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Development of Intensity Duration Frequency (Idf) Curves for Rainfall Prediction in Some Selected States in South-West Nigeria

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Abstract

Rainfall Intensity-Duration-Frequency (IDF) relationship remains one of the most widely used tools in hydrology and water resources engineering, especially for planning, designing and operations of water resource projects. The target of this study is to develop IDF curves for the prediction of rainfall intensity in some selected states in South-West Nigeria.

Forty (40) year's annual maximum rainfall data ranging from 1974 to 2013 was employed for the study. To ascertain the data quality, test of homogeneity using residual mass curve and test of hypothesis was employed. Rainfall depth at selected durations were estimated using the empirical reduction formula given by Indian Meteorological Department (IMD) while the mathematical relationship between rainfall intensity and rainfall durations was determined using the curve fitting tool in MATLAB. Thereafter, rainfall intensities for 2minutes, 5 minutes, 10 minutes, 15 minutes, 30 minutes, 60 minutes, 120 minutes, 180 minutes, 240 minute and 320 minutes were estimated coupled with the mean and standard deviation of the data for different durations. The popular Gumbel probability distribution model was employed to calculate the rainfall frequency factor for selected return periods (T= 2, 5, 10, 25, 50 and 100yrs). The rainfall intensity corresponding to a specified return period was computed using the linear relationship between the magnitude of a hydrological event X_T and the departure ΔX_T. To assess the best fit model that can be employed to predict rainfall intensity for various return periods at ungauged locations, four empirical IDF equations, namely; Talbot, Bernard, Kimijima and Sherman equations were employed. The model with the least calculated sum of minimized Root Mean Square Error (RMSE) was acclaimed the best fit empirical model.

Results obtained revealed that the Talbot model was the best fit model with calculated sum of minimized error of 5.666066E-07 and 6.424229E-07. The model was thereafter employed to predict the rainfall intensity for different durations at 2, 5, 10, 25, 50 and 100yrs return periods.

1. Introduction

Rainfall intensity–duration–frequency (IDF) curves are graphical representations of the probability that a given average rainfall intensity will occur within a given period of time by providing mathematical relationship between the rainfall intensity, i, the duration, d, and the return period T (or equivalent to the annual frequency of exceedance f) [1].

As reported in the works of [2], IDF curves allow for the estimation of the return period of an observed rainfall event or conversely of the rainfall intensity corresponding to a given return period. He stated that design storms derived from IDF curves are commonly adopted in water resources engineering for designing of urban drainage systems, evaluating the endurance of hydraulic structures, and assessing regional flood vulnerabilities. Rainfall intensity–duration–frequency (IDF) curves play an important role in water resources Engineering and Management. Other applications range from assessing rainfall events, classifying climatic regimes, deriving design storms and assisting in designing urban drainage systems, etc. The deriving procedure of IDF curves requires long-term historical rainfall observations, whereas lack of fine-timescale rainfall records (e.g. sub-daily) often results in less reliable IDF curves.

An intensity duration frequency curve is a mathematical model that expresses the relationship between intensity, duration, and return period of precipitation [3, 4]. The Engineering application of rainfall intensity is mainly in the estimation of design discharge for flood control structures [5, 6]. Generation of intensity duration frequency curves for flood prediction especially within the West Coast has become imperative owing to the recent devastations caused by flood in various regions of Nigeria; perhaps being due to the lack of rainfall data and the subsequent design of most drainage structures without appropriate rainfall intensity values [7]. Rainfall Intensity-Duration-Frequency (IDF) relationship is one among the numerous tools use in hydrology and water resources engineering since it can provide concise information between the maximum intensity of rain that falls within a given period of time. IDF curves are used in combination with runoff estimation formulas such as the rational method in order to predict the peak runoff flow from exact point of basin.

Attempt has been made by several researchers to develop intensity duration frequency curves in different parts of the world. [8] Proposed a generalized IDF formula using the one hour, 10 years' rainfall depths as an index. [9] Further developed a generalized IDF formula for any location in the United States using three base rainfall depths: one hour 10 years, twenty four hours 10 years and one hour hundred years ($P_1^{10}, \mathrm{P}_{24}^{10}, \mathrm{P}_1^{100}$) which describe the geographical variation of rainfall. [7] Developed intensity duration frequency curves for Calabar Metropolis South-South, Nigeria.

In this study, an attempt was made to develop intensity duration frequency curves for some selected states within the West Coast of Nigeria, namely; Warri, Akure, Benin City and Ekiti and also generate models that can be employed to predict the intensity of rainfall for ungauged sites within the study locations.

2. Materials and Methods

2.1 Description of Study Area

Nigeria is located in West Africa between latitude 4^0 N and 14^0 N and between longitudes 2^0 E and 15^0 E. It has a total area of 925,796 km^2. The ecological zones of the country are broadly grouped into three, which are; Sahel, Savannah and the Guinea zones. Nigeria is affected by the Tropical Continental and Tropical Maritime air masses. The Tropical Continental is responsible for the dry season while the Tropical Maritime is responsible for the rainy season. The intervening periods of transition from the real onset and cessation of rain falls between February and April and between September and November respectively. Also, a depression is indicated in the rainfall amount during the month of August and this has been named “the little dry season” or “August break” or “midsummer” [10].

