

Numerical Investigation of MHD Free-Convective Flow of Chemically Reacting Fluid in a Vertical Channel Having Variable Viscosity

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Abstract

A time-dependent fully developed magnetohydrodynamic free-convective flow of viscous, incompressible, electrically conducting and chemically reacting fluid under the influence of externally applied magnetic field in a vertical parallel-plate channel in the presence of temperature-dependent viscosity has been investigated numerically by using semi-implicit finite difference scheme. The chemical reaction is assumed to be exothermic in nature which follows an Arrhenius kinetic law. The influence of the variable viscosity parameter, Hartmann number and skin friction in case of air ($Pr = 0.71$) and water ($Pr = 7$) has been analyzed by using graphs. Numerical results show that, velocity of both air and water increases with the increase in the value of variable viscosity parameter and non-dimensional time but decreases with increasing value of Hartmann number. Also, variable viscosity parameter enhanced the skin friction while Hartmann number reduced it. In addition, larger time is required to attain steady-state condition in the case of water in comparison to the case of air.

1. Introduction

The phenomena of convective flow in a channel region occurs in many industrial processes, science and technology such as geo-thermal energy utilization, thermal insulation engineering, insulation of high temperature gas-cooled reactor vessels, thermal energy storage system, heat exchangers, chemical processing equipment and petroleum reservoirs Singh et al. [1]. The research activities in the area of convective flow in an electrically conducting viscous fluid subject to an externally applied magnetic field in a channel have found attention due to a wide range of applications in science and technology such as plasma aerodynamics, nuclear engineering control, mechanical engineering, manufacturing processes, astrophysical fluid dynamics and magnetohydrodynamic (MHD) energy stems [2-4]. Many authors studied free convective hydromagnetic flows in a channel region with different geometries and thermal conditions. Unsteady free convective hydromagnetic flows in rotating systems have been studied by [5-9]. Unsteady natural convection flow between vertical parallel plates in the presence of magnetic field can be found in [10-13]. Recently, Uwanta and Hamza [14] investigated unsteady natural convection flow of reactive hydromagnetic fluid in a moving vertical channel.

All the above-mentioned studies, assumed that the viscosity of the fluids to be constant. This approximation works well as the fluid viscosity depend weakly on temperature or the temperature difference is small compared with the average temperature of the system. As the fluid viscosity strongly depend on temperature or temperature difference is very large, the viscosity is not

negligible and will influence the fluid flow and heat transfer behavior significantly [15]. A series of investigations have been conducted recently in the area of unsteady/steady flow of incompressible and electrically conducting fluid over a stretching problem in the presence of variable viscosity and thermal conductivity by many researchers. For example, Pal and Mondal [16] analyzed numerically the effects of temperature-dependent viscosity and thermal conductivity on MHD mixed convection flow in a non-Darcy porous medium over a stretching surface. Salawu and Dada [17] investigated the radiative heat transfer on MHD flow over a continuously stretching sheet with dissipation in a porous medium in the presence of variable viscosity and thermal conductivity. The transient MHD boundary layer flow in a rotating fluid due to a stretching surface in a porous medium with a temperature-dependent viscosity in the presence of thermal radiation has been reported by Rashad [18]. The role of variable viscosity nanofluid over a radially stretching convectively heated surface have been examined by Makinde et al. [19]. Devi and Prakash [20] studied numerically the effects of temperature-dependent viscosity and thermal conductivity on MHD flow over a slandering stretching sheet. Cai et al. [21] examined the MHD convective heat transfer over a permeable stretching wedge with variable viscosity and thermal conductivity. The influence of variable viscosity over a convectively heated plate in a Darcy porous medium and non-Darcy porous medium in the presence of n th order chemical reaction have been studied by [22, 23]. Recently, Manjunatha and Giresha [24] considered the MHD flow and heat transfer of a dusty fluid over an unsteady stretching sheet in the presence of variable viscosity and thermal conductivity. The motion of reactive variable viscosity fluid found applications in production of oil and gas from geological structures, the gasification of coal, the retorting of shale oil, filtration and surface catalysis of chemical reactions, ion exchange and chromatography [25].

The present work is devoted to analyze the combined effects of MHD and temperature dependent viscosity on flow of viscous reactive fluid in a vertical channel by using semi-implicit finite difference scheme.

