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Optimizing Model Factor For The Prediction Of Actual Behavior Of Full Scale Reinforced Concrete Beam

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Abstract

Scaling rules can be effectively applied to predict the behavior of reinforced concrete (RC) structure based on the properties of an identical model with scaled geometry. Physical or numerical modelling serves as a fundamental approach to explain the dynamics of engineering system. In this scientific article, optimising model factor for the prediction of actual behaviour of full-scale RC beam was investigated. Eight (8) model RC beams with scale factors ranging from 2 to 8 was produced and tested in bending based on Buckingham pi theorem. RC beam which had been investigated experimental by Kachlakev and McCurry [21] was used as the full-scale beam. Results showed that scale factors 2 to 5 were adequately large and gave the best prediction concerning similarity with the full-scale beam, the models proved satisfactorily yielding best results in terms of similarity to the actual prototype. When performing model tests for RC structure, the effect of the scale factors must at all times be taken into account and scale effect, the size effect has to be well thought out, and a clear difference must be reached between the two. As well, little variation in the material properties could alter the behaviour. Load-deflection curve for the prototype and model beam exhibits a similar trend, with an initial linear relationship

1. Introduction

Physical or numerical modelling serves as a fundamental approach to explain the behavioral dynamics of engineering system, elements, or structures, as highlighted by Roberto & Jorge [1]. One such instance of a model representing a whole structure or a portion of a structure is the physical modelling of a structure made of reinforced concrete (RC), as outlined by Coutinho et al. [2]. The choice of an appropriate scale for the model is paramount, ideally maximizing its size [3]; however, it is constrained by practical laboratory factors [4]. Laboratory testing is a crucial component in the conception of a product [3], even though theoretical and numerical analytical techniques are also very important tools. Before being put to use, their behavior needs to be confirmed by full-scale laboratory testing. By doing this, safety, performance, and desired reliability can be achieved [5- 6].

Holmes & Sliter [7] demonstrate how using scaled models saves cost and time. The authors anticipate that a single model test saved up to 1/3 of the cost of a corresponding full-scale structure and testing project. The cost and time of fully-scale laboratory testing is reduced by at least one-third when a model is used [7]. An entire laboratory test, including a combination of small-scale and full-scale models, would result in higher economic and temporal efficiency as reported by Taylor [8]. Similitude philosophy defines the requirements of developing a model of a full-scale and predicting the actual behavior using the model information, which was reported by Shuai et al. [9]. Similitude theory relates the small scale (model) to the actual size (prototype) RC element [10]. In many contemporary uses, the growing level of detail in engineering problems means that numerical and theoretical assessments are not sufficient (and entirely inappropriate for extremely complex constructions) for examining if a system's behavior matches the specifications of the design [11]. Because of the limitations of full-scale testing, there is considerable demand for laboratory tests [4]. As a result, the use of similitude method has increased rapidly. In fact, this method is used broadly in different disciplines and applications.

According to Zhu [12], similitude theory initially appeared in the 1800 years ego. In truth, Galilei & Weston [13] argued that the strength and RC element size do not reduce in the same proportion; the authors reported that as dimensions reduces, the strength increases. The unusual component of this statement is that the authors were already dealing with size effects in the 18th century. Rayleigh [14] is one of the first authors to discuss scientific models relative to dimensional analysis. Though Rayleigh [14]'s work attempted to highlight the significance of similitude procedures, particularly in engineering programme, as noted in Zhu [12]. Three (3) decades had passed until the publishing of another work that highlights the effectiveness of similitude techniques as discussed by Zohuri [15]. The technique of dimensional analysis was first used to solve basic and complex issues using a systematic approach. This led to a thorough understanding of the modelling of components with stress-strain properties and significant deflections. Simitses et al. [16] present a review in which similitudes and modelling approaches are based on dimension analysis. Yazdi & Rezaeepazhand [17] provide understanding into obtaining similitude criteria using both dimensional analysis application of governing equations. Zohuri [15] presents an overview of classical dimensional analysis before delving deeper into the issues, which exceed Buckingham's Pi theorem.