The South-West region presented in Figure 1 falls within the tropics. The vegetation is predominantly ever green High Rain Forest with Guinea Savannah to the north. Mineral and forest resources are plentiful. The proportion of the Muslims and the Christians are about the same;

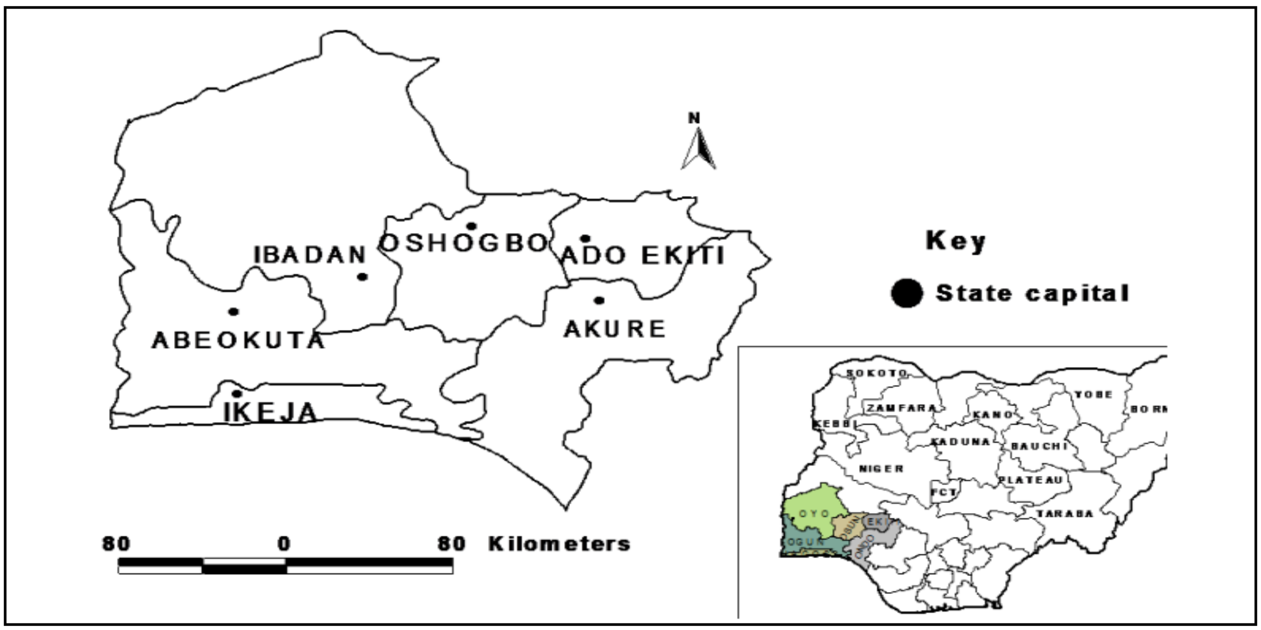

Figure 1: Map of South-West Nigeria

2.2 Data Collection

The data used for this study is annual daily rainfall data obtained from Nigeria Meteorological Agency (NIMET) Abuja. This agency is saddled with the sole responsibility of measuring, analyzing and storing meteorological data and forecasting the weather in Nigeria. Daily rainfall data were obtained and analyzed with the aid of Microsoft Excel Program in order to determine the annual maximum daily rainfall series (AMDS). To determine the rainfall intensity (I), the popular Gumbel probability distribution model was employed.

2.3 Preliminary Analysis of the Data

2.3.1 Test of Homogeneity

Owing to the method of data collection coupled with the conditions around the observation site and reliability of the measurement procedures; non-homogeneous observation may appear in the data series and can challenge the quality of the data and the overall outcome of the results since frequency analysis of data requires that the data be homogeneous and independent. Homogeneity test was carried out to establish the fact that the data used are from the same population distribution. The test is based on the cumulative deviation from the mean as expressed using the mathematical equation proposed by [11] as follows;

$$S_k = \sum_{i=1}^{k}\left(X_i - \bar{X}\right) \quad k = 1, \text{----------}\ n \tag{1}$$

Where;

X_i = The record for the series X_1 X_2 -----------X_n

$\bar{X}$ = The mean

S_{ks} = the residual mass curve

For a homogeneous record, it is expected that the data points fluctuate around the zero line as defined by the residual mass curve. To perform the homogeneity test, a software package (Rainbow) for analyzing hydrological data was employed.