1.2 Governing Equations of the Model

Consider the unsteady free-convective flow of an incompressible, electrically conducting and chemically reacting fluid between two vertical parallel walls in the presence of variable viscosity under the influence of a transversely magnetic field of strength B_0 as shown in Fig.1. It is assumed that, the magnetic Reynolds number is very small so that the induced magnetic field is neglected. The fluid temperature dependent variable viscosity is of the type given by Makinde and Chinyoka [26] $\bar{\mu} = \mu_0 e^{-\gamma(T' - T_0')}$, where μ_0 is the initial fluid dynamic viscosity at the temperature T_0' . Following Jha et al. [27], the non-dimensional governing equations under the Boussinesq's approximation can be written as;

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial}{\partial y'} \left(\bar{\mu} \frac{\partial u'}{\partial y'} \right) + g \beta (T' - T_0') - \frac{\sigma B_0^2 u'}{\rho} \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{QC_0^* A}{\rho C_p} e^{\left(\frac{-E}{RT'}\right)} \quad (2)$$

The initial and boundary conditions of the present problem are

$$\left. \begin{aligned} t' \leq 0 : u' = 0, T' \rightarrow T_0', \quad 0 \leq y' \leq H \\ t' > 0 : u' = 0, T' = T_0', \quad \text{at } y' = 0 \\ u' = 0, T' = T_0', \quad \text{as } y' \rightarrow H \end{aligned} \right\} \quad (3)$$

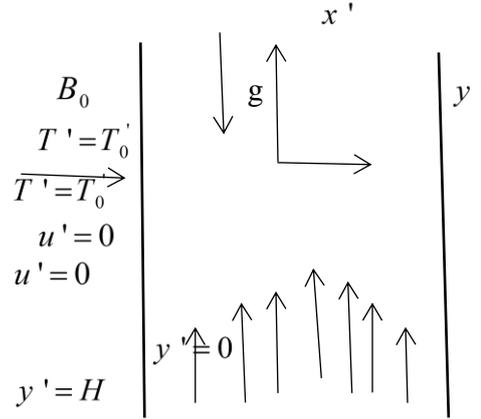


Fig.1: Geometry of the Problem

Where σ is the conductivity of the fluid, B_0 is the electromagnetic induction, β is the coefficient of thermal expansion, Q is the heat of reaction, A is the rate constant, E is the activation energy, R is the universal gas constant, ν is the kinematic viscosity, C_0^* is the initial concentration of the reactant species, g is the gravitational force, C_p is the specific heat at constant pressure, k is the thermal conductivity and ρ is the density of the fluid.

To solve Equations (1) and (3), we employ the following dimensionless variables and parameters

$$\left. \begin{aligned} y = \frac{y'}{H}, \quad t = \frac{t' \mu_0}{H^2}, \quad \lambda = \frac{QC_0^* AEH^2}{RT_0^2} e^{\left(\frac{-E}{RT_0}\right)}, \quad U = \frac{u' \mu_0 E}{g \beta H^2 RT_0^2}, \\ M^2 = \frac{\sigma B_0^2 H^2}{\nu \rho}, \quad \theta = \frac{E(T' - T_0)}{RT_0^2}, \quad \varepsilon = \frac{RT_0}{E}, \quad \text{Pr} = \frac{\mu_0 \rho C_p}{k}, \quad \alpha = \frac{bRT_0^2}{E} \end{aligned} \right\} \quad (4)$$

Using (4), the Equations (1) to (3) take the following form:

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial y} \left(e^{-\alpha \theta} \frac{\partial U}{\partial y} \right) + \theta - M^2 U \quad (5)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} + \frac{\lambda}{\text{Pr}} e^{\left(\frac{\theta}{1+\varepsilon \theta}\right)} \quad (6)$$

The initial and boundary conditions in dimensionless form are

$$\left. \begin{aligned} u = 0, \quad \theta = 0, \quad 0 \leq y \leq 1, \quad t \leq 0 \\ t > 0: u = 0, \quad \theta = 0, \quad \text{at } y = 0 \\ u = 0, \quad \theta = 0, \quad \text{as } y = 1 \end{aligned} \right\} \quad (7)$$

Where α , M , Pr , λ and ε are variable viscosity parameter, magnetic Hartmann number, Prandtl number, Frank-Kamenetskii parameter, and activation energy parameter. The other non-dimensional quantities are the skin friction (C_f), and the heat transfer rate, (Nu), are given as

$$C_f = \left. \frac{\partial U}{\partial y} \right|_{y=0,1}, \quad Nu = \left. \frac{\partial \theta}{\partial y} \right|_{y=0,1} \quad \text{where } C_f \text{ is the skin friction and } Nu \text{ is the Nusselt number.}$$