Full-scale experimental testing is time consuming and costly; it can be problematic to carry out in some situations as reported by Jian et al. [3]; in certain instances, the value of the information collected is insufficient to justify the expense and time required [10]. For these factors, it is advantageous to develop an accurate model of the actual system, that is, a reliably model of predicting the full-scale that can be study at a reduced cost and time. The aim of this paper is to provide a strength model factor for the prediction of actual behavior of full scale reinforced concrete beam, which has had limited reports. The strength model represents a direct model covering materials similar to those used in the prototype [15], offering predictive understanding into the prototype's behavior under various loads until failure. To effectively model a RC structure in terms of strength, it necessitates the use of model concrete and model-reinforcing elements, with each material adhering to similitude conditions requisite for reflecting the prototype materials [18].

2. Material and Method

2.1 Similitude Method

The Buckingham pi theorem as reported by Buckingham [19] and Alessandro et al. [20] was used to actualize the similitude approach, the procedure concerned with the relationship between physical quantities in the following way:
If y_1, y_2, ... y_n are quantities that are interrelated by a functional form, this relationship can be expressed as

$$f(y_{1,}\ y_{2,} \dots \dots .. y_N) = 0 \quad (1)$$

Equation (1) can be expressed as

$$\Phi\left(\Pi_{1,}\ \Pi_{2,} \dots \dots .. \Pi_N\right) = 0 \quad (2)$$

where the pi terms are dimensionless terms of the physical variables y_1 through y_n

Buckingham further revealed that by chosen, for example, variable $y_{1,}$ then

$$y_1 \ = K y_2^a . y_3^b . y_4^c \ etc \quad (3)$$

where K = Constant, and a, b, c are to calculated.

Considering a deformable body, the displacement u of the body will certainly involve length, elastic modulus E, point load, Q, and moment, M.

$$f(u, Q, l, E, M) = 0 \qquad f\left(y_{1,}\ y_{2,}\ y_{3,}\ y_{4,}\ \right) = 0 \quad (4)$$

$$or \qquad u = \Phi(\mathrm{Q}, \mathrm{l}, \mathrm{E}, \mathrm{M}) \quad (5)$$

$$or \qquad u = \mathrm{K} Q^a l^b E^c\ M^{\ d} \quad (6)$$

where a, b, c are constant to be calculated

by introducing the fundamental dimensions (Length L, Time T, force F)

$$L = \mathrm{K} F^a L^b (F L^{-2})^c\ (FL)^{\ d}$$

$$L^1 = \mathrm{K} F^a L^b F^c L^{-2c}\ F^d L^d$$

$$L^1 = \mathrm{K} F^{a+c+d} \quad L^{b-2c+d}$$

Equating the indices, we have

$$0 = a + c + d \ \Longrightarrow c = -a - d$$

$$1 = b - 2c + d \ \Longrightarrow b = 1 + 2c - d$$

$$b = 1 + 2(-a - d) - d$$

$$b = 1 - 2a - 3d$$

Substituting into (6)

$$u = \mathrm{K} Q^a l^{1-2a-3d} E^{-a-d}\ M^{\ d}$$

$$u = \mathrm{K} Q^a . l^1 . l^{-2a} . l^{-3d} E^{-a} . E^{-d} . M^{\ d}$$

$$\frac{u}{l} = \mathrm{K}\left(\frac{Q}{l^2 E}\right)^a . \left(\frac{M}{l^3 E}\right)^d$$

$$\frac{u}{l} = \mathrm{K}\left(\frac{Q}{l^2 E}\right)^a . \left(\frac{M}{l^3 E}\right)^d$$

$$\Phi\left(\frac{u}{l}, \frac{Q}{l^2 E}, \frac{M}{l^3 E}\right) = 0 \qquad or \qquad \Phi(\Pi_1, \Pi_2, \Pi_3) = 0 \quad (7)$$

$$where\ \Pi_1 = \frac{u}{l}, \quad \Pi_2 = \frac{Q}{l^2 E}, \quad \Pi_3 = \frac{M}{l^3 E}$$