To further confirm that the rainfall data are statistically homogeneous, test of hypothesis was done as follows:

H0: Data are statistically homogeneous

H1: Data are not homogeneous

The null and alternate hypothesis were tested at 90%, 95% and 99% confidence interval that is 0.1, 0.05 and 0.01 degree of freedom

2.3.2 IDF Theory

The magnitude of a hydrological event X_T may be represented as the mean $\bar{X}$ plus the departure DX_T of the variate from the mean as presented in the works of [12]

$$X_T = \bar{X} + DX_T \tag{2}$$

The departure is related to the frequency factor by;

$$DX_T = K_T s \tag{3}$$

Where; K_T is the frequency factor and σ is the standard deviation of rainfall intensities. The departure DX_T and the frequency factor K_T are functions of the return period and the type of probability distribution that predefined the data used. Combining equation 2.1 and 2.2 gives

$$X_T = \bar{X} + K_T\sigma \tag{4}$$

Which is approximated by;

$$P_T = P_{ave} + K_T\sigma \tag{5}$$

Where P_T is the desired rainfall peak value for a specific frequency, P_{ave} is the average of maximum rainfall intensity. The rainfall intensity I(mm/hr) of a specified return period T is given as;

$$I = \frac{P_T}{T_d} \quad \text{where } T_d \text{ is duration (hrs)} \tag{6}$$

2.3.3 Generation of IDF Curve

Rainfall intensity is defined as the ratio of the total amount of rain (rainfall depth) falling during a given period to the duration of the period It is expressed in depth units per unit time, usually as mm per hour (mm/hr). The step by step procedure for generating the IDF curves is presented as follows;

i. Forty years (40) daily rainfall data for the period 1974 to 2013 were collected and the annual maximum daily series (AMDS) were determined using Microsoft excel program and thereafter arranged in ascending order of magnitude.

ii. The rainfall depth for selected rainfall durations were estimated using the empirical reduction formula given by Indian Meteorological Department (IMD) reported in the works of Rashid et al. (2018) as follows; $p_t = P_{24}\left(\frac{t}{24}\right)^{\frac{1}{3}}$ where; P_t is the required rainfall depth (mm) at t-hr duration, P_{24} is the daily rainfall depth (mm) and t is the duration of rainfall for which the rainfall depth is required in (hr)

iii. Rainfall intensities were determined for the calculated rainfall depths at selected durations using; $I = \frac{R}{T}$ Where; I; is the rainfall intensity (mm/hr), R; is the amount of rainfall (mm) and T; is the duration of the rainfall (hrs).

iv. The computed rainfall intensities were then ranked in descending order of magnitude with the highest intensity taking the value of 1 in the rank. The return periods or recurrence intervals (T) were calculated using the Weibull's formula given as; $T = \frac{n+1}{m}$ where; T is the recurrence interval (yrs), n is the highest rank and m is the ranked value of each rainfall intensity.

v. The Exceedence Probability (EP) was determined using the linear relationship between probability and recurrence interval given as; $P = \frac{1}{T}$ where; P is the probability and T is the recurrence interval

vi. To determine the mathematical relationship between rainfall intensity and rainfall durations, the curve fitting tool in MATLAB was employed. The input and output data were first converted to the required format (.DAT file) while the "cftool" function was employed to activate the curve fitting platform where the modelling was done. Selected goodness of fit statistics, namely; coefficient of determination (R^2), adjusted R^2 value (Adj. R^2) and Root Mean Square Error (RMSE) were employed to monitor the adequacy of the modelling process and the model with the highest R^2, (Adj. R^2) and lowest RMSE was selected as the best fit model

vii. Using the best fit model, rainfall intensities for 2minutes, 5 minutes, 10 minutes, 15 minutes, 30 minutes, 60 minutes, 120 minutes, 180 minutes, 240 minute and 320 minutes were estimated coupled with the mean and standard deviation of the data for different durations.

viii. The rainfall frequency factor (K_T) for selected return periods (T= 2, 5, 10, 25, 50 and 100yrs) were computed using the quantile equation of the popular Gumbel probability distribution model presented as follows;

$K_t = -\frac{\sqrt{6}}{\pi}\left\langle 0.5772 + \ln\left[\ln\left(\frac{T}{T-1}\right)\right]\right\rangle$ Where T; is the selected return periods

ix. The rainfall intensity corresponding to a specified return period was computed using;

$X_T = \bar{X} + K_T\sigma$ where; X_T; is the rainfall intensity corresponding to a specified return period T (yrs), $\bar{X}$; is the mean rainfall intensities and σ; is the standard deviation of rainfall intensities. Finally, the IDF curves were generated from the plot of rainfall intensities against duration for corresponding return period.

