conductivity) to effectively predict the DO (output). The ANOVA approach was used for the data, a regression model was generated, the Cobb Douglas model was modified and solved using MathLab(2015) the output elasticities were generated and the modified Cobb Douglas model was also generated. The DO was found to be in the range of 12.78616mg/L to 17.28149mg/L for the treatment which has exceeded the maximum for any fish to survive. Further an R^2 value of 71.39% was obtained which shows that the various components of the model were able to account for 71.39% of the variation in the given period of 11 weeks. A comparative analysis was done, the percentage variation of the DO_{CD} and DO_{REG} are <0.005 which gives good credence to the modified Cobb Douglas model

1. Introduction

The In the struggle to enhance the nutritional status of the population and to reduce rural poverty, particularly in developing nations, aquaculture has a significant potential [12]. African and Middle Eastern native tilapia has transformed from obscurity to become one of the most productive and widely traded food fish in the world. One of the food industries with the fastest growth rates worldwide is the production of farmed tilapia. About 71% of the world's total tilapia production comes from the Nile tilapia (Oreochromis niloticus), the most cultivated fresh water species among farmed tilapia [8]. Every aquatic creature depends on water for survival.

A water quality variable in aquaculture is any property of water that influences fish or other aquatic animals' chances of surviving, reproducing, growing, or being managed in any way [5]. Fish carry out their physiological functions in the water medium in all types of culture systems, including breathing, waste excretion, feeding, maintaining salt balance, and reproduction. As a result, any aquaculture system's total performance is influenced in part by the water quality[2]. Low output, profit, and product quality are the results of stressed fish and poor water quality[9]. When impurities in the water that can harm cultivated species' growth, development, reproduction, or even cause their death are present, productivity is diminished. In order to maintain a generally stress-free environment that satisfies the physical, chemical, and biological standards for the fishes' normal health and growth performance, fish farmers are required to control water quality [10], [11].

Fish fanning is used to help farmers maximize fish growth and productivity in order to maximize profits. For better growth performance, water is a major factor in the growth process. Therefore, it should come as no surprise that the programs for managing water quality have a significant impact on the success of fish farming. Fish culture operations' success or failure is largely determined on the water quality. For fish farming to be effective, factors including temperature, dissolved gases, pH, nutrition, and the possibility of harmful materials present must be taken into account[14]**.** The maintenance of a high-quality, healthy aquatic environment is greatly influenced by the physical, chemical, and biological components of the aquatic ecosystem. In fish culture, dissolved oxygen (DO) is regarded as one of the most crucial water quality criteria. Low DO in culture ponds is frequently associated with elevated levels of carbon dioxide (CO2) and unionized ammonia (NH3), both of which are toxic to fish. This combination significantly increases the defenselessness of fish to diseases. Chronic low levels of DO in fish culture cause stress to cultivated fish [6]. This results in reductions in feeding, feed conversion, and growth.

Fish in saline water may have major difficulties growing or even surviving if DO concentration is less than 5mg/L, whereas aquatic life can no longer survive below 2mg/L. According to a study [5], the limited solubility, quick uptake by phytoplankton, fish, and other creatures, as well as the sluggish rate of atmospheric replenishment into undisturbed water, all contribute to the dynamic nature of DO. Any water body's DO concentration changes over time and is influenced by a variety of physical, biological, and chemical elements, including pH, temperature, atmosphere, pressure, and salinity.

The impact of DO on freshwater fish-physiology was a focus in 2022 [7]. A detailed literature survey is given based on DO and freshwater fish swimming, feeding, disease management, survival, respiration, metabolism, growth, reproduction, health parameters, immunity and stress of freshwater fishes. Before now, deep learning models, such as recurrent neural network (RNN), long short-term memory (LSTM), and gated recurrent unit (GRU), are often used to predict the trend of time series, but it is unclear which one of them is more suitable for prediction of DO in fishery ponds. However the use of other tools like RNN model, LSTM model, and GRU model were utilized to build three DO predicting models [13]. This study is set to use the Cobb Douglas model of equation (1), which will be modified, in evaluating dissolved oxygen dynamics in fresh water fiber fish tank.