According to Buckingham, subsequently

$$\Pi_1 = \Phi(\Pi_2, \Pi_3 \dots .. \Pi_n) \quad (8)$$

then, Next, concerning the prototype (p) and its corresponding model

$$\frac{\Pi_{1p}}{\Pi_{1m}} = \frac{\Phi(\Pi_{2p}, \Pi_{3p} \dots .. \Pi_{np})}{\Phi(\Pi_{2m}, \Pi_{3m} \dots .. \Pi_{nm})}$$

If perfect similarity exists, all dimensionless pi variables in the prototype and model are the same. Therefore,

$$\Pi_{1p} = \Pi_{1m} \quad (9)$$

2.2 Development of Scaling Factors

In model analysis, it's typical to establish the scaling factor for a specific parameter 'i' (which could represent stress, E, length, l , etc.) as the quotient of the prototype value i_p divided by the model value i_m, [19],

$$\mathrm{S}_i = {}^{i_p}\!/_{i_m} \quad (10)$$

So, when considering length ' l ', the scaling factor 'S' is defined as follows:

$$S_l = {l_p}/{l_m} \quad (11)$$

This criterion directly governs the geometric resemblance to the prototype. Another crucial aspect, particularly in structural analysis, is the material property (modulus of elasticity). Using this as a scaling factor,

$$S_E = {E_p}/{E_m} \quad (12)$$

From Eq. (9)

$\Pi_{1p} = \Pi_{1m}, which\ means\ that;$

$$\Pi_{2p} = \Pi_{2m} \quad or \quad \frac{Q_p}{l^2{}_p E_p} = \frac{Q_m}{l^2{}_m E_m} \quad (13)$$

$$S_Q = \frac{Q_p}{Q_m} = \frac{E_p}{E_m}\left(\frac{l_p}{l_m}\right)^2 = S_E . S_l^2 \quad (14)$$

Also,

$$\Pi_{3p} = \Pi_{3m} \quad or \quad \frac{M_p}{l^3{}_p E_p} = \frac{Q_m}{l^3{}_m E_m} \quad (15)$$

$$S_M = \frac{M_p}{M_m} = \frac{E_p}{E_m}\left(\frac{l_p}{l_m}\right)^3 = S_E . S_l^3 \quad (16)$$

2.3 Reinforced concrete models

To determine the scaling factors, dimensional evaluation is once again necessary. In RC applications, it's advantageous to consider stress 'S_σ' and length 'S_l' as the required scaling factors [9]. Once more, it's preferable to limit the parameters considered for the analysis to u, σ, Q, l, M, and ρ, where ρ represents the density of concrete. Consequently, the governing equation can be expressed as follows:

$$f(u, \sigma, Q, l, M, \rho) = 0$$

$$u = \Phi(\sigma, \mathrm{Q}, \mathrm{l}, \mathrm{M}, \rho)$$

$$u = \mathrm{K}\sigma^a . Q^b . l^c . M^d \rho^e \quad (17)$$

by introducing the fundamental dimensions (Length L, force F)

$$L = \mathrm{K}(Fl^{-2})^a . F^b . L^c . (FL)^d (\mathrm{F}L^{-3})^e$$

$$L = \mathrm{K}(Fl^{-2})^a . F^b . L^c . (FL)^d (\mathrm{F}L^{-3})^e$$

$$L = \mathrm{K}F^{a+b+d+e} \quad L^{-2a+c+d-3e}$$

Equating the indices, we obtain

$$a + b + d + e = 0 \Longrightarrow b = -a - d - e$$

$$-2a + c + d - 3e = 1 \Longrightarrow c = 1 + 2a - d + 3e$$

Substituting into Eq (17)

$$u = \mathrm{K}\sigma^a . Q^{-a-d-e} . l^{1+2a-d+3e} . M^d \rho^e$$

$$u = \mathrm{K}\sigma^a Q^{-a} . l^{2a} \ l . M^d Q^{-d} \ l^{-d} \rho^e Q^{-e} l^{3e}$$

$$\frac{u}{l} = \mathrm{K}\left(\frac{\sigma . l^2}{Q}\right)^a \left(\frac{M}{Ql}\right)^d \left(\frac{\rho l^3}{Q}\right)^e$$

$$\frac{u}{l} = \mathrm{K}\left(\frac{\sigma . l^2}{Q}\right)^a \left(\frac{M}{Ql}\right)^d \left(\frac{\rho l^3}{Q}\right)^e$$

$$\Phi\left[\frac{u}{l}, \frac{\sigma l^2}{Q}, \frac{M}{Ql}, \frac{\rho l^3}{Q}\right] = 0$$

$$or \quad \Phi(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0$$

$$\Pi_1 = \frac{u}{l}, \quad \Pi_2 = \frac{\sigma l^2}{Q}, \quad \Pi_3 = \frac{M}{Ql}, \quad \Pi_4 = \frac{\rho l^3}{Q}$$

