



Steady State Free Convection with Heat and Mass Transfer in the Presence of Variable Thermal Conductivity

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Abstract

The steady state free convection flow in a vertical channel with heat and mass transfer in the presence of uniform magnetic field applied normal to the flow region and temperature dependent variable thermal conductivity has been investigated in this paper. The nonlinear partial differential equations together with the boundary conditions are reduced to dimensionless form. Approximate solutions have been derived for velocity, temperature, concentration, Skin friction, Nusselt number, and Sherwood number are presented graphically and discussed quantitatively for various parameters embedded in the problem.

1. Introduction

The free convection flow with heat mass transfer problem has been studied by several researchers in the recent decades. A situation in flow formation which is not time dependent is called steady state flow. This type of flow has applications in many engineering devices like boiler, turbine, condenser and water pump that run nonstop for many months before they are shut down for maintenance. Several scholars considered steady natural convection flow through channels due to its significance in engineering technology; especially; in cooling/heating application. For example, it is used in computer engineering where electronic cabinets containing circuits are design in channel forms so as to enhance cooling of the computer system; in civil engineering, channels are used for irrigation purposes, measuring discharge of water in a river, studying the spread of pollutants and so on. In relation to this, the effects of variable thermal conductivity and variable viscosity on steady free convective heat transfer flow process along an isothermal vertical plate in the presence of heat

sink was presented by Mahanti and Gaur [1]. Takhar *et al.* [2] studied the flow and mass transfer characteristics of a viscous electrically conducting fluid on a continuously stretching surface with non-zero slot velocity. Chantasiriwan [3] discussed steady state determination of temperature dependent thermal conductivity. Heat transfer with variable conductivity in a stagnation-point flow towards a stretching sheet was reported by Chiam [4]. Makinde [5] investigated a note on the Hydromagnetic Blasius flow with variable thermal conductivity. The governing fluid flow equation with boundary condition have been transformed into set of coupled ordinary differential equations with the help of similarity transformations and solved shooting method with Runge-Kutta-Fehlberg integration scheme and concluded that the wall skin friction is an increasing function of the magnetic field parameter, the heat transfer rate increases with Prandtl number and magnetic but drops with increasing values of the thermal conductivity, both thermal and velocity field parameter. Boundary layer thickness lessens with a rise in magnetic field parameter intensity, and the fluid temperature increases with a rise in thermal conductivity but diminishes with a rise in Prandtl number parameter. The influence of temperature dependent viscosity and thermal conductivity on natural convection flow through a vertical channel was considered by Ajibade and Ojeagbaze [6]. The governing equations for temperature, velocity and concentration fields were solved analytically using the differential transformation method. The results were verified with results from the exact and numerical methods, excellent agreement was observed. The results of fluid flow within the channel reveals the increasing fluid viscosity and fluid temperature within the channel while it decreases fluid velocity.

Jha [7] investigated the effect of applied magnetic field on transient free convection flow in a vertical channel. The effect of temperature dependent thermal conductivity on hydromagnetic free convection flow along vertical flat plate with heat conduction was derived by Rahman *et al.* [8].

Sparrow *et al.* [9] discovered the combined effects of steady free and forced convective laminar flow and heat transfer between vertical walls. Jha and Aina [10] studied role of suction/injection on steady fully developed mixed convection flow in a vertical parallel plate microchannel. Hamza [11] investigated free convection slip flow of an exothermic fluid in a convectively heated vertical channel. Numerical solutions of the problem are obtained by using unconditionally stable and convergent implicit finite difference scheme. Due to nonlinearity of the governing equations, approximated solutions for steady state version of the problem have been derived for momentum, energy, skin friction and the rate of heat transfer by using perturbation series method. Muhim [12] discussed effect of variable thermal conductivity and the inclined magnetic field on MHD plane Poiseuille flow in a porous channel with non-uniform plate temperature. The reduced similarity equations were then solved by finite difference technique.

Furthermore, Souayeh *et al.* [13] investigated heat transfer characteristics of fractionalized Hydromagnetic fluid with chemical reaction in permeable media. The governing fluid flow equations with boundary conditions have been transformed into set of coupled ordinary differential equations with the reduced similarity transformations and solved Fourier sine with Laplace transforms. The effects of physical parameters were examined. Aung *et al.* [14] obtained their results for steady free convective flow between vertical walls by considering different physical situations of transport processes. Paul *et al.* [15] deliberated the transient free convective flow in a vertical channel having constant temperature and constant heat flux on the channel walls. The effects of the thermophoresis, viscous dissipation and joule heating on steady MHD flow over an inclined radiative isothermal permeable surface with variable thermal conductivity was accomplished by Reddy [16]. Jha *et al.* [17] investigated the analytical solution for the transient free convection flow in a vertical channel as a result of symmetric heating. Singh *et al.* [18] deliberated the transient free convection flow between two vertical parallel plates. Saeed *et al.* [19] discussed Heat and Mass transfer of free convection flow over a vertical plate with chemical reaction under wall slip effect. Solutions for the fluid velocity, temperature and concentration are achieved in closed forms by applying Laplace transform. Combined effect of variable and thermal conductivity on free

convection flow of a viscous fluid in a vertical channel using differential transformation method was considered by Umavathi and Shekar [20]. Ajibade and Ojeagbase [21] investigated steady natural convection heat and mass transfer flow through a vertical porous channel with viscosity and thermal conductivity. The variability in viscosity and thermal conductivity are considered linear function of temperature. The governing equations are transformed into a set of coupled nonlinear ordinary differential equations. Results obtained were compared with exact solution when some of the flow conditions were relaxed and results from differential transformation method show an excellent agreement with the exact solution which was obtained analytically.