2. Material and Methods

To examine the relationship between inputs and outputs in the production process, economists frequently utilize the Cobb Douglas model in equation (1) which is a production function. The Cobb Douglas model makes the assumption that a production process' output is a function of its inputs. The model is stated thus:

$$Q = AX^{a}Y^{b} \tag{1}$$

Where X and Y are the input, Q is the output, A is a constant factor, and a and b are the output elasticities of the inputs which controls how the inputs and outputs interact.

Output elasticities measures the responsiveness of output to a change in the levels of either inputs used in production. The Cobb Douglas model is a versatile tool that may be used to examine various production processes, including the construction of fish ponds which can be modified as the need arises.

2.1 Data Collection and Input Data Structure

The secondary data collection is based on the study of Adams [1] with qualitative attributes that was experimentally carried out using a well-setup fibre fish tank for rearing Nile tilapia (Oreochromis niloticus) with three treatment administered in a bi-weekly basis. Data collection instrument were employed. The parameters inputted were broken down into various chemical elements of pH, Temperature, Turbidity, Ammonia, Transparency, TDS (Total dissolved solid), EC (Electrical conductivity) to effectively predict the DO for each treatment under investigation. The data obtained and the restructured data are shown below.

Table1: Treatment (Chicken Manure Only) Bi-Weekly for 6months [1]

Time Bi-weekly	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Y
2	6.77	27.719	32.72	0	179.9	362.3	0.83	3.577
4	9.246	28.364	32.96	0.8	109.22	217.3	1.07	12.797
6	9.826	28.412	32.64	0.8	110.28	218.56	4.5	14.642
8	10.206	28.647	32.06	0.9	113.92	225.7	9.14	16.425
10	10.088	29.003	20.68	0.9	128.69	258	14.05	16.736
12	9.47	28.633	29.79	0.6	116.25	232.1	19.74	17.094
14	8.704	29.397	36.08	0.4	116.75	319.8	26.57	15.842
16	8.924	28.794	38.42	0.3	161.75	323.8	40.38	16.603
18	8.598	28.506	35.8	0.4	160.92	313.7	59.38	16.875
20	8.577	28.361	35.02	0.3	161.08	322.2	79.27	17.222
22	8.463	28.139	33.32	0.3	162.22	324.4	111.48	17.211
24	8.314	28.119	30.61	0.3	162.78	324.9	143.47	16.481
Total	**100.416**	**314.375**	**357.38**	**6**	**1548.86**	**3080.46**	**509.05**	**177.928**

(Source: Adams 2021)

where
X_1 = pH; X_2 = Temperature, X_3 = Turbidity, X_4 = Ammonia, X_5 = Transparency, X_6 = TDS (Total dissolved solid), X_7 = EC (Electrical conductivity)

The zero (0) in the first row makes the computation not feasible, hence the row and the column has to be eliminated and then restructured statistically for the model to be applied.

Table 2**:** Total Output and Inputs

Time Bi-weekly	K_1	K_2	L_1	L_2	M	E	Q
4	9.246	28.364	32.96	109.22	217.3	1.07	12.797
6	9.826	28.412	32.64	110.28	218.56	4.5	14.642
8	10.206	28.647	32.06	113.92	225.7	9.14	16.425
10	10.088	29.003	20.68	128.69	258	14.05	16.736
12	9.47	28.633	29.79	116.25	232.1	19.74	17.094

14	8.704	29.397	36.08	116.75	319.8	26.57	15.842
16	8.924	28.794	38.42	161.75	323.8	40.38	16.603
18	8.598	28.506	35.8	160.92	313.7	59.38	16.875
20	8.577	28.361	35.02	161.08	322.2	79.27	17.222
22	8.463	28.139	33.32	162.22	324.4	111.48	17.211
24	8.314	28.119	30.61	162.78	324.9	143.47	16.481
Total	100.416	314.375	357.38	1548.86	3080.46	509.05	177.928

2.2. Equation Formulation and Computation

The Cobb Douglas model assumes that the output of a production process is a function of the inputs used in the production process. The model can be expressed as:

$$Y = AX^{a}Y^{b} \tag{2}$$

Where Y is the output, A is a constant factor, X is the input, a and b are parameters that determine the relationship between the inputs and outputs.

The Cobb Douglas model is a flexible model that can be used to analyze different types of production processes, including fish pond production.

