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A Proposed $Z^{1/3}$ – Dependence Formula for Nuclear Potential Radii with Constant Parameter for both Heavy and Light Nuclei

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Abstract

The size of the nucleus is the fundamental static properties of nucleus which has been studied to acquire some information on the complexity of nuclear many-body problems. This study built a picture of atomic nucleus as a finite-sized central object with positive charge +Ze, equals in magnitude with negative charge (-e) of orbiting leptons. From this picture a simple relation, that could be applied to calculate the size of the fields originated from the positively charged protons, is deduced. This simple relation, $R_Z = r_Z Z^{1/3}$, which is a function of proton charge Z, successfully provides a constant radius parameter for both light and heavy nuclei. The relation could also be applied to search information on the size of the unknown atomic nuclei. For this reason, the A – dependence formula, $R = r_0 A^{1/3}$, for measuring the size of atomic nuclei could be replaced by the Z – dependence formula. This support the idea that the charge radii for the atomic nuclei are more directly related to its charge number, R_Z, rather than its mass number, R. Therefore, this work offers an easy way to measure or predict the nuclear charge radius for both light and heavy nuclei from the assumption of nuclear finite sized model. This development in nuclear size measurement could improve simplifying the complexity in nuclear structure and dynamic.

1. Introduction

The size of the nucleus, often characterized by charge radius, is the fundamental static properties of nucleus as its spin, binding energy, parity, mass, effective interaction and so on [1-3]. Nowadays, nuclear size data constitute one of the most precise and extensive arrays of experimental information available for the interpretation of nuclear characteristics. Based on charge distributions, the nuclear size has been obtained by combined analysis of two types of experimental data: (*i*) fast electron scattering and muonic atom X-rays and (*ii*) K_α X-ray isotope shift and optical isotope shift. These experimental data measured different properties of the nuclear ground-state charge distributions. Therefore, accurate knowledge of the nuclear radii was obtained from the combination of data from different experimental methods [4,5]. The nuclei near the β-stability line have been measured by the electron scattering experiments, the scattering of ions and particles from nuclei and the muon X-ray transition technique and the corresponding charge radii have been accurately obtained. Results from these measurements indicated that β-stable nuclei behave like a solid sphere with constant density. Moreover, short lived isotopes cannot be used as target at rest due to their unstable property. Instead, direct reaction with radioactive isotope beam can be done in inverse kinematics, where the role of beam and target are interchanged. The recent development of radioactive isotope beam, the modern techniques for optical isotope shift measurement and fast developments especially in ultra-high-sensitivity laser spectroscopy techniques have made it possible to reach unstable isotopes far from the β-stability line. From these techniques, the charge radii for unstable nuclei mainly based on the interaction and reaction cross sections have been determined. This revealed information on the complexity of nuclear many-body problems such as neutron skin effect, halo (the proton-rich or the neutron-rich) nuclear and the nuclear shape variances near the neutron and proton drip-lines [5-9]. It is also an important observable in the evolution of charge radius the deformation of ^{203}Fr through a comparison between the measured charge radii and the theoretical predictions. Recent measurement of the nuclear charge radius for the two-neutron halo ^{6}He indicates that its halo is a di-neutron "orbiting" the ^{4}He core. Halo nuclei with diffuse outer neutron distributions have been known to exist at the limits of β-stability lines for many of the lighter elements [10,11]. Halo nuclei present a huge challenge to experimenters and theorists struggling to understand their many strange properties. The charge radii of borromean (halo) nuclei are determined from the electron-ion scattering experiment by folding of a three-body density functional. The charge radii of light exotic nuclei $^{6,8}He$, ^{11}Li, ^{14}Be, $^{17,19}B$ have been predicted theoretically from the calculations of charge form factors. The charge form factors of both light and heavier exotic nuclei have been calculated within the plane wave Born approximation.

The discrepancy between the experimental and theoretical predictions of nuclear charge radii made from microscopic and macroscopic models are refined through Bayesian training of neural networks. Recently, experimental data have been employed in the artificial neural networks and a new simple formula has been estimated by least-square fitting and describes the *rms* nuclear charge radii well [12,13]. Theoretically both microscopic and macroscopic theories are important tools for calculating the nuclear charge radius. The relativistic continuum Hartree–Bogoliubov theory which is an extended version of relativistic mean field theory gives much better results for both spherical and deformed nuclei near the drip-lines. The Hartree–Fock method with an effective nucleon-nucleon interaction with Skyrme forces is successfully used to calculate the charge radii of β-isotopes nuclei far away from closed-shell configurations. Besides, recent work attempts to deduce charge radii based on the α decay cluster and proton emission data [5,7,9].