3. Results and Discussion

Descriptive statistics of annual maximum rainfall data employed for this study is presented in Table 1

Table 1: Descriptive Statistics of annual maximum daily series

S/n	Descriptive Statistics	Akure	Ekiti
1	Mean	84.0125	82.5725
2	Variance	560.56625	1117.625122
3	Standard Deviation	23.67628032	33.4309007
4	Skewness	1.367428534	3.250393214
5	Kurtosis	1.442170598	14.42121624
6	Median	77.8	74.35
7	Maximum	150.1	246.3
8	Minimum	58.6	49.5

Result of homogeneity test conducted on the annual maximum daily rainfall series for the stations employed in the study are presented in Figures 2 and 3 respectively

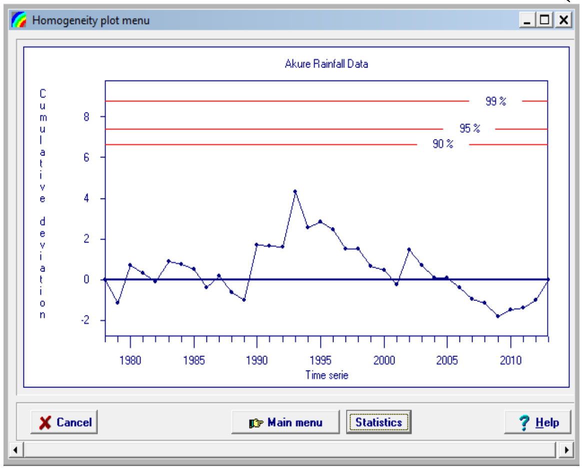

Figure 2: Homogeneity test of Akure Data

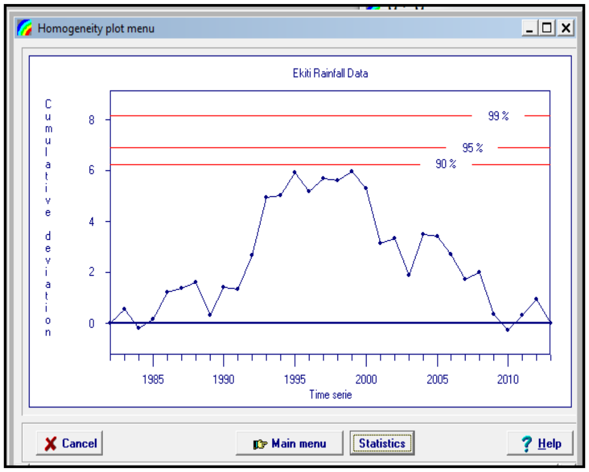

Figure 3: Homogeneity test of Ekiti Data

For homogeneous records, it is expected that the data points fluctuate around the center line of the residual mass curve. Results of Figures 2 and 3 revealed that the data used are statistically homogeneous since the data points were observed to fluctuate around the center line of the residual mass curve. To further confirmed the homogeneous nature of the data, homogeneity statistics was done to test the strength of the null hypothesis over the alternate hypothesis. Based on the computed statistics, the null hypothesis (H0) was accepted, and it was concluded that the rainfall data used are statistically homogeneous at 90%, 95% and 99% confidence level.

Using the homogeneous rainfall data, the rainfall depth for selected rainfall durations were estimated using the empirical reduction formula given by Indian Meteorological Department (IMD). Thereafter, the corresponding rainfall intensities at the selected durations were computed. The mathematical relationship between the calculated rainfall intensities (I) and the selected durations (d) was determined using the curve fitting tool in MATLAB and presented in Table 2

Table 2: Relationship between intensity and duration

Akure	$I = \frac{\left(12.59d^2 + 22.91d - 0.07236\right)}{\left(d^2 + 0.1096d - 0.001176\right)}$ (7)	$R^2 = 0.993$
		Adj. $R^2 = 0.9922$
		RMSE = 7.421
Ekiti	$I = \frac{\left(-0.2327d^3 + 14.57d^2 + 19.99d - 0.2706\right)}{\left(d^2 + 0.07827d - 0.001386\right)}$ (8)	$R^2 = 0.9989$
		Adj. $R^2 = 0.9987$
		RMSE = 2.046

Using the mathematical relationship presented in Table 2, the predicted intensities at the selected durations were computed and the values were regressed against the observed rainfall intensities in order to develop a regression equation between the observed and predicted rainfall intensity. Results of regression model is presented in Figures 4 and 5 respectively