Since this work is based on the modified Cobb Douglas function in analyzing six (6) inputs parameters of dissolved oxygen in fiber glass tank, the function $Y = AX^{a}Y^{b}$ can be modified to look in this manner due to the large number of variables being considered.

$$Q = f(K_1, K_2, L_1, L_2, M, E) \tag{3}$$

$$Q = AK_1^{\alpha}, K_2^{\beta}, L_1^{\gamma}, L_2^{\mu}, M^{\phi}, E^{\omega}) \tag{4}$$

Where Q = Dissolved Oxygen (Total Output)

K_1 = pH; K_2 = Temperature; L_1 = Turbidity; L_2 = Transparency; M =TDS (Total dissolved solid); E = EC (Electrical conductivity) and α, β, ϒ, μ, ϕ and ω are the output elasticities of the various inputs.

Further, because of the elasticities and for ease of computation, production function which is a generalization of the Cobb Douglas production function was translog thus:

$$\log Q = \log A + \alpha \log K_1 + \beta \log K_2 + \gamma \log L_1 + \mu \log L_2 + \varphi \log M + \omega \log E \tag{4}$$

$$y = \beta_0 + \alpha x_1 + \beta x_2 + \gamma x_3 + \mu x_4 + \varphi x_5 + \omega x_6 + \varepsilon \tag{5}$$

where

$$y = \log Q \tag{6}$$

$$\beta_0 = \log A, x_1 = \log K_1, x_2 = \log K_2, x_3 = \log L_1, x_4 = \log L_2, x_5 = \log M, x_6 = \log E,$$
$$\varepsilon = error \tag{7}$$

From Multivariate Linear Regression fit model

$$\sum y = \sum \beta_0 + \alpha \sum x_1 + \beta x_2 + \gamma \sum x_3 + \mu \sum x_4 + \varphi \sum x_5 + \omega \sum x_6 \tag{8}$$

$$\sum x_1 y = \beta_0 \sum x_1 + \alpha \sum x_1^{2} + \beta \sum x_1 x_2 + \gamma \sum x_1 x_3 + \mu \sum x_1 x_4 + \varphi \sum x_1 x_6 + \omega \sum x_1 x_6 \tag{9}$$

$$\sum x_2 y = \beta_0 \sum x_2 + \alpha \sum x_1 x_2 + \beta \sum x_2^{2} + \gamma \sum x_2 x_3 + \mu \sum x_2 x_4 + \varphi \sum x_2 x_5 + \omega \sum x_2 x_6 \tag{10}$$

$$\sum x_3 y=\beta_0\sum x_3+\alpha\sum x_1x_3+\beta\sum x_2x_3+\gamma\sum x_3{}^2+\mu\sum x_3x_4+\varphi\sum x_3x_5+\omega\sum x_3x_6 \quad (11)$$
$$\sum x_4 y=\beta_0\sum x_4+\alpha\sum x_1x_4+\beta\sum x_2x_4+\gamma\sum x_3x_4+\mu\sum x_4{}^2+\varphi\sum x_4x_5+\omega\sum x_4x_6 \quad (12)$$
$$\sum x_5 y=\beta_0\sum x_5+\alpha\sum x_1x_5+\beta\sum x_2x_5+\gamma\sum x_3x_5+\mu\sum x_4x_5+\varphi\sum x_5{}^2+\omega\sum x_5x_6 \quad (13)$$
$$\sum x_2 y=\beta_0\sum x_6+\alpha\sum x_1x_6+\beta\sum x_2x_6+\gamma\sum x_3x_6+\mu\sum x_4x_6+\varphi\sum x_5x_6+\omega\sum x_6{}^2 \quad (14)$$

The six months bi-weekly data obtained from the experiment performed (secondary data) were transformed and substituted into equations 8-14. From this and using standard coefficient matrix and adjoint canonical form, we obtained the matrix

$$M_0=\begin{bmatrix} 11 & 10.55270 & 16.01623 & 16.57591 & 23.56536 & 26.85009 & 14.99384:13.28082 \\ 10.55270 & 10.13376 & 15.36553 & 15.88957 & 22.58604 & 25.73729 & 14.24057:12.73780 \\ 16.01623 & 15.36553 & 23.32029 & 24.13430 & 34.11690 & 39.09430 & 21.82478:19.33729 \\ 16.57591 & 15.88957 & 24.13430 & 25.02898 & 35.53128 & 40.48009 & 22.67380:20.01039 \\ 23.56536 & 22.58604 & 34.31169 & 35.53128 & 50.54589 & 57.58271 & 32.56041:28.46906 \\ 26.85009 & 25.73729 & 39.09430 & 40.48009 & 57.58271 & 65.60051 & 37.03958:32.43504 \\ 14.99384 & 14.24057 & 21.82478 & 22.67380 & 32.56041 & 37.03958 & 24.54979:18.31092 \end{bmatrix}$$