Nuclear charge radius can represent the overall characteristics of the entire nucleus as its knowledge plays a key role in testing theoretical models of nuclei and in describing the internal and external structure such as halo, neutron skin effect, shape variances potential, nucleon density, single particle orbitals, wavefunctions, and so on. Therefore, any developments in its measurement techniques can improve the understanding of the complex dynamic of atomic nucleus [14-17]. When nucleus is treated as a solid sphere with constant density $\rho_0 \simeq 0.15\ fm^{-3}$ and nucleon number A, then its size can be determined from the relation: $R = r_0 A^{1/3}$, where r_0 is the range of the nuclear force [18,19]. In turn, the root-mean-square charge radius of such a uniform distribution is given by $R_{rms} = \langle r^2 \rangle^{1/2} = \sqrt{3/5}R$. However, the conventional A – dependent formula is not globally valid for all nuclei as the ratio $R/A^{1/3}$ is far from being constant especially when there is a significant difference between proton and neutron numbers [20,21]. For example, $R/A^{1/3}$ = 1.312 fm for ^{40}Ca and $R/A^{1/3}$ = 1.234 for ^{48}Ca while $R/A^{1/3}$ = 1.217 for ^{190}Pb and $R/A^{1/3}$ = 1.201 fm for ^{214}Pb [22-26]. However, various nuclear bulk properties, such as the nuclear binding energy, the Wigner's isobaric mass multiplet equation, the separate neutron shells and proton shells in a nucleus, and the shell model harmonic oscillator potential strength $\hbar\omega_p$ and $\hbar\omega_n$, etc, are all isospin-dependent. Thus, it is not surprised that such a simple isospin-independent A-dependence formula is unable to satisfactorily describe the global variation in nuclear charge with mass number. Another evidence of the violation of A-dependence formula is found in the measurements of isotope shift (associated with an addition of two neutrons) within an isotopic chain are often found to be significantly smaller than that predicted by the A-dependence formula. In contrast, there is also evidence that the observed charge radius difference between neighboring isotones, (associated with the addition of two protons) is usually much larger than that predicted by the A-dependence formula.

The variation in the ratio $R/A^{1/3}$ from both theoretical and experimental data make the measurement of nuclear charge radius a great challenge to nuclear physicist. In view of these reasons, $Z^{1/3}$ dependence formula has been introduced in order to obtain constant nuclear parameter, r. The charge radii for the most atomic nuclei may be more directly related to its charge number, $R_Z = r_Z Z^{1/3}$, rather than its mass number, $R = r_0 A^{1/3}$ and in contrast to the $A^{1/3}$ law, the radius parameter of $Z^{1/3}$-law, r_Z, keeps almost a constant value of 1.63 *fm* for various atomic nuclei. This modification also reflects on the Coulomb energy term in the semi-empirical nuclear mass formula [6,8]. Therefore, this work adopt the $Z^{1/3}$-dependence formula for nuclear charge radii and determine from the finite-size nuclear potential, the new potential charge radii in order to obtained constant value of the parameter r_Z, for both light and heavy nuclei.

2. Methodology

For a nucleus with spherically symmetric charge distribution $\rho(r) = 3Ze/4\pi R^3$, the effective interaction can best be described by the lepton-nuclear potential energy $U(r)$, where within a nuclear radius $r \leq R$, the expression is described by

$$U(r,R) = -Z^{2/3}\gamma(3-\alpha^2) \quad (1)$$

where $\gamma = ke^2/2r_Z$ and $\alpha = r/R_Z$. Outside the nucleus, $r > R$, this expression reduces to Z/r potential

$$U(r) = -\frac{Zke^2}{r} \quad (2)$$

The potential (1) has a constant value of $U(R) = -Z^{2/3}\gamma$ inside the nucleus. Figure 1 shows the nature of finite-size of atomic nucleus of charge $+Ze$ and electron with charge $-e$, orbiting at distance r from the nucleus