2. Methodology

2.1 Model Formulation

The governing Equations (5) and (6) with the boundary conditions (7) are solved numerically by using semi-implicit finite difference scheme given by Makinde and Chinyoka [26]. The forward difference formulas are used for all time derivatives and approximate both the second and first derivatives with second order central differences. The semi-implicit finite difference equation corresponding to Equations (5) and (6) are as follows:

$$-r_1 \mu U_{j-1}^{(N+1)} + (1 + 2r_1 \mu + M^2 \Delta t) U_j^{(N+1)} - r_1 \mu U_{j+1}^{(N+1)} = r_2 \mu U_{j-1}^{(N)} + (1 - 2r_2 \mu - M^2 \Delta t) U_j^{(N)} + r_2 \mu U_{j+1}^{(N)} - r_3 (\theta_{j+1}^N - \theta_{j-1}^N) (U_{j+1}^N - U_{j-1}^N) + \Delta t \theta_j^N \quad (8)$$

$$-r_1 \theta_{j-1}^{(N+1)} + (\text{Pr} + 2r_1) \theta_j^{(N+1)} - r_1 \theta_{j+1}^{(N+1)} = r_2 \theta_{j-1}^{(N)} + (\text{Pr} - 2r_2) \theta_j^{(N)} + r_2 \theta_{j+1}^{(N)} + \lambda \Delta t \exp\left(\frac{\theta_j^{(N)}}{1 + \varepsilon \theta_j^{(N)}}\right) \quad (9)$$

Where $r_1 = \xi \Delta t / \Delta y^2$, $r_2 = (1 - \xi) \Delta t / \Delta y^2$, $r_3 = \mu \alpha \Delta t / 4 \Delta y^2$, $0 \leq \xi \leq 1$. We chose $\xi = 1$, the detailed reasons to this particular selection was documented by Makinde and Chinyoka [26]

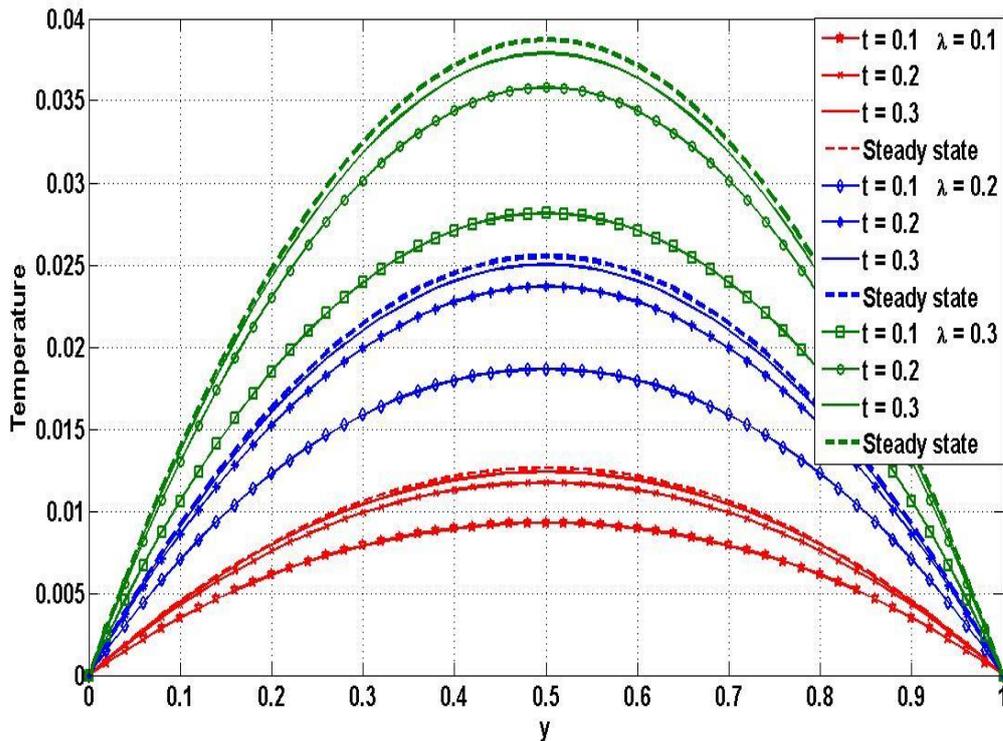


Fig.2 Variation of unsteady and steady-state temperature with λ for $M = 0$, $\alpha = 0$