Using MathLab (2015) to solve the matrix, we have the solution
$\beta_o=-1.4875 \quad \alpha=0.2563 \quad \beta=2.0394 \quad \gamma=0.0208 \quad \mu=0.4522 \quad \phi=0.1160 \quad \omega=0.0983$
Recall $\beta_o=-1.4875$ but $LogA=-1.4875$
$A=10^{-1.4875}-1.4875$
$A=0.03255$
A=0.03255

Substituting these values in equation (3), we have the production function otherwise known as the Cobb Douglas Model for Dissolved Oxygen equation 15:

$$Q=0.03255\,K_1^{0.2563}K_2^{2.0394}L_1^{0.0208}L_2^{0.4522}M^{0.1160}E^{0.0983} \quad (15)$$

Various values of K_1, K_2, L_1, L_2, M and E can be inputted to obtain corresponding values of Q. Therefore the function, in addition to being a monitoring device can be used as a forward-planning tool to project into the future, particularly to determine the amount of DO in a fiber glass fish tank.

3. Results and Discussion

In order to get the output (Cobb Douglas DO) in treatment, the data of table 2 was inputted into the six factor (6) Cobb Douglas model in equation 15

$$Q=0.03255\,K_1^{0.2563},K_2^{2.0394},L_1^{0.0208},L_2^{0.4522},M^{0.1160},E^{0.0983}.$$

Also a linear regression model was developed to check the accuracy of the Cobb Douglas Model of the results of DO from the secondary data. The model is as presented in equation 16.

$$Q = -60.2 + 1.89K_1 + 1.88K_2 + 0.041L_1 + 0.072L_2 - 0.029M + 0.0394E \quad (16)$$

Table 3 is the comparison of the Cobb Douglas model (DO_{CD}) and the regression model (DO_{REG}). The result shows that from week 1 to week 11 the DO increases from 12.78616mg/L to 17.28149mg/L which has exceeded the maximum for any fish to survive.

Table 3: Dissolved Oxygen for Cobb Douglas and Regression Models

WEEK	DO_{CD}	DO_{REG}	%error
1	12.786	13.555	-0.0601
2	14.950	14.903	0.0031
3	16.274	16.277	-0.00002
4	16.532	16.577	-0.0027
5	17.075	15.167	0.1118
6	16.293	16.415	-0.0075
7	16.424	16.221	0.0124
8	16.507	15.938	0.0345
9	16.833	16.142	0.0411
10	17.013	16.727	0.0168
11	17.282	17.583	-0.0174

Figure 1 shows the plot of dissolved oxygen for Cobb Douglas model (blue line) and dissolved oxygen for the regression (red line) for the eleven weeks. From observation the two plot lines intersect from week two. At week five the regression for dissolved oxygen was low and at week 6 they both intersect following a linear path.