Sehra *et al.* [22] studied convection heat mass transfer and MHD flow over a vertical plate with chemical reaction, arbitrary shear stress and exponential heating. Alim *et al.* [23] analyzed the heat generation effects on MHD natural convection flow along a vertical wavy surface with variable thermal conductivity. Rout *et al.* [24] established the MHD heat and mass transfer of chemical reaction fluid flow over a moving vertical plate in presence of heat source with convective surface boundary condition. Uwanta and Omokhuale [25] investigated the effects of variable thermal conductivity on heat mass transfer with Jeffery fluid. The problems of steady and unsteady flows having combined heat mass transfer by free convection with and without chemical reaction have been studied extensively by different scholars. Hamza *et al.* [26] have studied the problem of unsteady/steady hydromagnetic convective flow between two vertical walls heated symmetrically/asymmetrically in the presence of variable thermal conductivity. The aim of the present work is motivated to study steady state free convection with heat and mass transfer in the presence of variable thermal conductivity.

2.0. Methodology

2.1. Formulation of the Problem

Consider steady, natural convections, heat and mass transfer flow of an electrically, conducting incompressible viscous fluid, having temperature dependent thermal conductivity between two vertical walls under the influence of a uniform transverse magnetic field of strength B_o .

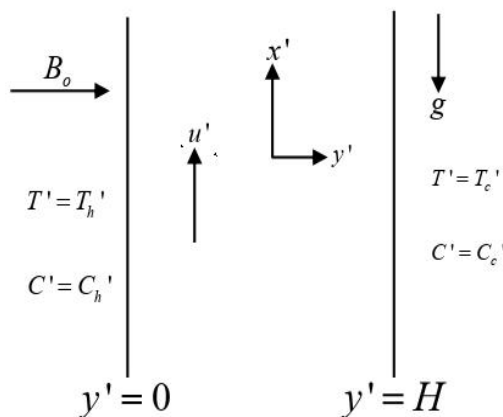


Figure 1: Geometry of the Problems

It is assumed that both the fluid and the walls are at rest and maintained at a constant temperature T_m' and the mass concentration C_m' . At time $t' > 0$, the wall is maintained at uniform temperature T_h' and uniform concentration C_h' which are higher than T_c' and C_c' respectively. We choose a Cartesian coordinate system with x' axis along the upward direction and the y' axis normal to it. Thermal conductivity (k_f) which obeys linear temperature law according to $k_f = k_m [1 + \delta(T' - T_m')]$

, where k_m is the fluid free thermal conductivity and δ is a constant dependent on the fluid ($\delta > 0$ for lubrication oils, hydromagnetic working fluids and $\delta > 0$ for air or water). Under these assumptions, along with Boussinesq's approximation, the governing equations for momentum, energy, and continuity in laminar incompressible boundary layer flow can be written as:

$$\frac{\partial U'}{\partial t'} = \nu \frac{\partial^2 U'}{\partial y'^2} + g\beta(T' - T'_m) - \frac{\sigma B_o^2 U'}{\rho} + g\beta^*(C' - C'_m) \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{1}{\rho c_p} \frac{\partial}{\partial y'} \left[k_f \frac{\partial T'}{\partial y'} \right] \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (3)$$

The corresponding initial and boundary conditions are prescribed as follows:

$$\begin{aligned} t' \leq 0; \quad U' = 0, \quad T' = T'_m, \quad C' = C'_m, \quad \text{for } 0 \leq y' \leq H \\ t' > 0; \quad U' = 0, \quad T' = T'_h, \quad C' = C'_h, \quad \text{at } y' = 0 \\ U' = 0, \quad T' = T'_c, \quad C' = C'_c, \quad \text{at } y' = H \end{aligned} \quad (4)$$

Where ν, σ, ρ are kinematic viscosity, conductivity of the fluid and density respectively, g, β, β^* are gravitational force, coefficient of the thermal expansion and concentration expansion coefficient respectively, D, B_o, c_p are the chemical molecular diffusivity, the electromagnetic induction and specific heat at constant pressure respectively.

The non-dimensional quantities introduced in the above equations are as follows:

$$\begin{aligned} y = \frac{y'}{H}, \quad t = \frac{t'\nu}{H^2}, \quad U = \frac{U'\nu}{g\beta(T'_h - T'_m)H^2}, \quad Sc = \frac{\nu}{D}, \quad Pr = \frac{\nu\rho c_p}{k_m}, \\ M^2 = \frac{\sigma B_o^2 H^2}{\nu\rho}, \quad Gc = \frac{g\beta^*(C'_h - C'_m)}{g\beta(T'_h - T'_m)}, \quad \lambda = \delta(T'_h - T'_m), \\ \theta = \frac{T' - T'_m}{T'_h - T'_m}, \quad C = \frac{C' - C'_m}{C'_h - C'_m}, \quad R = \frac{T'_c - T'_m}{T'_h - T'_m}, \quad Rc = \frac{C'_c - C'_m}{C'_h - C'_m} \end{aligned} \quad (5)$$

Applying (5) to (1), (2), (3), (4), the following governing equations in non-dimensional form are obtained:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial y^2} + \theta - M^2 U + GcC \quad (6)$$

$$Pr \frac{\partial \theta}{\partial t} = (1 + \lambda\theta) \frac{\partial^2 \theta}{\partial y^2} + \lambda \left(\frac{\partial \theta}{\partial y} \right)^2 \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2}, \quad (8)$$

with the following initial and boundary conditions in dimensionless form:

$$\begin{aligned}
 t \leq 0; \quad U = 0, \theta = 0, C = 0, & \quad \text{for all } y \\
 t > 0; \quad U = 0, \theta = 1, C = 1, & \quad \text{at } y = 0 \\
 U = 0, \theta = R, C = Rc, & \quad \text{at } y = 1
 \end{aligned} \tag{9}$$

Where Pr is the Prandtl number, Sc is the Schmidt number, Gc is the mass Grashof number, M is the Magnetic field parameter, λ is the variable thermal conductivity, Rc is the mass buoyancy parameter and R is the temperature buoyancy parameter.