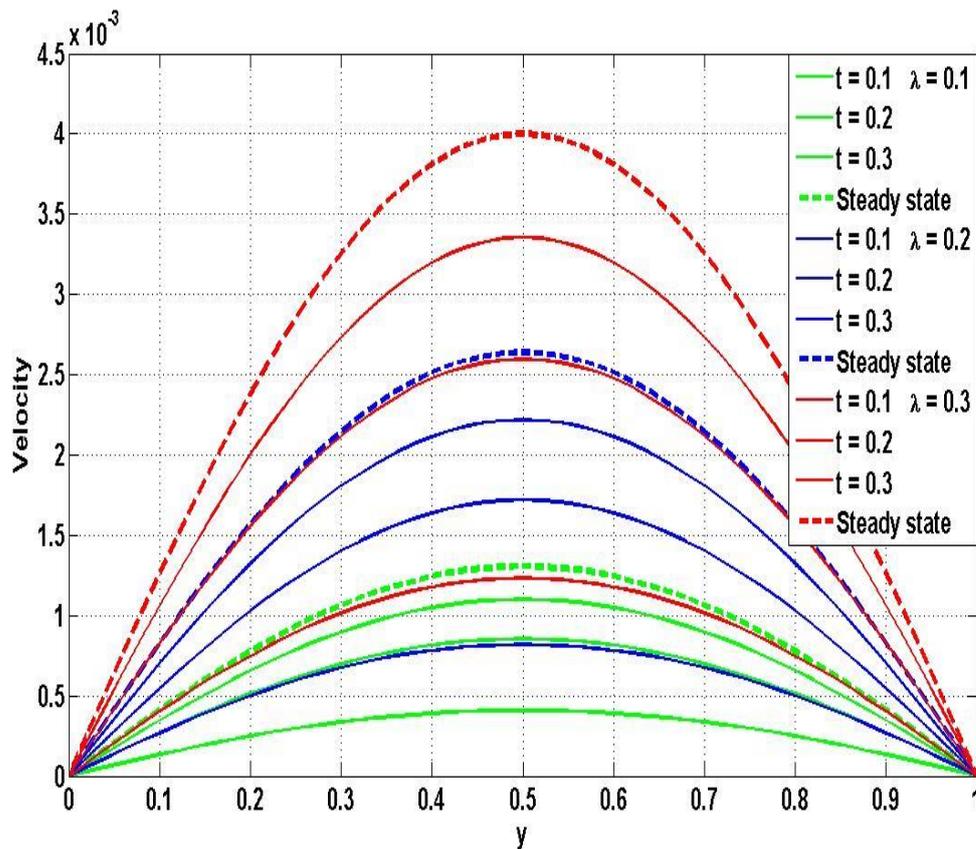


Fig.3. Variation of unsteady and steady-state velocity with λ for $M = 0$, $\alpha = 0$

3. Results and Discussion

The numerical results are obtained by solving Equations (8) and (9) using the method described in the previous section for various values of physical parameters to describe the physics of the problem. The dimensionless parameters that govern the flow are magnetic (M), variable viscosity (α), Frank-Kamenetskii (λ), non-dimensional time (t), activation energy parameters (ε) and Prandtl number (Pr). In this paper, the values of the Prandtl number (Pr) are chosen to represent air ($Pr = 0.71$) and water ($Pr = 7$). Unless otherwise stated the following parameters values are used for the computation $M = 1$, $\alpha = 0.1$, $\lambda = 0.1$, $\varepsilon = 0.01$ and $t = 0.1$. Results obtained are displayed graphically for velocity, temperature and skin friction for various flow parameters. Fig.2 and 3 represent the comparison with work of Jha et al. [27] when $Pr = 0.71$, $M = 0$ and $\alpha = 0$. It is clear that excellent agreement between the present numerical solutions and that of Jha et al. [27] exists.