Likewise, it can be demonstrated that

$\Pi_{2p} = \Pi_{2m}$

$$S_{Q=}\frac{\sigma_p l_p^2}{Q_p} = \frac{\sigma_m l_m^2}{Q_m}$$

$$S_{Q=}\frac{Q_p}{Q_m} = \frac{\sigma_p}{\sigma_m}\left(\frac{l_p}{l_m}\right)^2 = S_\sigma . S_l^2 \quad (18)$$

$$S_{M=}\frac{M_p}{M_m} = \frac{Q_P l_p}{Q_m l_m}$$

$$S_{M=}\frac{M_p}{M_m} = S_\sigma . S_l^2\left(\frac{l_p}{l_m}\right) = S_\sigma . S_l^3 \quad (19)$$

$$S_\rho = \frac{\rho_p}{\rho_m} = \frac{Q_p}{Q_m}\left(\frac{l_m}{l_p}\right)^3 = S_\sigma / S_i \quad (20)$$

2.4 Model and prototype beams

The beam presented in Figures 1 and 2 was considered as prototype and which had been investigated experimental by Kachlakev & McCurry [21], Kachlakev et al. [22] and Hamid et al. [23]. The prototype RC beam has an effective length of 5.485 m with a cross-section of 0.305 m x 0.770 m with specific reinforcement details as presented in Table 1. A two-point load configuration was applied, with a spacing of 1.825 meters between the loads. The flexural steel bar is depicted in Figures 1 and 2. No reinforcement was provided; the beam was constructed and tested at the laboratory of Oregon State University. The beam was reported failed at 476 kN and 482 kN by Kachlakev et al. [22] and Hamid et al. [23], respectively. The yield stress of steel and the compressive strength concrete was reported was reported as 414 and 20.7MPa, respectively.

2.5 Experimental Design

The experimental design adopts Kachlakev et al. [22] beam by scaling down the beam with 2, 3, 4, 5, 6, 7, and 8 scale factors. The study intends to provide a strength model factor for the prediction of the actual behavior of a full-scale reinforced concrete beam. The beam from Kachlakev et al. [22] was taken as the prototype beam. Thus, the materials considered in this investigation were Portland limestone cement that met EN 197-1[24] requirements and fine and coarse aggregates with specific gravities of 2.6 and 2.7, respectively, according to ASTM C128 [25]. To guarantee acceptable workability and strength development, a water-cement ratio (w/c) of 0.55 was employed, following Neville's [26] recommendations. The reinforcements in metric sizes of the prototype and model beams, as well as geometry configurations, are shown in **Table 1**. A total of eight (8) model beams with model factors of 2, 3, 4, 5, 6, 7, and 8 were produced for this investigation.

The experimental design followed the similitude principles outlined in the research, which were derived using the Buckingham Pi theorem and dimensional analysis. The scaling factors for various parameters, such as length (S_L), modulus of elasticity (S_E), load (S_Q), and moment (S_M), were determined based on the relationships between the prototype and model properties.