2.2. Analytical Solutions

The analytical solutions have played significant role in validating and exploring computer routines of complicated problems. We reduce the governing equations of this problem into a form that can be solved analytically by setting $\partial U / \partial t = 0, \partial \theta / \partial t = 0$ and $\partial C / \partial t = 0$, into (6), (7) and (8), the equations gives:

$$\frac{d^2U}{dy^2} + \theta - M^2U + GcC = 0 \tag{10}$$

$$(1 + \lambda\theta) \frac{d^2\theta}{dy^2} + \lambda \left(\frac{d\theta}{dy} \right)^2 = 0 \tag{11}$$

$$\frac{1}{Sc} \frac{d^2C}{dy^2} = 0 \tag{12}$$

The boundary conditions are:

$$\begin{aligned}
 U = 0, \theta = 0, C = 0, & \quad \text{for all } y \\
 U = 0, \theta = 1, C = 1, & \quad \text{at } y = 0 \\
 U = 0, \theta = R, C = Rc, & \quad \text{at } y = 1
 \end{aligned} \tag{13}$$

The steady state solutions of (10) to (12) subject to (9), we use perturbation method of the form:

$$\begin{aligned}
 U &= U_0 + \lambda U_1 \\
 \theta &= \theta_0 + \lambda \theta_1 \\
 C &= C_0 + \lambda C_1
 \end{aligned} \tag{14}$$

Substituting (13) into (10) to (12), the solution of the governing equations is obtained as:

$$U = A_1 \cosh My + A_2 \sinh My + A_3 y + A_4 + \lambda B_1 \cosh My + \lambda B_2 \sinh My + \lambda B_3 y^2 + \lambda B_4 y + \lambda B_5 \tag{15}$$

$$\theta = (R-1)y + 1 - \lambda \frac{(R-1)^2 y^2}{2} + \lambda b_1 y + \lambda b_2 \tag{16}$$

$$C = (Rc-1)y + 1 + \lambda f_1 y + \lambda f_2 \tag{17}$$

The Skin friction from the velocity is given by:

$$\begin{aligned}
 \left. \frac{dU}{dy} \right|_{y=0} &= MA_2 + A_3 + M \lambda B_2 + \lambda B_4 \\
 \left. \frac{dU}{dy} \right|_{y=1} &= MA_1 \sinh M + MA_2 \cosh M + A_3 + M \lambda B_1 \sinh M + M \lambda B_2 \cosh M + 2\lambda B_3 y + \lambda B_4
 \end{aligned} \tag{18}$$

Similarly, the Nusselt number become:

$$\left. \frac{d\theta}{dy} \right|_{y=0} = (R-1) + \lambda b_1 \tag{19}$$

$$\left. \frac{d\theta}{dy} \right|_{y=1} = -(R-1)^2 \lambda + (R-1) + \lambda b_1$$

While, the Sherwood number is:

$$\left. \frac{dC}{dy} \right|_{y=0} = (Rc-1) + \lambda f_1 \tag{20}$$

Where,

$$b_1 = \frac{R^2 - 2R + 1}{2}, \quad A_1 = -A_4, \quad A_2 = \frac{A_4 \cosh M - (A_3 + A_4)}{\sinh M}, \quad A_3 = \frac{(R-1) + Gc(Rc-1)}{M^2},$$

$$A_4 = \frac{1 + Gce_2}{M^2}, \quad B_1 = -B_5, \quad B_2 = \frac{B_5 \cosh M - (B_3 + B_4 + B_5)}{\sinh M}, \quad B_3 = -\frac{(R-1)^2}{2M^2},$$

$$B_4 = \frac{b_1 + Gcf_1}{M^2}, \quad \text{and} \quad B_5 = \frac{b_2 + Gcf_2}{M^2}$$

3.0. Results and Discussion

In this paper, the numerical solutions for different values of the magnetic field M, Schmidt number Sc, mass Grashof number Gc, buoyancy force distribution R, mass buoyancy Rc and thermal conductivity λ were obtained. The following parameter values are fixed throughout the computation except where otherwise stated, $y = 0:0.01:1$, $Sc = 0.22$, $R = 0.5$, $Gc = 1$, $Rc = 0.1$, $\lambda = 0.7$, $M = 1$.

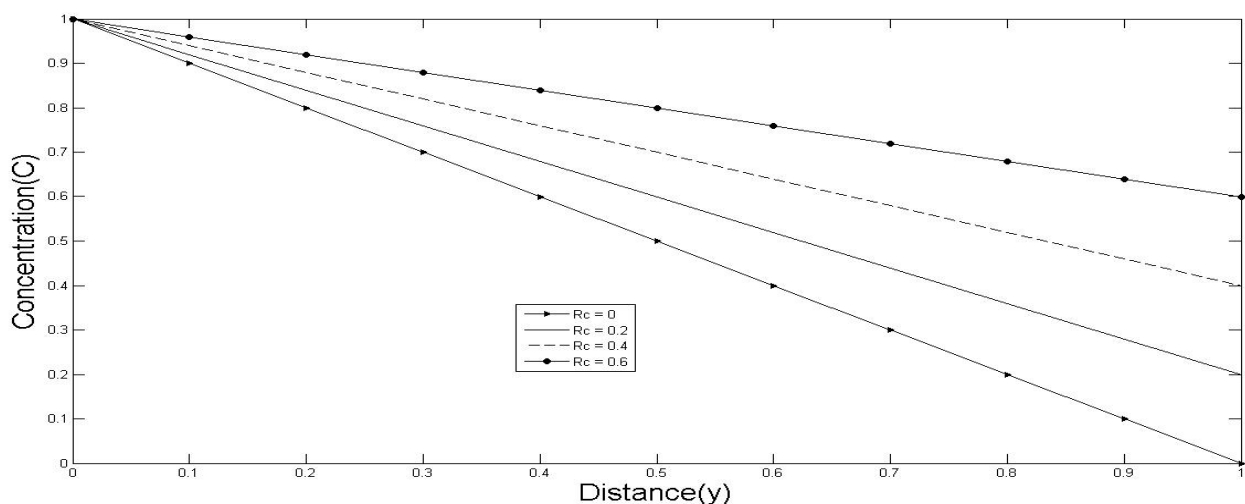


Figure 2: Effect of various value of Rc on concentration profile.