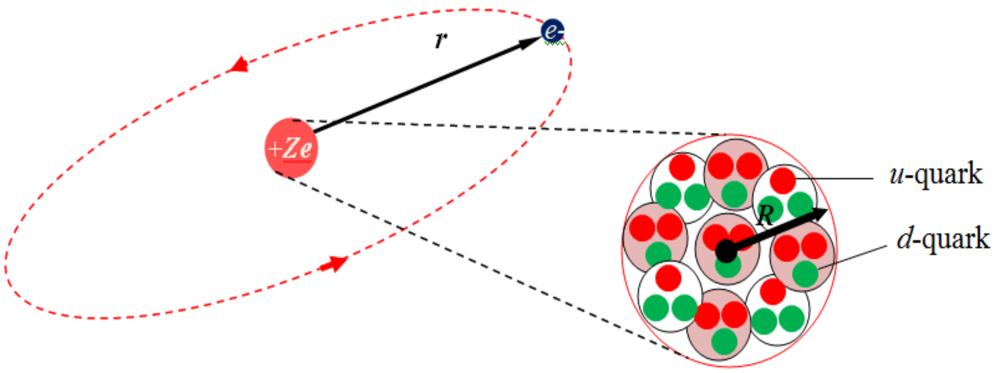

Figure 1: The finite sized atomic nucleus of charge $+Ze$ orbited by an electron with charge $-e$ at distance r from the nucleus

3. Results and Discussion

The finite – size nuclear potential energy $U(R)$ is computed for various nuclei by using equations (1) and the results are presented in Table 1.

Table 1: The values of finite-size potential energies of various atomic nuclei

r/R (fm)	The Finite-Size Potential (MeV)							
	$^{1}H_{1}$	$^{7}Li_{3}$	$^{23}Na_{11}$	$^{39}K_{19}$	$^{63}Cu_{29}$	$^{85}Rb_{37}$	$^{107}Ag_{47}$	$^{133}Cs_{55}$
0.0	-3.0000	-6.2403	-14.8383	-21.3611	-28.3174	-33.3111	-39.0709	-43.3874
0.1	-2.9900	-6.2195	-14.7888	-21.2899	-28.2230	-33.2001	-38.9407	-43.2427
0.2	-2.9600	-6.1570	-14.6404	-21.0763	-27.9398	-32.8670	-38.5499	-42.8089
0.3	-2.9100	-6.0530	-14.3931	-20.7203	-27.4679	-32.3118	-37.8988	-42.0857
0.4	-2.8400	-5.9074	-14.0469	-20.2218	-26.8071	-31.5345	-36.9871	-41.0734
0.5	-2.7500	-5.7202	-13.6017	-19.5810	-25.9576	-30.5352	-35.8150	-39.7717
0.6	-2.6400	-5.4914	-13.0577	-18.7978	-24.9193	-29.3138	-34.3824	-38.1809
0.7	-2.5100	-5.2210	-12.4147	-17.8721	-23.6922	-27.8703	-32.6893	-36.3007
0.8	-2.3600	-4.9090	-11.6728	-16.8041	-22.2763	-26.2047	-30.7358	-34.1314
0.9	-2.1900	-4.5554	-10.8319	-15.5936	-20.6717	-24.3171	-28.5217	-31.6728
1.0	-2.0000	-4.1602	-9.8922	-14.2407	-18.8783	-22.2074	-26.0473	-28.9249
1.1	-1.7900	-3.7234	-8.8535	-12.7455	-16.8960	-19.8756	-23.3123	-25.8878
1.2	-1.5600	-3.2449	-7.7159	-11.1078	-14.7250	-17.3218	-20.3169	-22.5614
1.3	-1.3100	-2.7249	-6.4794	-9.3277	-12.3653	-14.5458	-17.0610	-18.9458
1.4	-1.0400	-2.1633	-5.1439	-7.4052	-9.8167	-11.5478	-13.5446	-15.0409
1.5	-0.7500	-1.5601	-3.7096	-5.3403	-7.0793	-8.3278	-9.7677	-10.8468
1.6	-0.4400	-0.9152	-2.1763	-3.1330	-4.1532	-4.8856	-5.7304	-6.3635
1.7	-0.1100	-0.2288	-0.5441	-0.7832	-1.0383	-1.2214	-1.4326	-1.5909
1.8	0.2400	0.4992	1.1871	1.7089	2.2654	2.6649	3.1257	3.4710
1.9	0.6100	1.2689	3.0171	4.3434	5.7579	6.7733	7.9444	8.8221
2.0	1.0000	2.0801	4.9461	7.1204	9.4391	11.1037	13.0236	14.4625
2.1	1.4100	2.9329	6.9740	10.0397	13.3092	15.6562	18.3633	20.3921
2.2	1.8400	3.8274	9.1008	13.1015	17.3680	20.4308	23.9635	26.6109
2.3	2.2900	4.7634	11.3265	16.3056	21.6156	25.4275	29.8241	33.1190
2.4	2.7600	5.7410	13.6512	19.6522	26.0520	30.6462	35.9452	39.9164
2.5	3.2500	6.7603	16.0748	23.1412	30.6772	36.0870	42.3268	47.0030
2.6	3.7600	7.8211	18.5973	26.7726	35.4911	41.7499	48.9688	54.3788
2.7	4.2900	8.9236	21.2187	30.5464	40.4939	47.6349	55.8714	62.0439
2.8	4.8400	10.0676	23.9391	34.4626	45.6854	53.7419	63.0344	69.9983
2.9	5.4100	11.2533	26.7583	38.5212	51.0657	60.0710	70.4578	78.2419
3.0	6.0000	12.4805	29.6765	42.7222	56.6348	66.6222	78.1418	86.7747
3.1	6.6100	13.7494	32.6936	47.0656	62.3927	73.3955	86.0862	95.5968
3.2	7.2400	15.0598	35.8097	51.5515	68.3393	80.3908	94.2911	104.7081
3.3	7.8900	16.4119	39.0246	56.1797	74.4747	87.6082	102.7564	114.1087