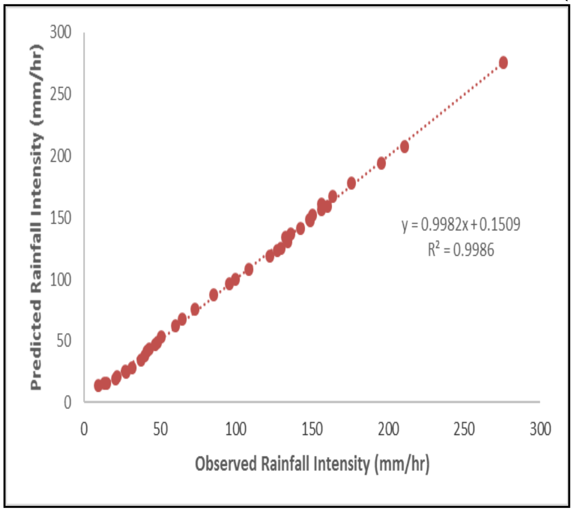

Figure 4: Observed versus predicted intensity for Akure

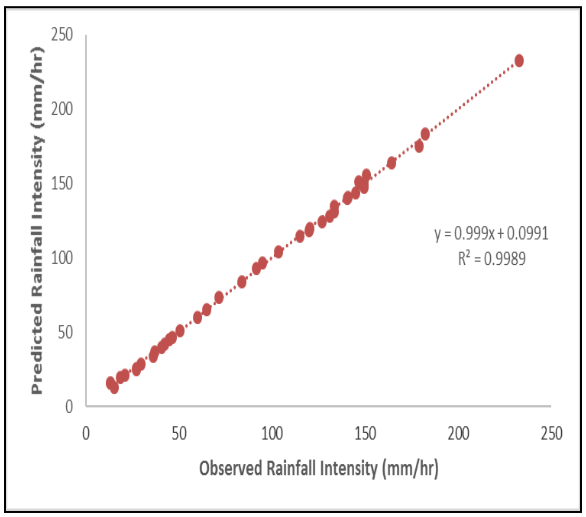

Figure 5: Observed versus predicted intensity for Ekiti

With a coefficient of determination R^2 value 0.9986 and 0.9989, it was concluded that; results of Table 2 are adequate. Using the selected models, rainfall intensities for 2minutes, 5 minutes, 10 minutes, 15 minutes, 30 minutes, 60 minutes, 120 minutes, 180 minutes, 240 minute and 320 minutes were estimated coupled with the mean and standard deviation. Using the calculated rainfall intensities and employing the procedures based on the popular Gumbel probability distribution model outlined for the calculation of rainfall intensity, the rainfall intensities for the various stations at the selected return periods were calculated.

Employing the computed rainfall intensity, intensity duration frequency (IDF) curves were generated for each station based on 2, 5, 10, 25, 50 and 100yrs return period and results are presented in Figures; 6 and 7 respectively