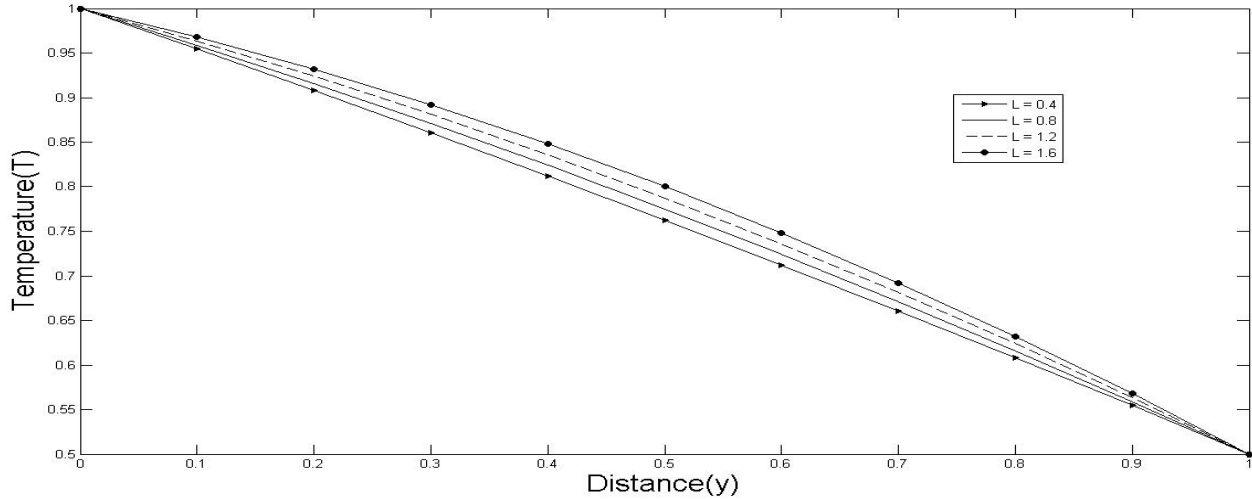


Figure 3: Effect of various value of $(L=\lambda)$ on temperature profile.

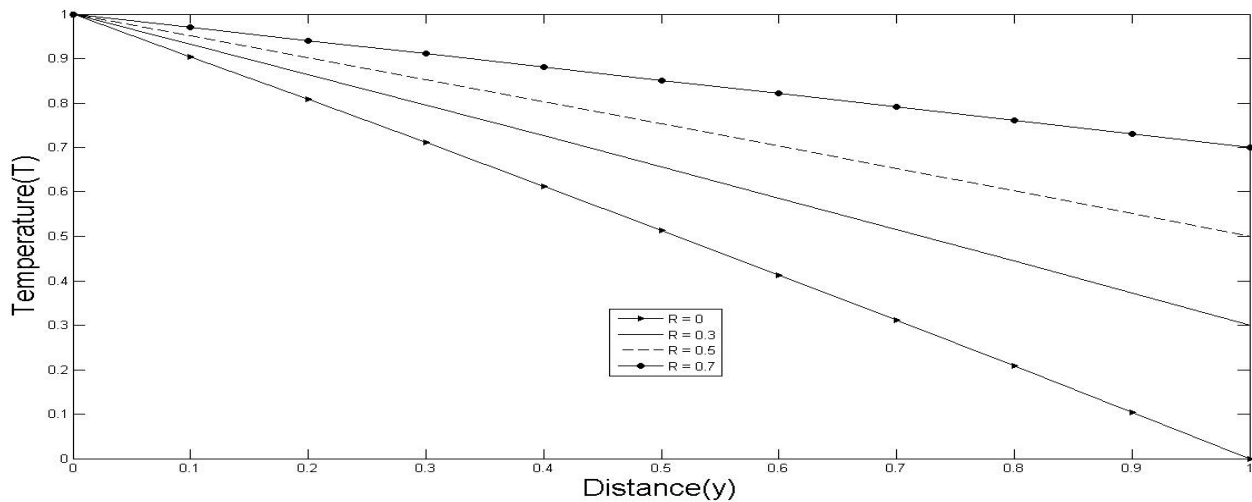


Figure 4: Effect of various value of R on temperature profile.

Figure 2 represents the concentration profile for various value of mass buoyancy ($R_c = 0, 0.2, 0.4, 0.6$). In Figure 2, it is observed that, the concentration increases with increase in mass buoyancy parameter R_c .

The temperature profiles have been studied and illustrated in Figure 3 and Figure 4 for various value of thermal conductivity ($\lambda = 0.4, 0.8, 1.2, 1.6$) and buoyancy force distribution ($R = 0, 0.3, 0.5, 0.7$), parameters shown in Figures 2 and 3 respectively. Figure 3 and 4 shows that, the temperature increase with increase of thermal conductivity parameter λ and similarly for the buoyancy force distribution parameter R .