2.6 Determination of model beam dimensions

The model beams were designed and produced following the similitude principles outlined in the research. The geometric properties, such as length, width, height, and reinforcement diameters, were scaled down according to the respective scale factors. For instance, model beam B (scale factor 2) had dimensions of 2743 mm length, 153 mm width, and 384 mm height, with reinforcement details as shown in Table 1. The process is demonstrated as:

Sample B (scale factor of 2)

$$\frac{l_p}{l_m} = S_l \Longrightarrow l_m = \frac{l_p}{S_l} = \frac{5485}{2} = 2743\ mm$$

$$b_m = \frac{b_p}{2} = \frac{305}{2} = 153\ mm$$

$$\emptyset_m = \frac{\emptyset_p}{2} = \frac{16}{2} = 8\ mm$$

The geometries of samples C, D, E, F, G, H, and I were computed as samples B. The samples A, B, C, D, E, F, G, H, and I are grouped as geometric properties of the prototype, which must be 2, 3, 4, 5, 6, 7, and 8 times the models, respectively. A two-point load bending test was conducted for each model beam as depicted in Figure 1. The two-point bending test proposed for the model beams follows the recommendations of ASTM C78/C78M [27] for examining the load-deflection behavior and failure loads of RC beams, which is a widely accepted testing method. The loads and corresponding defections were recorded for each model beam during the testing.

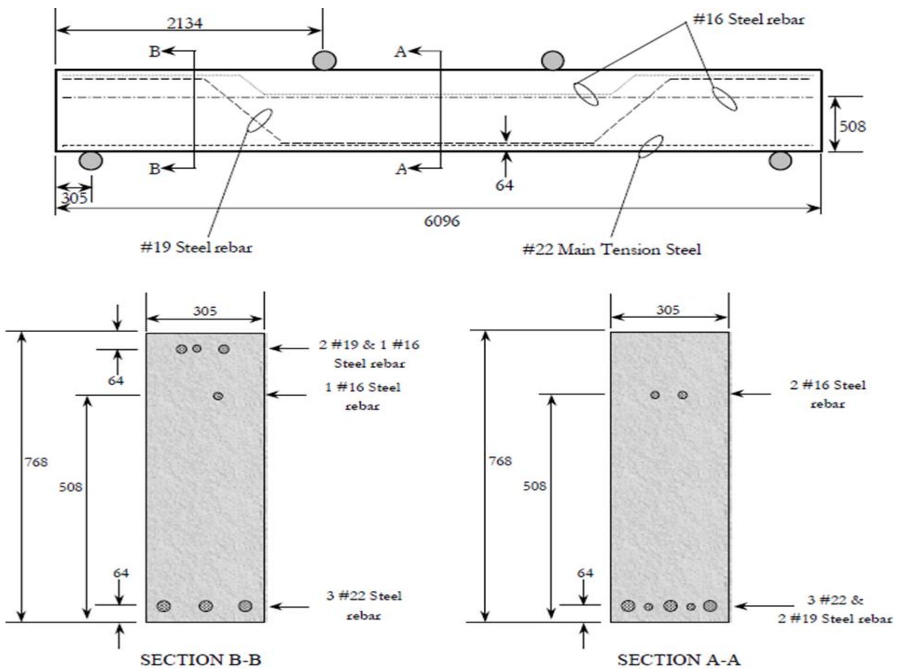


Figure 1: Typical steel reinforcement locations [21]

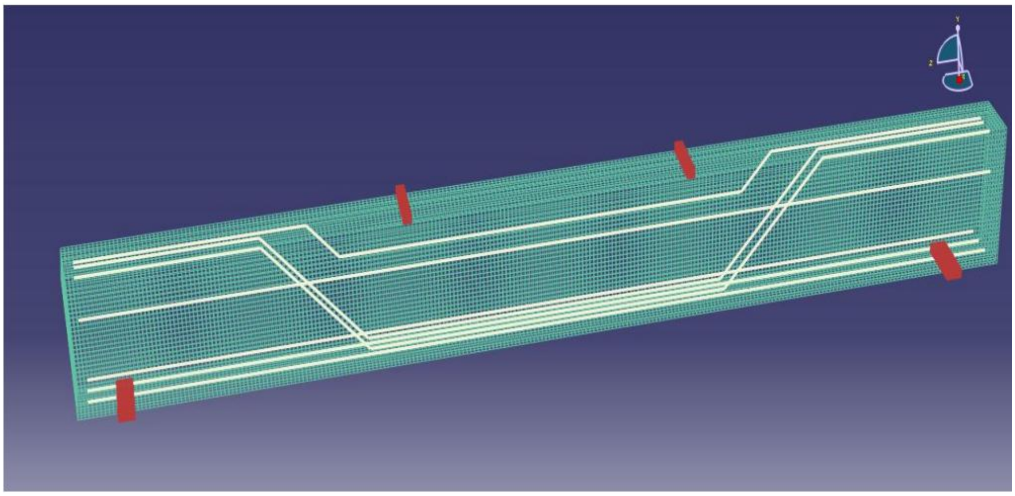


Figure 2. Finite element modeling parts in ABAQUS [23]