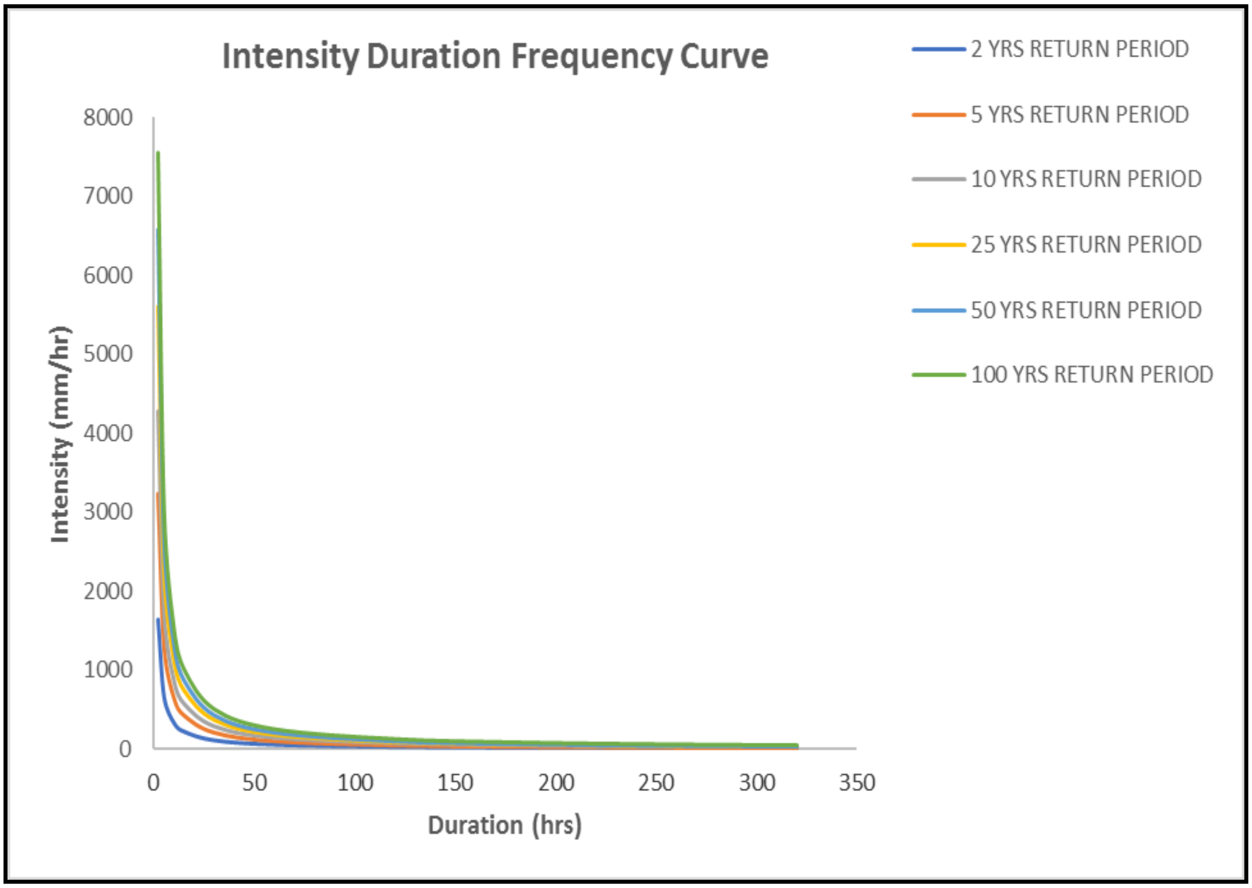

Figure 6: IDF curve for Akure based on Gumbel distribution

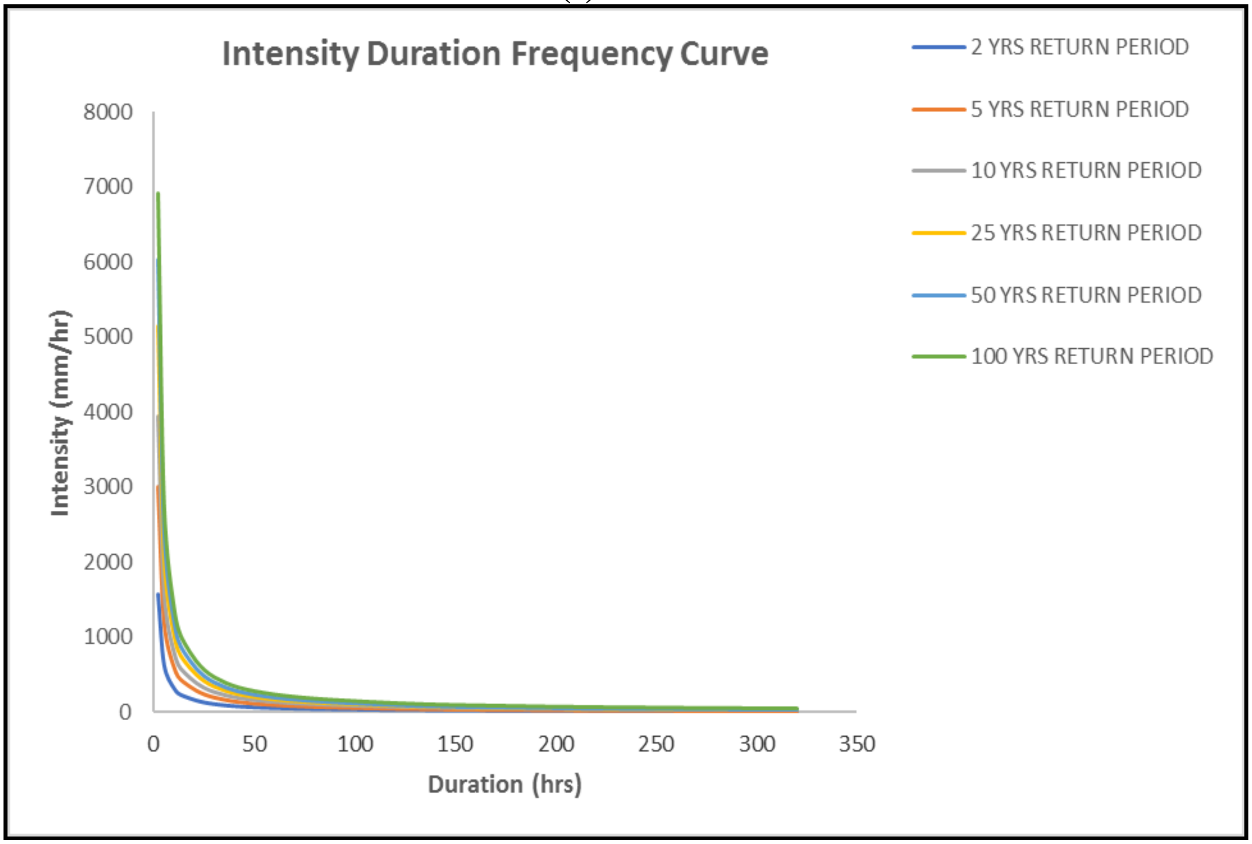

Figure 7: IDF curve for Ekiti based on Gumbel distribution

Results of Figures 6 and 7 indicate that rainfall intensity decreased with increase in duration for a given return period and increased with return period for a given duration of rainfall. Similar results were also reported by [7].

3.2. Fitting of Empirical IDF Equations

Empirical IDF equations represent the relationship between maximum rainfall intensity (as dependent variable) and other parameters of interest such as rainfall duration and frequency (as independent variables). There are several commonly used equations present in the literature of hydrology applications used for the analysis of rainfall intensity. For this study, four basic forms of equations used to describe the rainfall intensity duration relationship presented in Table 3 were employed

Table 3: Definition of selected empirical rainfall intensity equations

S/N	Empirical Equation	Reference
1	$i = \frac{a}{d + b}$ (9)	Talbot Equation [14]
2	$i = \frac{a}{d^e}$ (10)	Bernard Equation [13]
3	$i = \frac{a}{d^e + b}$ (11)	Kimijima Equation [14]
4	$i = \frac{a}{(d + b)^e}$ (12)	Sherman Equation [14]

Where;
i; is the rainfall intensity (mm/hr); d; is the duration (hr);
a, b and e; are the constant parameters related to the empirical equation.