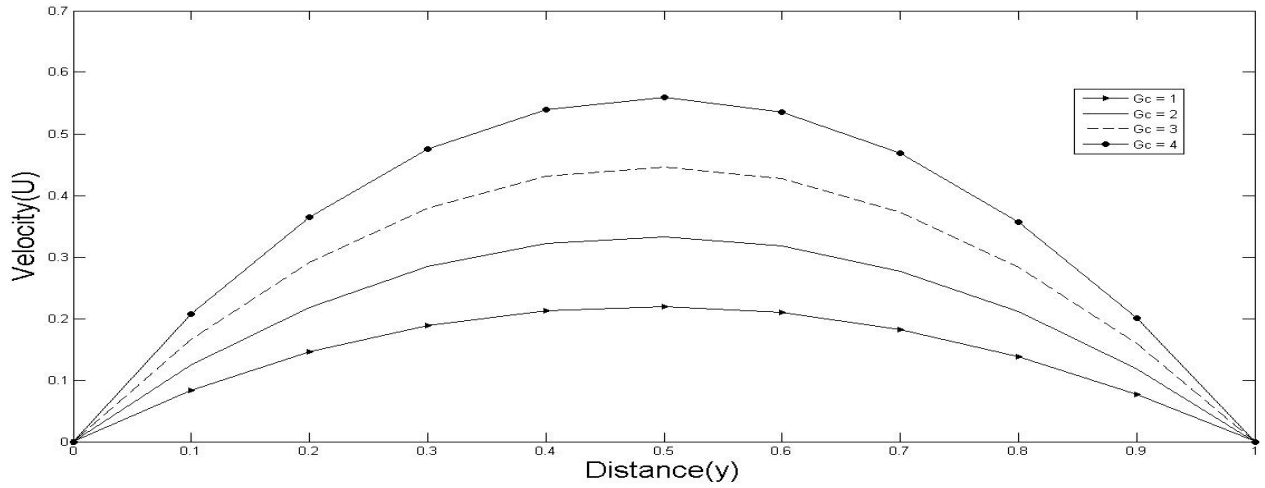


Figure 5: Effect of various value of G_c on velocity profile

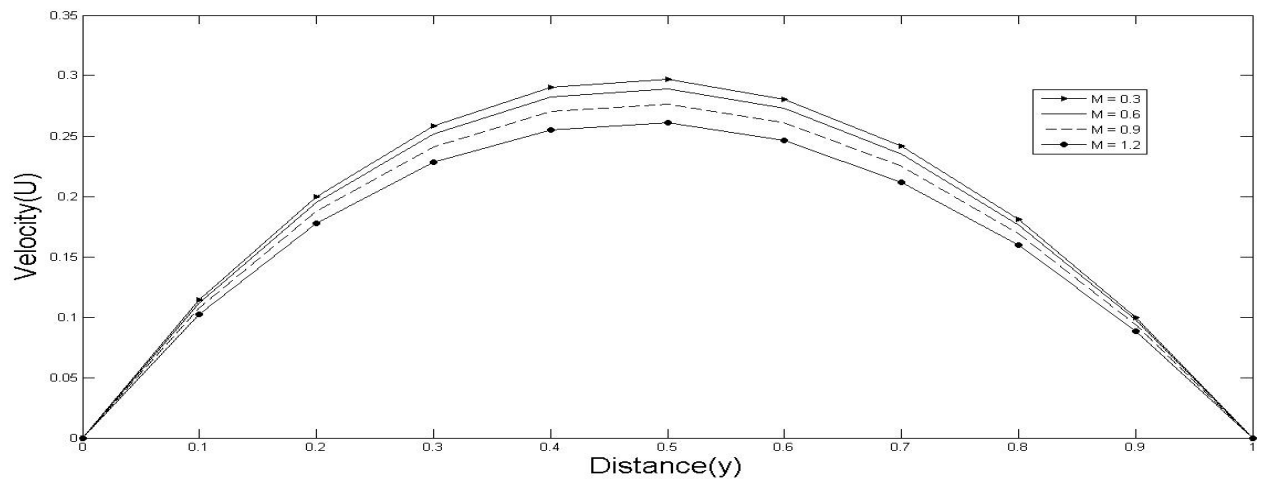


Figure 6: Effect of various value of M on velocity profile.

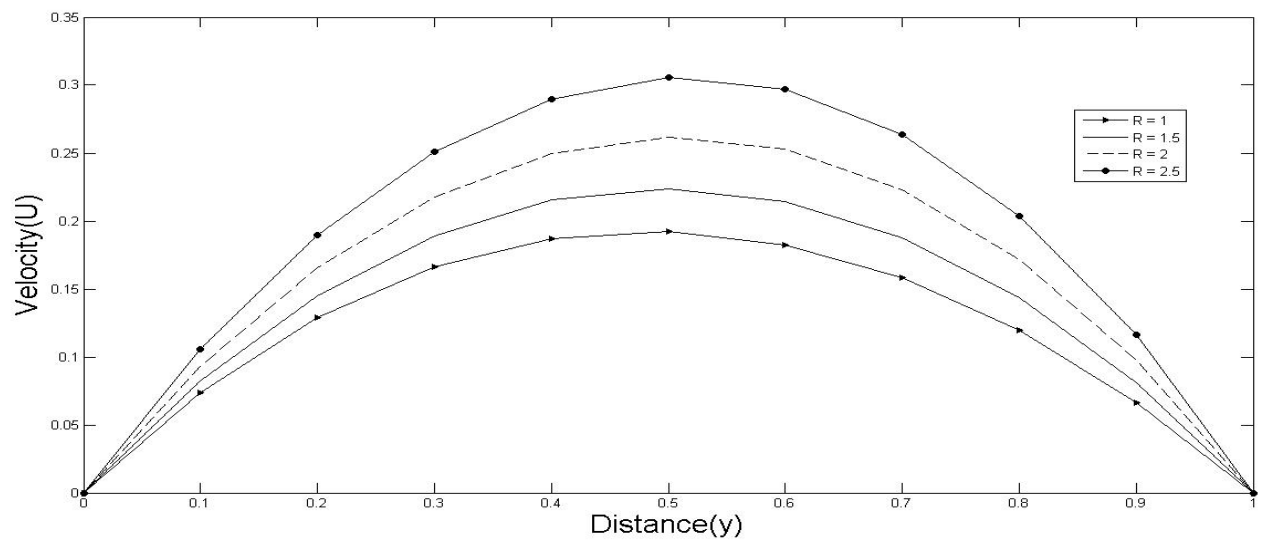


Figure 7: Effect of various value of R on velocity profile.