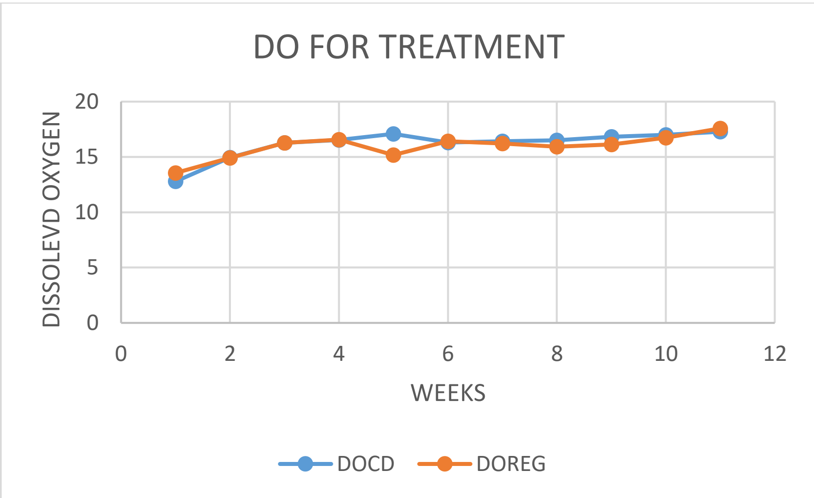

Figure1: Dissolved Oxygen plot for treatment one

Further from the model summary and Table 4, it was observed too that the model fits in well with a coefficient of determination (R^2) of 71.39%. This shows that the various components of the model were able to account for 71.39% of the variation in the given period of 11 weeks. The estimated standard deviation (S) of the error in this model is 1.08566. The Degree of Freedom (DF) which indicates the number of independent pieces of information involving the response data needed to calculate the sum of squares for the regression was calculated to be 1 while that of the error was calculated to be 4 with a total DF of 10. Also the total sum of squared (SS) distance was calculated to be approximately 16.4785. From this, the SS Regression which was a portion of the variation explained by the model, was estimated to be 11.7638 while the SS Error which was the portion not explained by the model and was therefore attributed to the errors, was estimated to be 4.7147 The Mean Square Regression (MSR) of the model was estimated to be 1.96063 while the Mean Square of the Error (MSE) also known as Mean Squared Deviation (MSD) which is a risk function was estimated to be 1.17866.

Table 4: Multivariate Linear Regression results for treatment

Source	Sum of Squares	Degree of Freedom(DoF)	Mean square	F-Value	P-value
regression	11.7638	6	1.96063	1.66	0.324
X_1	2.3986	1	2.39860	2.04	0.227
X_2	0.9111	1	0.91109	0.77	0.429
X_3	0.1270	1	0.12703	0.11	0.759
X_4	0.0676	1	0.06756	0.06	0.823
X_5	0.0431	1	0.04312	0.04	0.858
X_6	2.3077	1	2.30771	1.96	0.234
Error	4.7147	4	1.17866		
Total	16.4785	10			

The model fitness evaluation is presented in Table 5. It was observed that the p-values for each of the variables $x_1, x_2, x_3, x_4, x_5, x_6$ are not <0.05 .This unreliability result is also seen in the multicollinearity of the high level of variance inflation factors of the regression coefficient for x_4 x_5 and x_6 which are 469.57, 4463.77 and 15.03.

Table 5: Coefficient Estimate for Model Representing Treatment

Term	Coefficient estimate	SE Coefficient	T-Value	P-Value	VIF
Constant	-60.2	60.0	-1.00	0.372	-
X_1	1.89	1.33	1.43	0.227	6.90
X_2	1.88	2.14	0.88	0.429	5.59
X_3	0.041	0.125	0.33	0.759	2.87
X_4	0.072	0.303	0.24	0.823	469.57
X_5	-0.029	0.151	-0.19	0.858	463.77
X_6	0.0394	0.0282	1.40	0.234	15.03

The exclusion of these three variables will help to produce a more stable regression model and reducing the p-values. It is also observed that the coefficient estimate factors have both negative and positive values where the positive values indicate a favorable effect on the model and the negative value indicates antagonistic effect on the response. Figure 2 shows the Residual plots for DO from the REGRESSION analysis using the MINITAB software. It shows that the points fall within the lines on the normal probability plot indicating collinearity between the model and the input parameters.