These empirical equations show that rainfall intensity decreases with rainfall duration for a given return period. To estimate the parameters of the equations, non-linear regression technique using Microsoft excel solver was employed. The non-linear regression solver was applied to estimate the parameters of the four empirical IDF equations that are used to represent intensity-duration relationships. The value of the constants corresponding to a minimum root mean square error (RMSE) between the computed rainfall intensity and the corresponding return period was selected as the exact value. To select the model that best fit each location, the sum of minimized error was employed and the model with the least sum of minimized error was selected as the best fit model. Based on the results of the estimated parameters, it was observed that; Talbot model had the least sum of minimized error and was employed to estimate the rainfall intensities at the selected stations. The computed parameters of the Talbot model for the selected stations is presented in Tables 4 and 5 respectively

Table 4: Estimated parameters of Talbot Equation (Akure)

Return Periods T (Years)	**A**	**b**	**Minimized RMSE (OF[a])**	**Sum of Minimized Error (SME)**
2	54.855	0.00008	5.00522E-08	5.666066E-07
5	107.657	0.00013	4.95653E-08	
10	142.485	0.00004	6.17467E-08	
25	187.045	0.00011	5.66533E-08	
50	219.812	0.00012	3.18804E-07	
100	251.675	0.00040	2.97851E-08	

Table 5: Estimated parameters of Talbot Equation (Ekiti)

Return Periods T (Years)	A	b	Minimized RMSE (OF[a])	Sum of Minimized Error (SME)
2	52.643	0.00028	3.36565E-08	6.424229E-07
5	102.394	0.00076	2.65028E-07	
10	131.821	0.00012	4.15756E-08	
25	193.185	0.00424	1.76790E-07	
50	204.797	0.00065	1.63839E-10	
100	241.261	0.00160	1.25209E-07	

Using the best-fit model, the following equations presented in Table 6 were generated in order to predict the actual rainfall intensity in Akure, and Ekiti.

Table 6: Best fit model for predicting rainfall intensity within South- West Nigeria

Return Period (T)	Model Equation (Akure)	Model Equation (Ekiti)
2	$i = \frac{54.855}{d + 0.00008}$ (13)	$i = \frac{52.643}{d + 0.00028}$ (19)
5	$i = \frac{107.657}{d + 0.00013}$ (14)	$i = \frac{102.394}{d + 0.00076}$ (20)
10	$i = \frac{142.485}{d + 0.00004}$ (15)	$i = \frac{131.821}{d + 0.00012}$ (21)
25	$i = \frac{187.045}{d + 0.00011}$ (16)	$i = \frac{193.185}{d + 0.00424}$ (22)
50	$i = \frac{219.812}{d + 0.00012}$ (17)	$i = \frac{204.797}{d + 0.00065}$ (23)
100	$i = \frac{251.675}{d + 0.0004}$ (18)	$i = \frac{241.261}{d + 0.00160}$ (24)

Where; d is rainfall duration (hr). Using the equations in Table 4.6, the rainfall intensity for Akure and Ekiti were predicted and results obtained are presented in Figures 8 and 9 respectively.