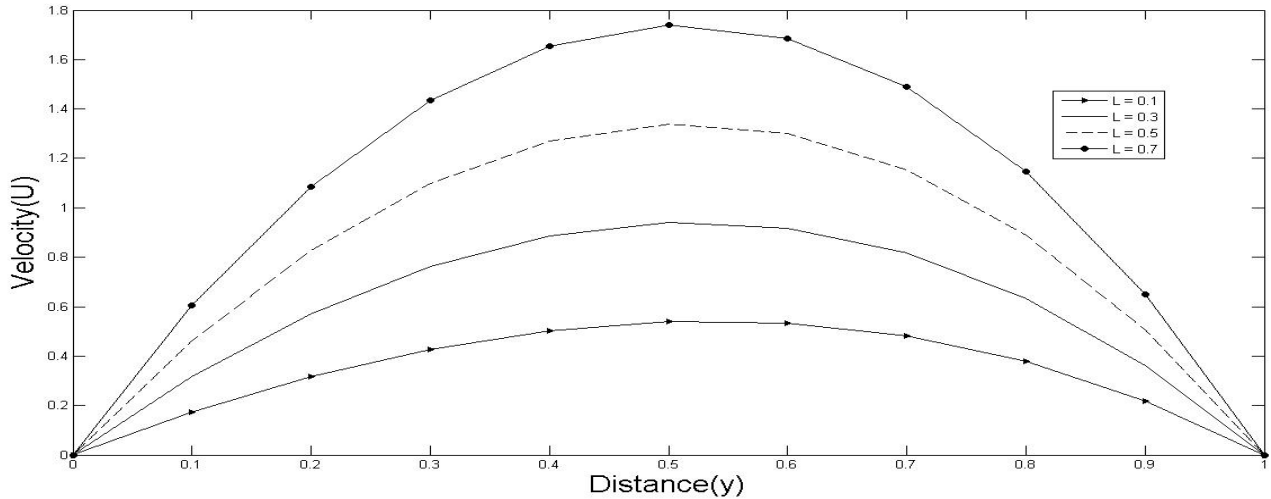


Figure 8: Effect of various value of ($L=\lambda$) on velocity profile.

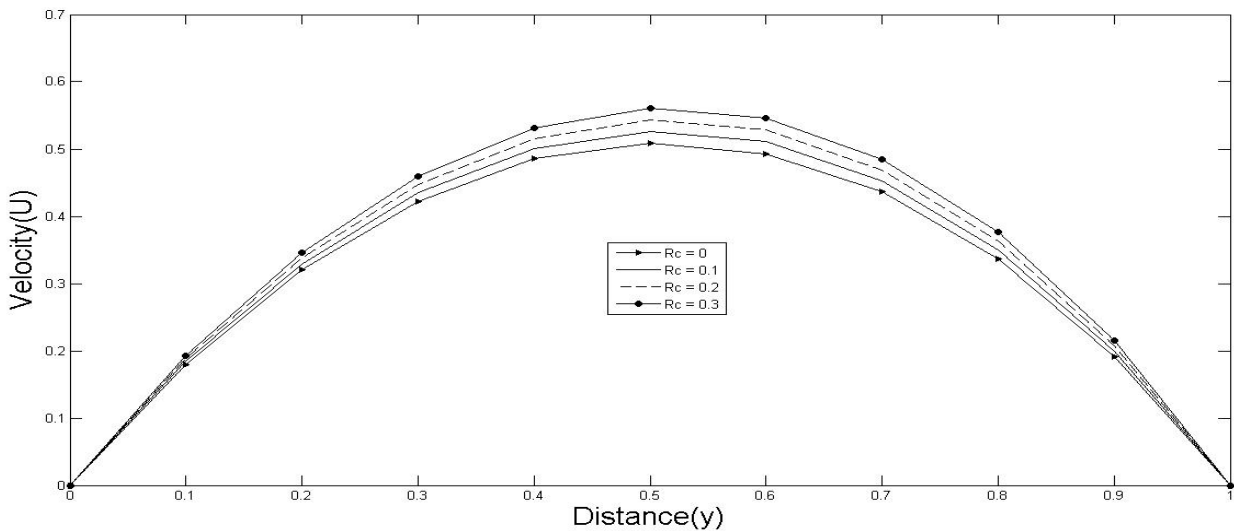


Figure 9: Effect of various value of Rc on velocity profile.

Figures 5 to 9 illustrates the velocity profiles for various value of mass Grashof number ($G_c = 1, 2, 3, 4$), magnetic field ($M = 0.3, 0.6, 0.9, 1.2$), buoyancy force distribution ($R = 1, 1.5, 2, 2.5$), thermal conductivity ($\lambda = 0.1, 0.3, 0.5, 0.7$) and mass buoyancy ($R_c = 0, 0.1, 0.2, 0.3$). Figure 5 shows that, the velocity increase when the mass Grashof number G_c increased. But, Figure 6 indicates that, the velocity decrease with increase in the magnetic field parameter M . Also, Figure 7 pointed that, the velocity increase whenever buoyancy force distribution parameter R increases just like Figure 8 which founds that, the velocity increase with increasing mass buoyancy parameter R_c . Similarly, Figure 9 reveals that, the velocity increase by increasing thermal conductivity parameter λ .

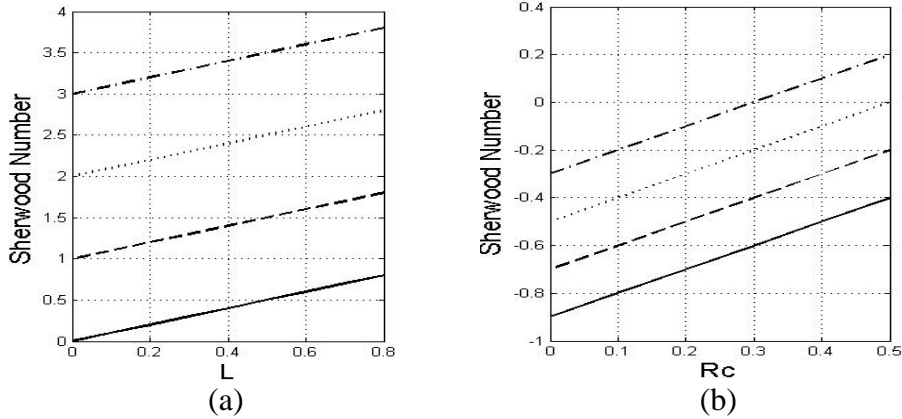


Figure 10: Effect of Sherwood number with ($L=\lambda$) and R_c at $y = 0$.