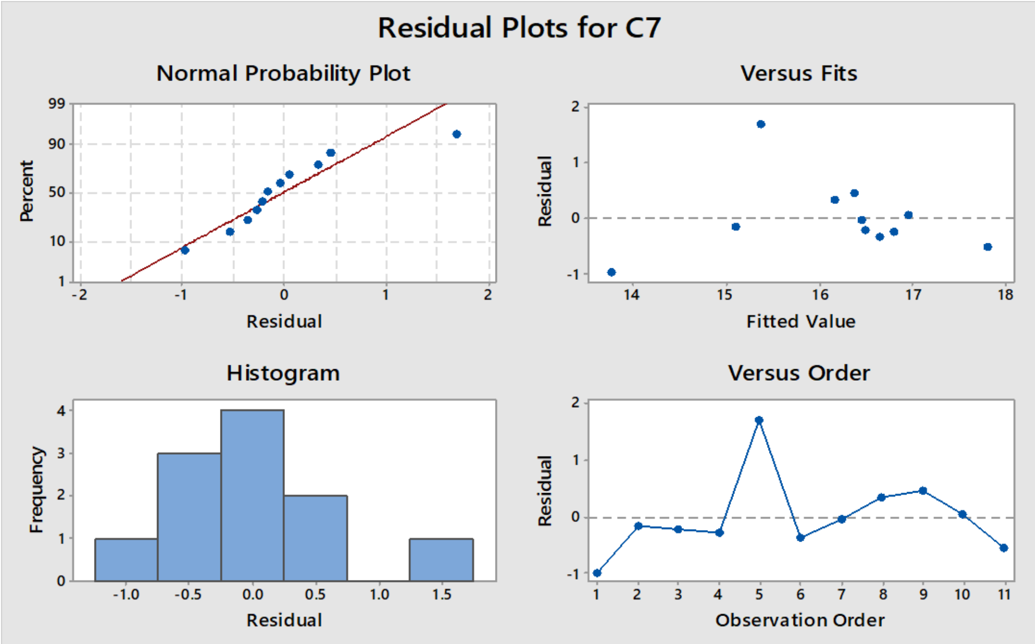

Figure 2: Residual Plot for Treatment One

Comparing the secondary data results (DO_E) with that of the Cobb Douglas (DO_{CD}) and the Regression (DO_{REG}) Models. Table 4 shows the experimental value (DO_E) for dissolved oxygen with the Cobb Douglas and Regression value for dissolved oxygen.

Table 4: Experimental Value and Model Results

WEEK	DO_E	DO_{CD}	DO_{REG}
1	12.797	12.786	13.555
2	14.642	14.950	14.903
3	16.425	16.274	16.277
4	16.736	16.532	16.577
5	17.094	17.075	15.167
6	15.842	16.293	16.415
7	16.603	16.424	16.221
8	16.875	16.507	15.938
9	17.222	16.833	16.142
10	17.211	17.013	16.727
11	16.481	17.282	17.583

From the graph in figure 3 the experimental DO_E, Cobb Douglas DO_{CD} and the regression DO_{REG} all converge/intersected. This shows that the Cobb Douglas model fits in well with the Experimental data from Adams (2021) and showed more reliability as compared to the regression model.

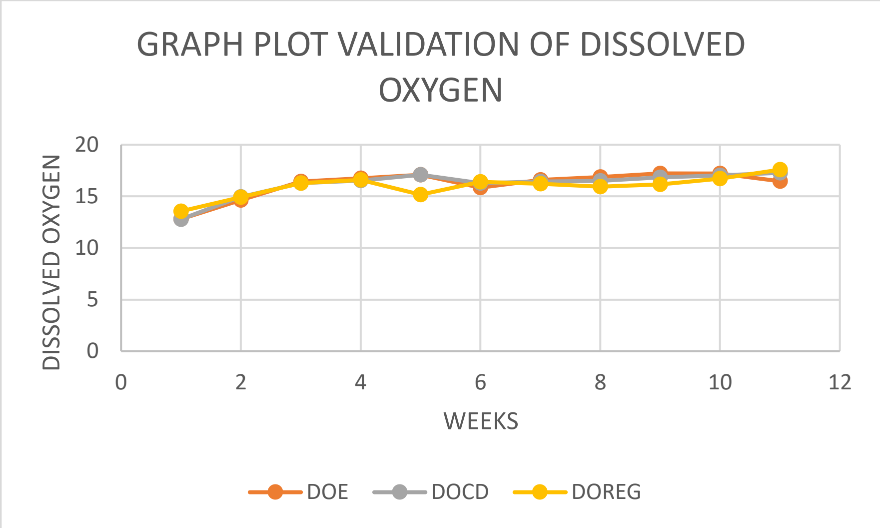

Figure 3: Dissolved oxygen plot for the three models

4. Conclusion

Models have been developed that show the interaction between dissolved oxygen and all available water quality parameters (pH, Temperature, Transparency, Total dissolved solid and Electrical conductivity, Turbidity, and Dissolve Oxygen). The percentage variation of the DO_{CD} and DO_{REG} are <0.005 showing a high degree of accuracy. While most DO for fish ponds and other tanks takes into account one to three water quality parameters, this research has looked into seven using the Cobb Douglas approach and the accuracy with experimental data (literature) was found to be accurate. It was observed also that the DO for this treatment exceeded that the fish will survive.

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