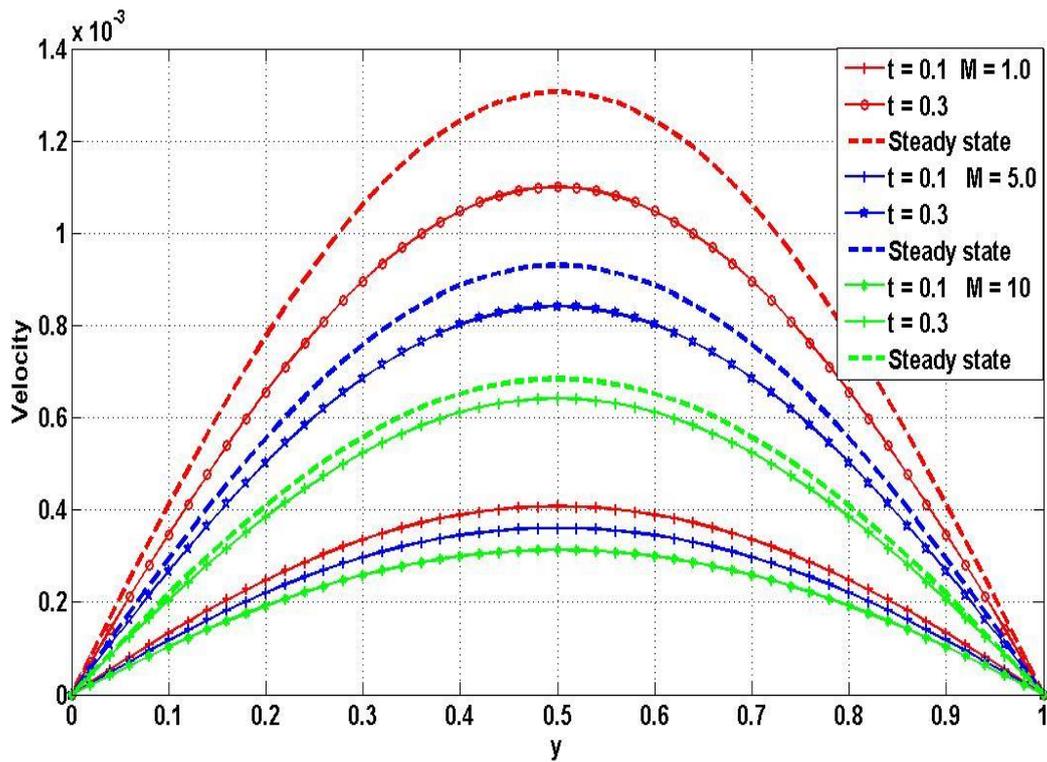


Fig.4. Variation of unsteady and steady-state velocity with M for Pr = 0.71

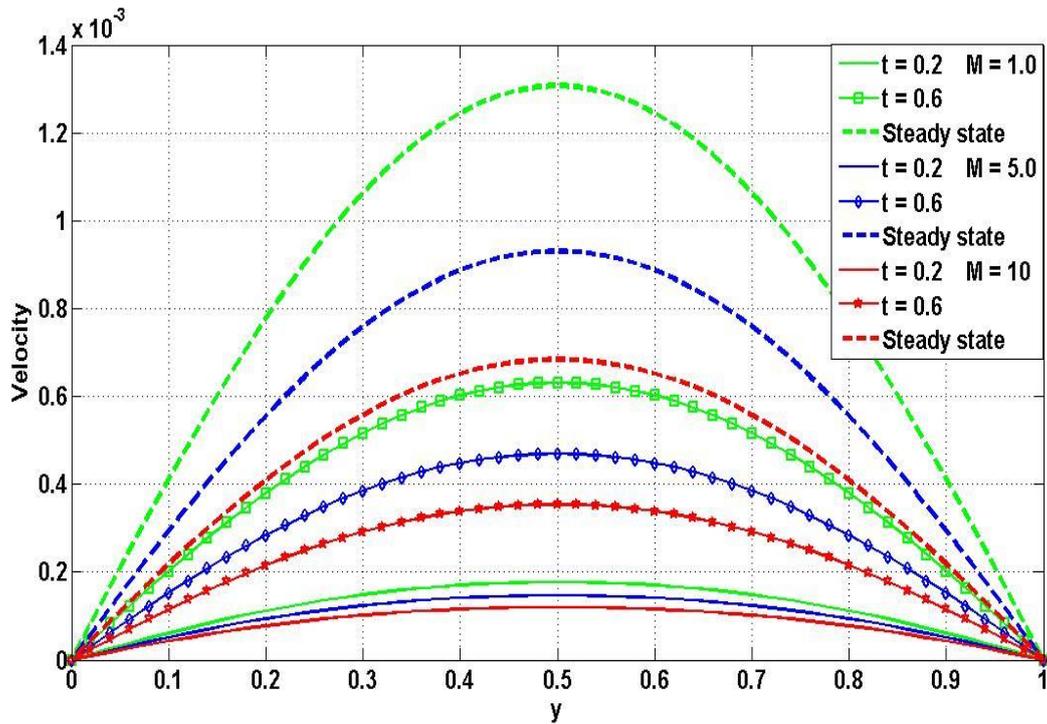


Fig.5. Variation of unsteady and steady-state velocity with M for Pr = 7

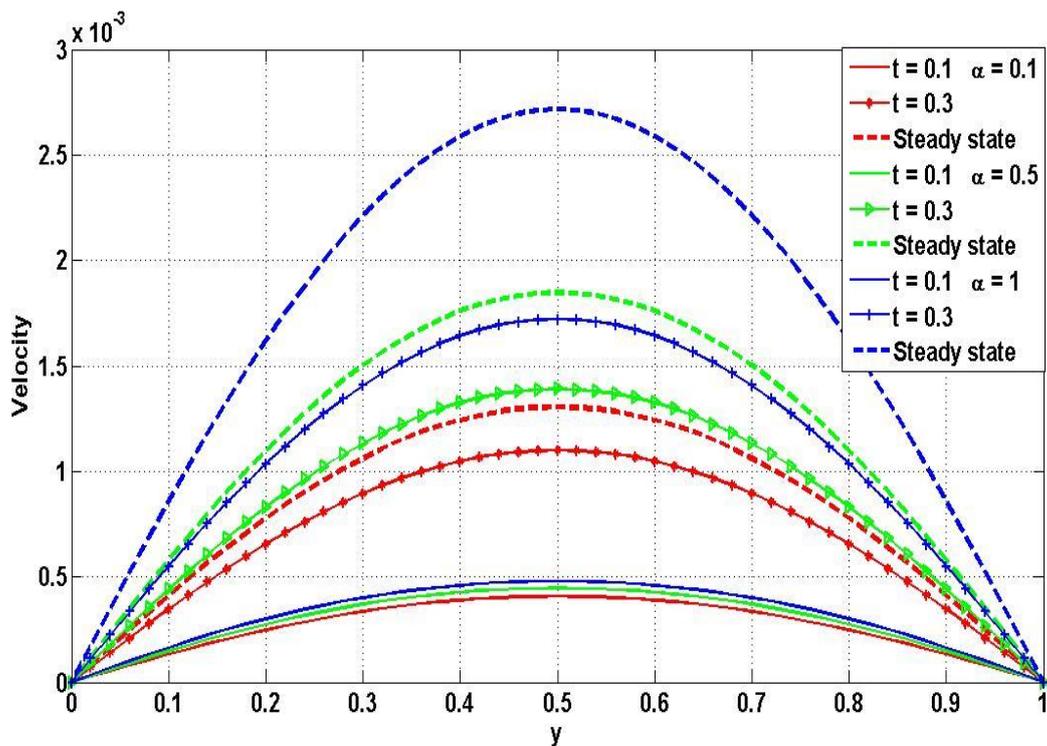


Fig.6. Variation of unsteady and steady-state velocity with α for $Pr = 0.71$

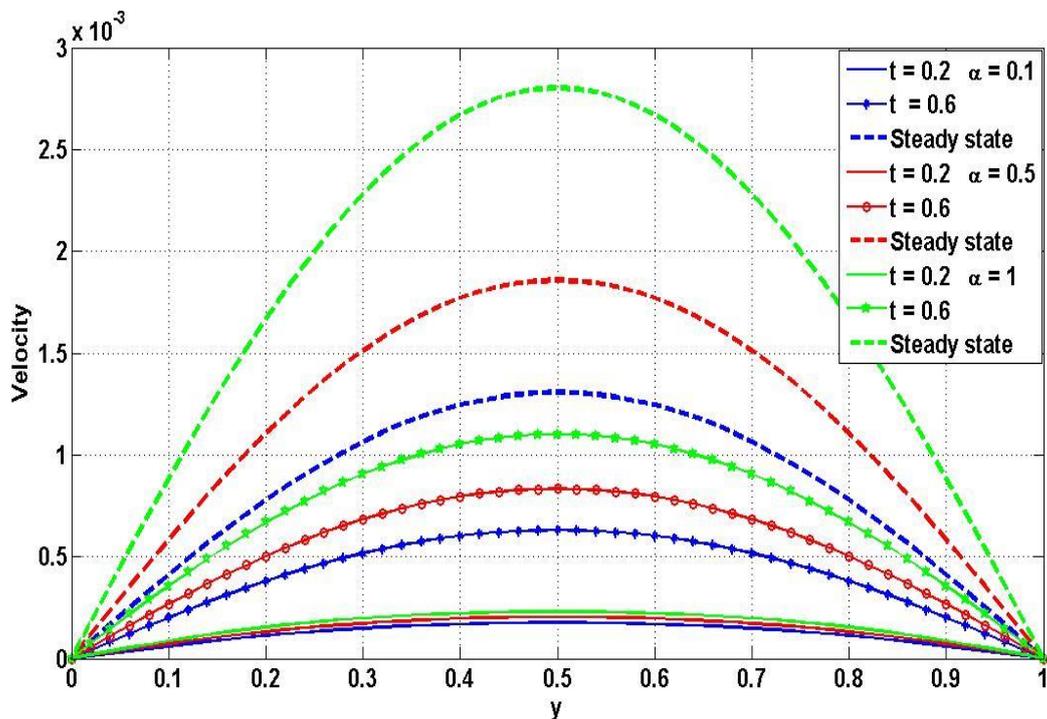


Fig.7. Variation of unsteady and steady-state velocity with α when $Pr = 7$

Figs.4 and 5 shows the influence of the Hartmann number (M) on unsteady and steady-state velocity profiles of the air ($Pr = 0.71$) and water ($Pr = 7$) respectively. These figures