Figure 2 gives the information on potential radii of selected nuclei as determined by finite – size potential model (1). In Figure 2 the negative region represents the range over which the potential extend and the values on the positive region have something to do with the charge of an electron which is equal in magnitude with that of nucleus. The intercepts on r axis represent the nuclear potential radius parameter,

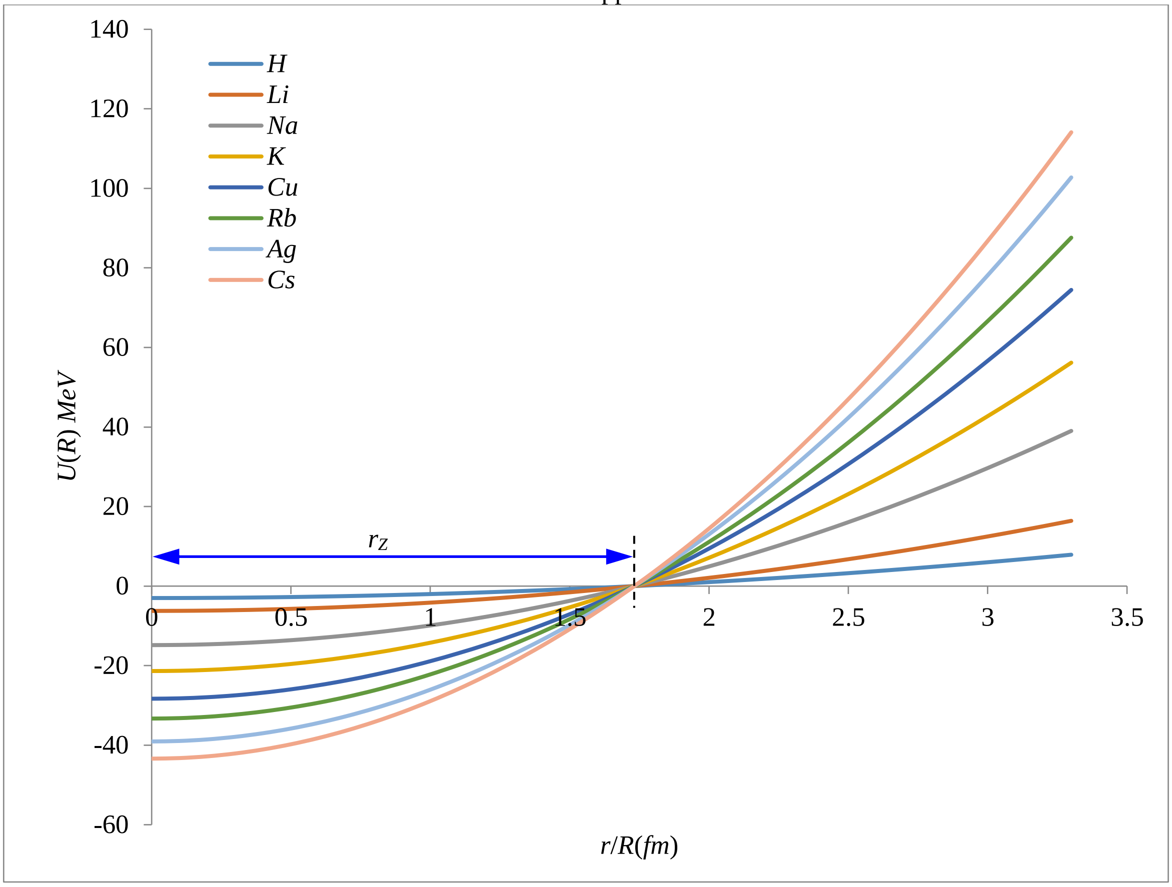

Figure 2: The plot of finite-sized potential energy of various atomic nuclei as a function of distance r from the nucleus

$$r_Z = 1.732\ fm = \sqrt{3}\ fm \tag{3}$$

Thus, the nuclear potential radius, r_Z estimated by this method has the constant value of $\sqrt{3}$ inside a region $r = 0$ (Figure 1), as described by coulomb potential (2). The charge radii of atomic nuclei, independent of neutron number, can follow remarkably very simple Z – dependence relations:

$$R_Z = r_Z Z^{1/3} \tag{4}$$

It can also be deduced from Table 2 that the Z – dependence nuclear charge radius can also be related to A – dependence nuclear radius through the relation

$$R_Z = 1.4434\ R \tag{5}$$

where $R = r_0 A^{1/3}$, with $r_0 = 1.2\ fm$.

Table 2: The values of nuclear potential radii, R_Z (4) as a function of parameter, r_Z

Nuclide: $^{A}X_{Z}$	$Z^{1/3}$	R_Z (fm)	R (fm)	R_Z/R
$^{7}Li_{3}$	1.4422	2.4980	1.7306	1.4434
$^{23}Na_{11}$	2.2224	3.8493	2.6669	1.4434
$^{39}K_{19}$	2.6684	4.6218	3.2021	1.4434
$^{63}Cu_{29}$	3.0723	5.3214	3.6868	1.4434
$^{85}Rb_{37}$	3.3322	5.7715	3.9986	1.4434
$^{107}Ag_{47}$	3.6088	6.2506	4.3306	1.4434
$^{133}Cs_{55}$	3.8029	6.5868	4.5635	1.4434