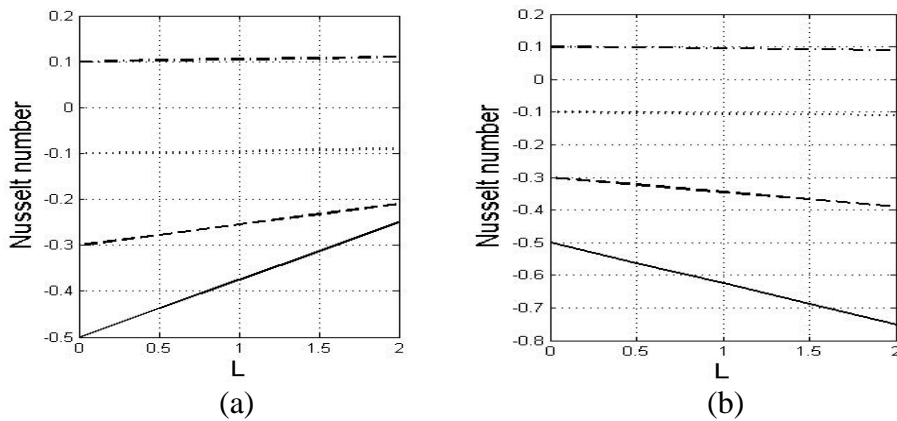


Figure 11: Effect of Nusselt number with ($L=\lambda$) and R at $y = 0$ and $y = 1$.

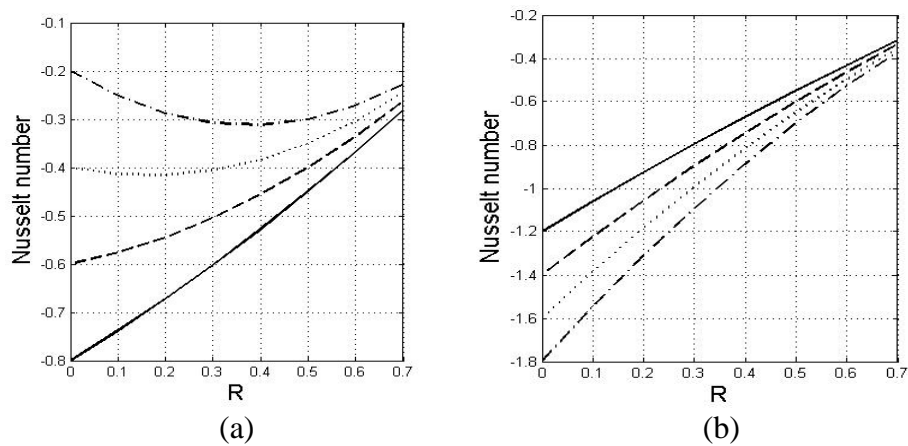


Figure 12: Effect of Nusselt number with R and ($L=\lambda$) at $y = 0$ and $y = 1$.

Figure 10 to 15 are for the Sherwood number, Nusselt number and skin friction for the analytical solution. Figure 10 illustrates the effect of mass buoyancy R_c and thermal conductivity λ on fluid Sherwood number, it is noticed that the Sherwood number in Figure 10, gets reduced with increase of mass buoyancy parameter R_c and thermal conductivity parameter λ .

Figure 11 and 12 displays the effect of buoyancy force distribution R and thermal conductivity λ on fluid Nusselt number for both $y = 0$ and $y = 1$ respectively and it is clearly seen that the Nusselt number in Figure 11, gets reduced with increase of buoyancy force distribution parameter R . While in Figure 12, fluid Nusselt number gets reduced by increasing thermal conductivity parameter λ at $y = 0$ and gets enhanced by increasing thermal conductivity parameter λ at $y = 1$.

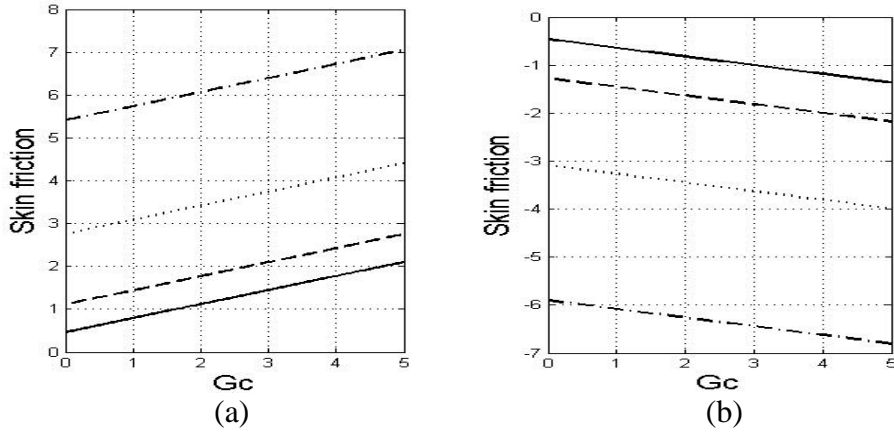


Figure 13: Effect of skin friction with Gc and R at $y = 0$ and $y = 1$.

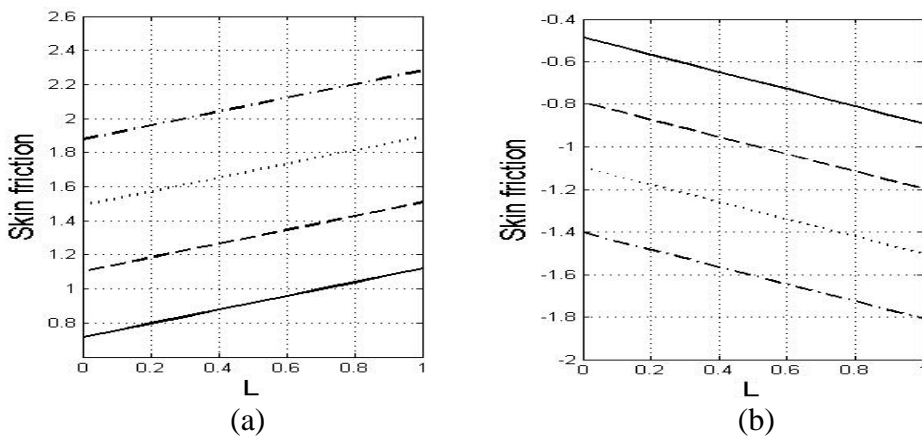


Figure 14: Effect of skin friction with $(L=\lambda)$ and Gc at $y = 0$ and $y = 1$.