Table 1 Beam Configuration

Sample ID	Scale Factor	Beam dimensions			Reinforcement	
		L (mm)	b (mm)	h (mm)	Bottom	Top
A	1	5485	305	768	3#22+2#19	2#16
B	2	2,743	153	384	1#20+1#12	2#8
C	3	1,828	102	256	1#12+1#10	2#5
D	4	1,371	76	192	1#10+1#6	2#4
E	5	1,097	61	154	1#8+1#5	2#3
F	6	914	51	128	1#6+1#5	2#3
G	7	784	44	110	1#5+1#4	2#2
H	8	686	38	96	#6	2#2

3. Results and Discussion

The model beams were loaded to failure at the Civil Engineering Laboratory, Niger Delta University. The capacity, mode of failure, and deflection results of beams are given in Table 2. Figures 3 down to 7 show load-defection curves for model and prototype beam, both beams exhibited the same trend, which is similar to those reported by Hamid et al. [23].

3.1 Load-deflection plot

The load-deflection plots presented in the study provide critical insights into the behaviour of the reinforced concrete beams under flexural loading, allowing for a comprehensive analysis of the scaling accuracy and the prediction capabilities of the proposed methodology. The deflection was observed and recorded at the midpoint of the bottom surface of the model beams, as this location is expected to experience the maximum deflection due to the applied load. The initial occurrence of cracking is marked by a change in the slope occurring at around 78.3 kN for the prototype, as reported by Kachlakev & McCurry [21], 18.4 kN, 8.2 kN, 4.3 kN, 2.89 kN, 1.92 kN, 1.45 kN, and 1.15 kN for samples B, C, D, E, F, G, and H, respectively, as shown in Figures 3 to 7. This change in slope is a clear indication of the transition from the uncracked state to the cracked state, marking a significant point in the structural behaviour of the beams.

Figure 3(a) presents the load-deflection curve for the prototype beam, as reported by Kachlakev and McCurry [21], while Figure 3(b) shows the load-deflection curve for the model beam with a scale factor of 2 (Sample B). Both curves exhibit a similar trend, with an initial linear relationship between load and deflection, followed by a nonlinear region as the beam approaches its ultimate capacity. Figures 4(a) to (b), 5(a) to (b) and 6(a) to (b) show the load-deflection curve for the model beams with a scale factor of 3, 4, 5, 6, 7, and 8, respectively. The curve exhibits a linear response up to the maximum load of 57.27, 29.7, 19, 13. 17, 9.5, and 7.1 kN, followed by a gradual decline in load-carrying capacity as the deflection increases. This behavior is consistent with the expected response of a reinforced concrete beam, where the initial linear response represents the elastic stage, and the gradual decline represents the post-yield, plastic deformation stage. The differences in the maximum loads and deflections between the model beams are in line with the scaling factors applied, as discussed earlier.