show that, at the initial stage the velocity of the water is less than that of the air but later reached equal steady-state. Larger values of the Prandtl number correspondingly decreases fluid thermal conductivity which results to weak diffusion of the thermal energy in the case of water than air. The velocity of both air and water decreases with increasing M . As expected, larger values of M correspond to higher degrees of drag

force (Lorenz force) which in turn reduced the velocity of the fluid. Also, it is observed from these two figures that time required to reach steady-state condition is higher in the case of water than that of air.

Figs.6 and 7 illustrates in particular the increase in fluid velocity of air and water respectively with increasing variable viscosity parameter (α). As expected, the fluid viscosity is reduced by increasing α , consequently, the molecular resistance to motion is decreased and hence fluid flow is enhanced. It is noticed that, both unsteady and steady-state velocity of air and water increases with increasing non-dimensional time. As explained above, at the initial stage velocity of air is higher than that of water.

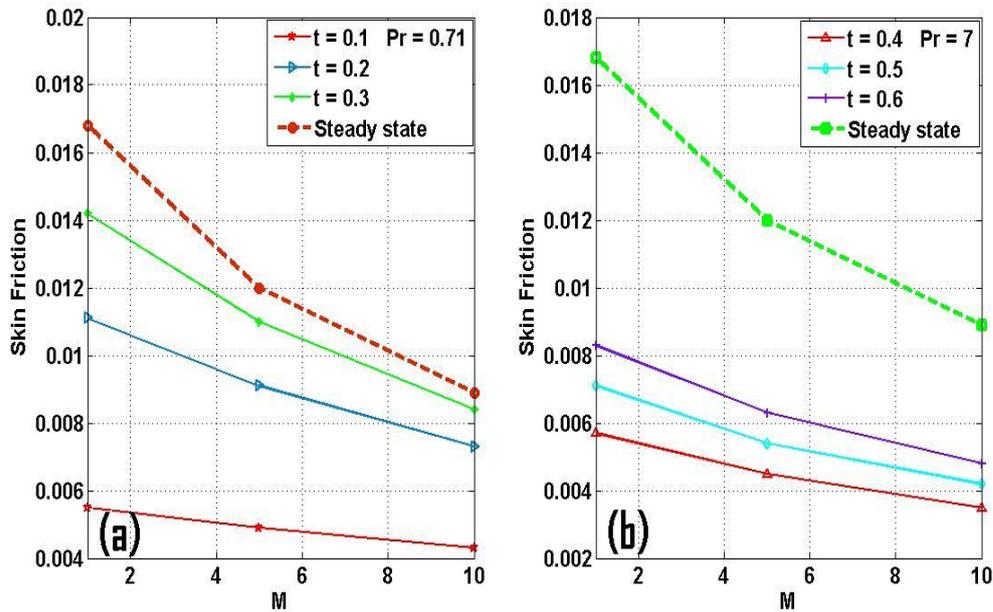


Fig.8. Variation of unsteady and steady-state skin friction at $y = 0$ with M of Pr = 0.71 and Pr = 7