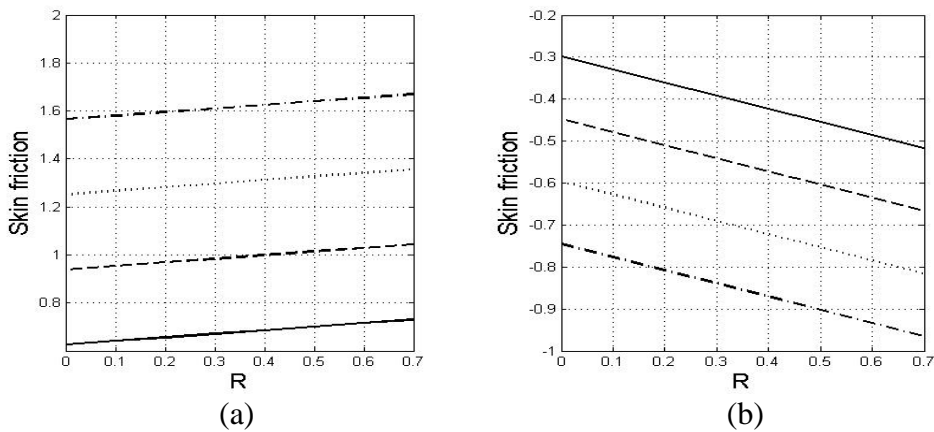


Figure 15: Effect of skin friction with R and Gc at $y = 0$ and $y = 1$.

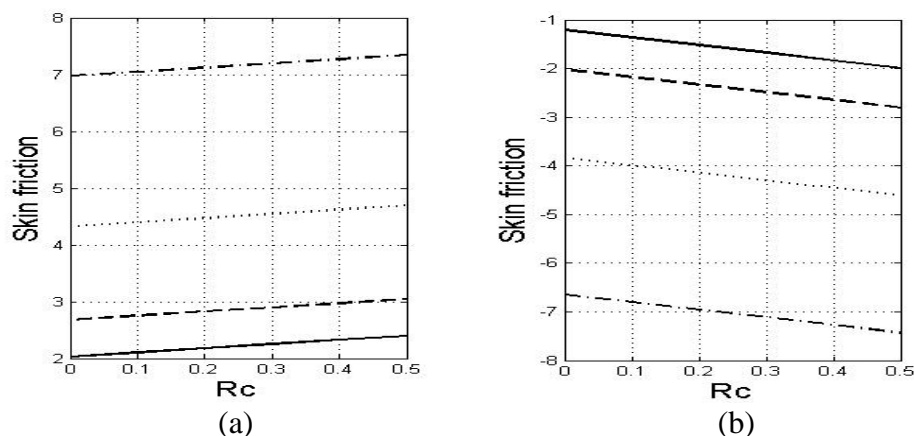


Figure 16: Effect of skin friction with Rc and R at $y = 0$ and $y = 1$.

Figures 13 to 16 represents the effect of thermal conductivity parameter λ , mass Grashof number Gc , buoyancy force distribution parameter R and mass buoyancy parameter Rc on the fluid skin friction for both $y = 0$ and $y = 1$ respectively. It is clearly seen that the fluid skin friction in Figure 13 (b) gets enhanced by increasing buoyancy force distribution parameter R just like Figure 14 (b) which found, that the fluid skin friction gets intensified with increase in mass Grashof number Gc . While in Figure 13 (a) the fluid skin friction gets reduced by increasing buoyancy force distribution parameter R . Similarly, Figure 14 (a) indicates that the fluid skin friction decrease with increase in the mass Grashof number Gc . Also, the fluid skin friction in Figure 15 (a) gets reduced by increasing mass Grashof number Gc . Likewise, Figure 16 (a) which founds that, the fluid skin friction decrease when the buoyancy force distribution parameter R increased. Further, Figure 15 (b) pointed that the fluid skin friction increase when the mass Grashof number Gc increased. Likewise, Figure 16 (b) reveals that the fluid skin friction increase with increasing buoyancy force distribution parameter R .

4.0. Conclusions

The present work analyzes the steady state free convection with heat and mass transfer in the presence of variable thermal conductivity. Dimensionless governing equations were solved analytically using the perturbation technique. Analytical solutions obtained are presented in graphs for the fluid flow and heat mass transfer characteristics for different values of parameters involved in the problem. From the study, the following conclusions were drawn:

- (1) Increase of, mass Grashof number Gc , buoyancy force distribution parameter R , mass buoyancy parameter Rc and thermal conductivity parameter λ enhances the velocity while reverse is the case with increase of magnetic field parameter M .
- (2) An increase in temperature is a function of an increase in buoyancy force parameter R , and thermal conductivity parameter λ , and the concentration increased due to increases in mass buoyancy parameter Rc .
- (3) At $y = 0$, Increase in buoyancy force distribution parameter R , and mass Grashof number Gc intensify the fluid skin friction. Similarly, at $y = 1$, skin friction gets reduced with increase of buoyancy force distribution parameter R and mass Grashof number Gc .
- (4) Increase in buoyancy force distribution R and thermal conductivity parameter λ diminishes Nusselt number at $y = 0$. Increase in thermal conductivity parameter λ also it enhances the fluid Nusselt number at $y = 1$, increase in buoyancy force distribution parameter R diminishes the Nusselt number at $y = 1$. Increase in thermal conductivity parameter λ , and mass buoyancy parameter Rc diminishes the Sherwood number.

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