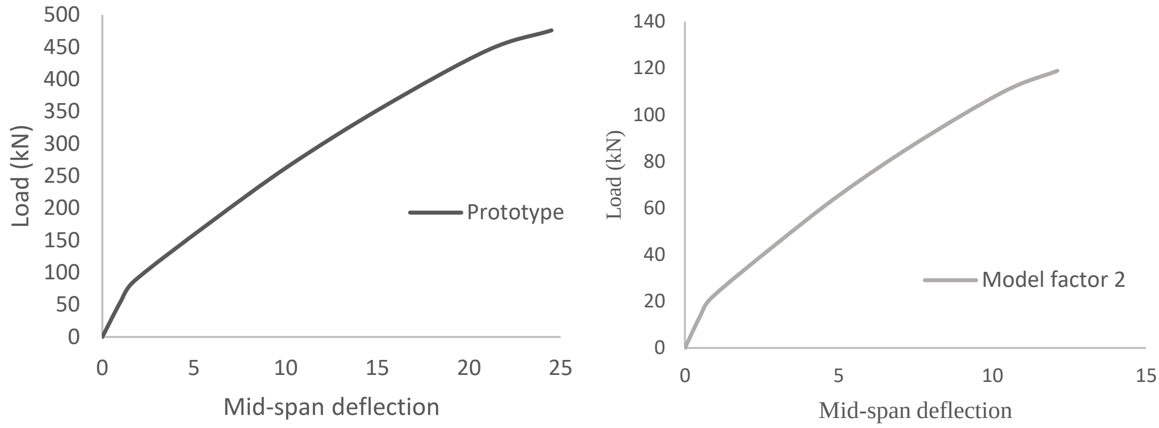
3.2 Comparison of Scale Ratios

Table 3 and Figure 7 revealed that the scale factors of 2, 3, 4, 5, and 6 adequately gave the best prediction concerning similarity with the prototype. It is clear that prototype and model beams behave almost the same; the error recorded (less than 5%) is presented in Table 3. These findings were similar to those of Noor & Boswell [28]. The study also revealed that when performing model tests for RC structure, the effect of the scale ratio must at all times be taken into account. In addition to the scale effect, the size effect has to be well thought out, and a clear difference must be reached between the two. The findings are similar to those of Marcilio & Roberto [10]. As well, little variation in the material properties could alter the behavior [29].

The results observed and presented in Tables 2 and 3 confirmed the importance of model testing for RC structures, which is in line with Holmes & Sliter [7]. Explicitly, the models scaled down by 2, 3, 4, 5, and 6 scale factors, exhibiting a commendable degree of reliability for the prototype RC beam. This finding holds a significant implication for the similitude technique, stressing the necessity of careful consideration for scale factors during the model study.

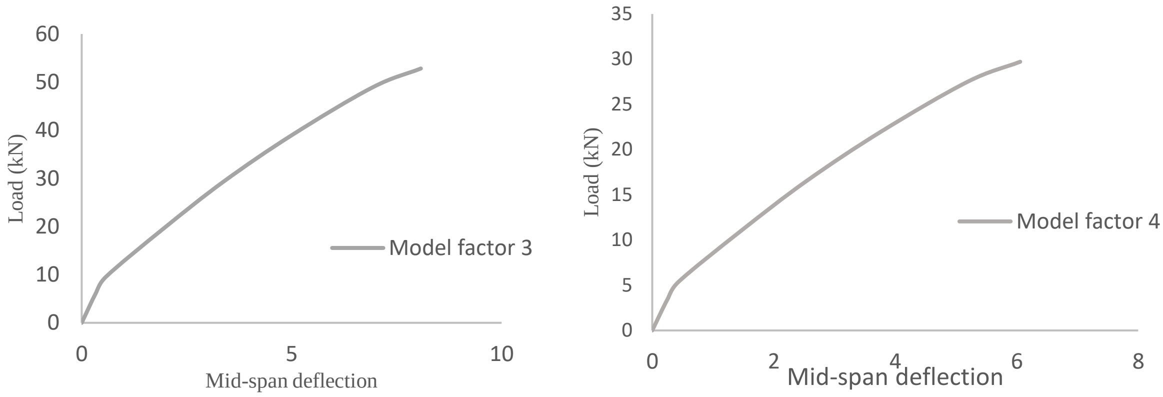
Table 2: Direct measured results

Sample ID	Scale Factor	First crack load kN	Deflection at first Load (mm)	Failure load kN	Deflection at failure Load (mm)	Failure mode
A	1	78.3	1.5	476	24.5	Shear
B	2	18.4	0.65	118.8	12.12	Flexural
C	3	8.2	0.45	52.78	8.14	Flexural
D	4	4.3	0.32	29.7	6.10	Flexural
E	5	2.89	0.29	19.0	4.8	Flexural
F	6	1.92	0.22	13.17	4.00	Flexural
G	7	1.45	0.2	9.5	3.35	Flexural
H	8	1.15	0.14	7.1	2.92	Flexural