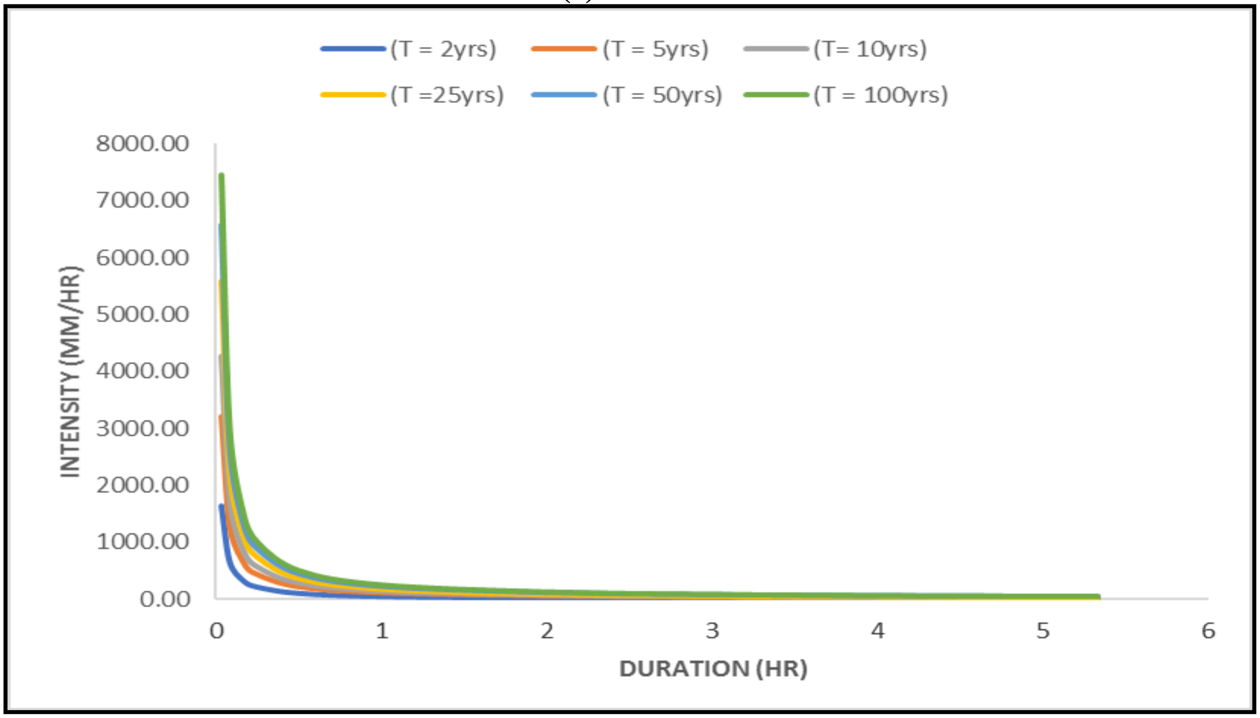

Figure 8: Predicted rainfall intensity for Akure based on Talbot Model

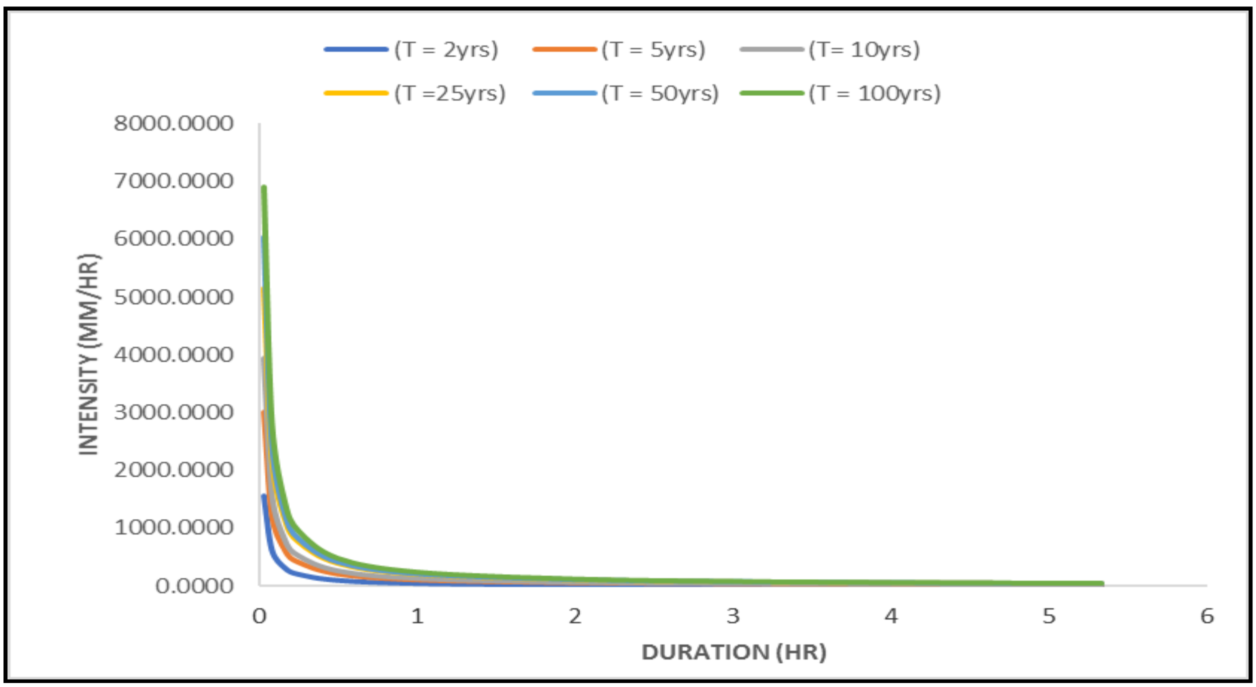

Figure 9: Predicted rainfall intensity for Ekiti based on Talbot Model

4. Conclusion

This study has been conducted to derive IDF curves/models for some selected states within the South-West of Nigeria. Results of the predicted rainfall intensities based on Talbot model showed a strong correlation with the rainfall intensity computed from the rainfall data using the popular Gumbel distribution model. Intensity-duration-frequency data are needed by hydrologists and engineers involved in planning and design of water resources projects. The achievement of sets of empirical equations generated for rainfall intensity prediction represents an important contribution of this study thus making it possible for a more rational work in several subjects related to hydrology.

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