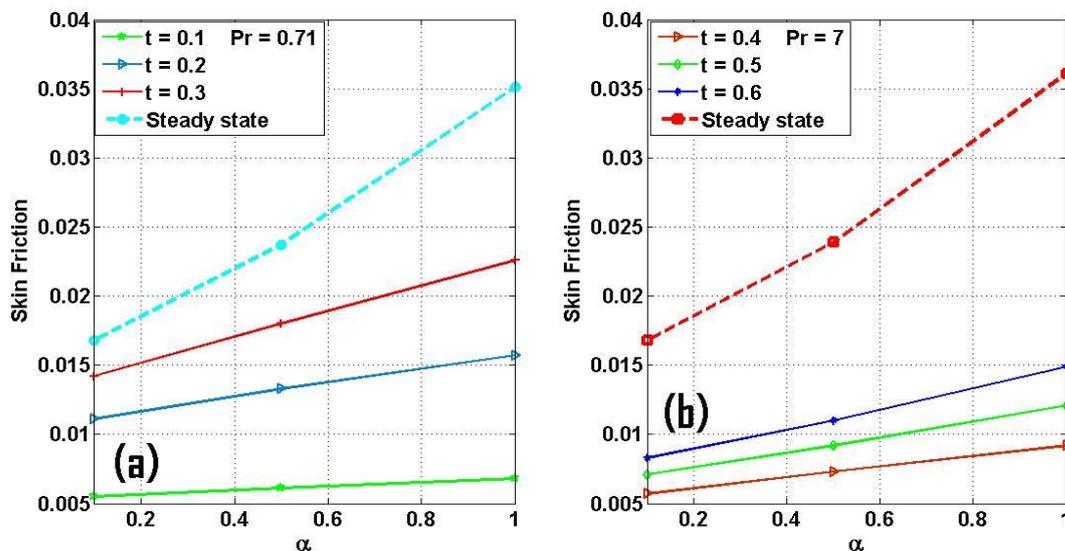


Fig.9. Variation of unsteady and steady-state skin friction at $y = 0$ with α of Pr = 0.71 and Pr = 7

The skin friction dependence on M for varying values of time (t) at the plate $y = 0$ is displayed in Fig.8a and 8b for air (Pr = 0.71) and water (Pr = 7) respectively. From the plots, it is seen that as M increases, the skin friction decreases. It is evident also that, skin friction increases with increasing time.

The skin friction dependence on α for varying values of time at the plate $y = 0$ is illustrated in Fig.9a and 9b for air ($Pr = 0.71$) and water ($Pr = 7$) respectively. It is reflected that, skin friction increases as α and t increases until a steady-state condition is achieved. It should be noted that the numerical values of skin friction at the plate $y = 0$ and $y = 1$ are same due to the symmetric condition of the flow.

4. Conclusion

The problem of unsteady hydromagnetic free-convective flow of a variable viscosity and viscous reactive fluid in a channel formed by two infinite vertical parallel plates in the presence of applied magnetic field is investigated numerically by using semi-implicit finite difference scheme. The results show that,

- (i) Increasing values of the variable viscosity parameter accelerate the fluid velocity while higher values of Hartmann number decelerate velocity of the fluid
- (ii) Skin friction is enhanced with increasing variable viscosity parameter whereas higher values of M reduced the skin friction.

Nomenclature

M	Hartmann number
g	acceleration due to gravity
Pr	Prandtl number
H	dimensionless gap between the plates
t'	dimensional time
t	dimensionless time
T'	dimensional temperature of the fluid
T_0'	initial temperature of the fluid and plate
C_0'	initial concentration of the fluid and plates
u'	dimensional velocity of the fluid
U	dimensionless velocity of the fluid
y'	dimensional co-ordinate perpendicular to the plate
y	dimensionless co-ordinate perpendicular to the plate
R	universal gas constant
x'	dimensional co-ordinate parallel to the plate
C_p	specific heat of the fluid at constant pressure
C_0^*	initial concentration of the reactant species
k	thermal conductivity of the fluid
E	activation energy

Greek Letters

ρ	density of the fluid
β	volumetric coefficient of thermal expansion
θ	dimensionless temperature
λ	Frank-Kamenetskii parameter
ν	kinematic viscosity
ε	activation energy parameter
α	dimensionless viscosity
μ	dimensional viscosity

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