The nuclear potential charge radius was measured previously from the study of the effect of nuclear finite-size on potential interaction, the results obtained described the nuclear potential radii as a function of nucleon number, A (= $Z + N$). This gives the constant potential radius parameter $r_C = \sqrt{3}r_0$, when compared with the A – dependence formula [27]. Here the value of r_0 is still not constant as it varied from light to heavy nuclei. Therefore, the present study improved this formula by relating the nuclear size directly with its proton charge, Z distribution since neutron, N carries no potential charge. This gives the constant radius parameter, $r_Z = \sqrt{3}\ fm$ for both light and heavy nuclei. Thus, when considering the size of nucleus as a measure of its potential charge distribution, one gets the constant radius parameter, r_Z. This solved the problems that have been a topic of debate in physics community. However, there is a discrepancy between the value $r_Z = \sqrt{3}\ fm$ measured from the present result and $r_Z = 1.63\ fm$ measured previously [6,8]. This is because the present study measured the nuclear potential radius which depends only on proton charge and therefore, it is greater than the nuclear charge radii. It is worth noting that the nuclear potential radii and the nuclear charge radii measured different quantity.

4. Conclusion

This study built a picture of the nucleus as an object with charge $+Ze$, equals in magnitude with the charge ($-e$) of orbiting leptons and deduced a simple relation that best describe the size nucleus as the wavefunctions of fields originated from the positively charged proton. The simple relation successfully provides a constant radius parameter for both light and heavy nuclei and could be applied to search for information on the size of any atomic nuclei. For this reason, the A – dependence radius R could be replaced by the Z – dependence radius, R_Z as the proton wavefunctions are beyond the nucleons boundary.

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