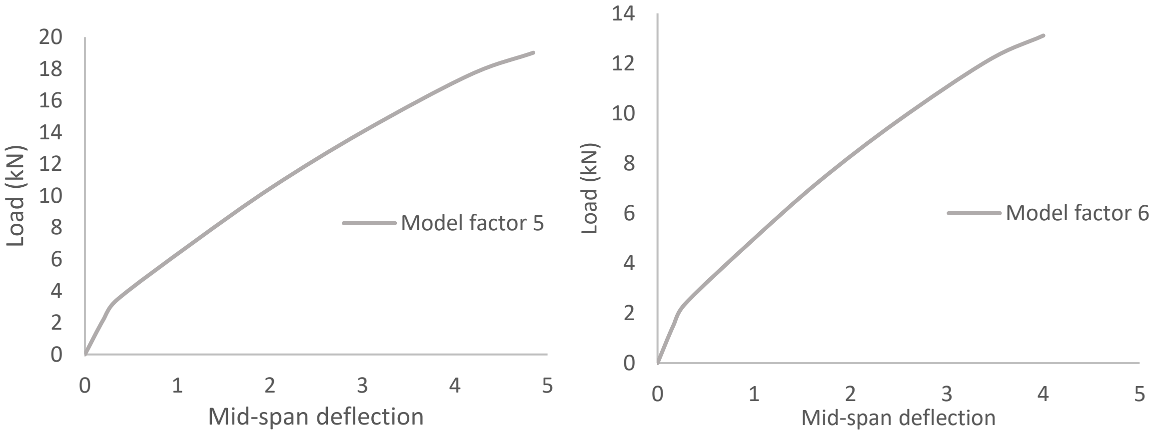
(a) (b)

Figure 3: Load-deflection curve (a) the Prototype beam [21]; (b) model beam scale down by a scale factor of 2



(a) (b)

Figure 4: Load-Deflection Curve. (a) model beam scales down by a scale factor of 3: (b) model beam scale down by a scale factor of 4.



(a) (b)

Figure 5: Load-Deflection Curve. (a) model beam scales down by a scale factor of 5; (b) model beam scale down by a scale factor of 6

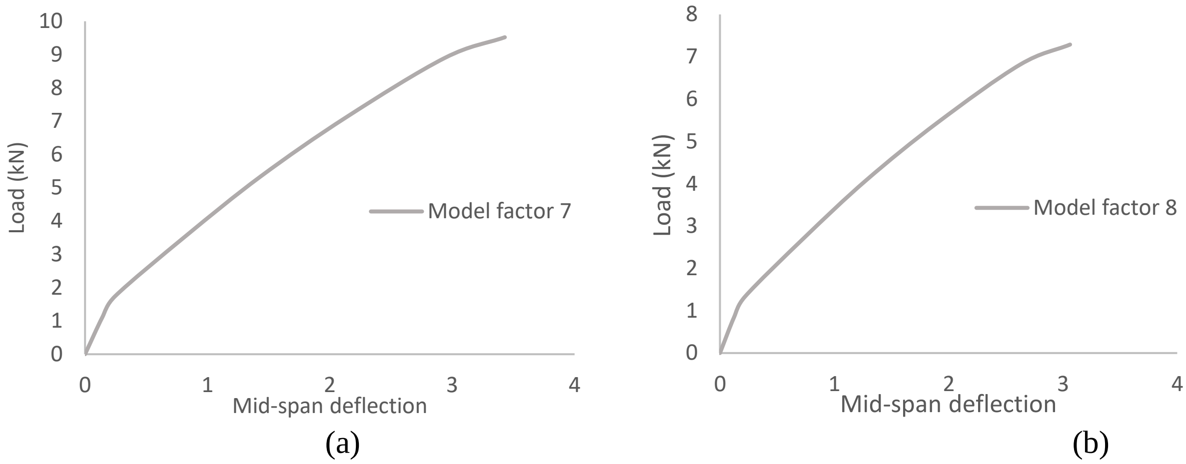


Figure 6: Load-Deflection Curve. (a) model beam scale down by a scale factor of 7; (b) model beam scale down by a scale factor of 8.

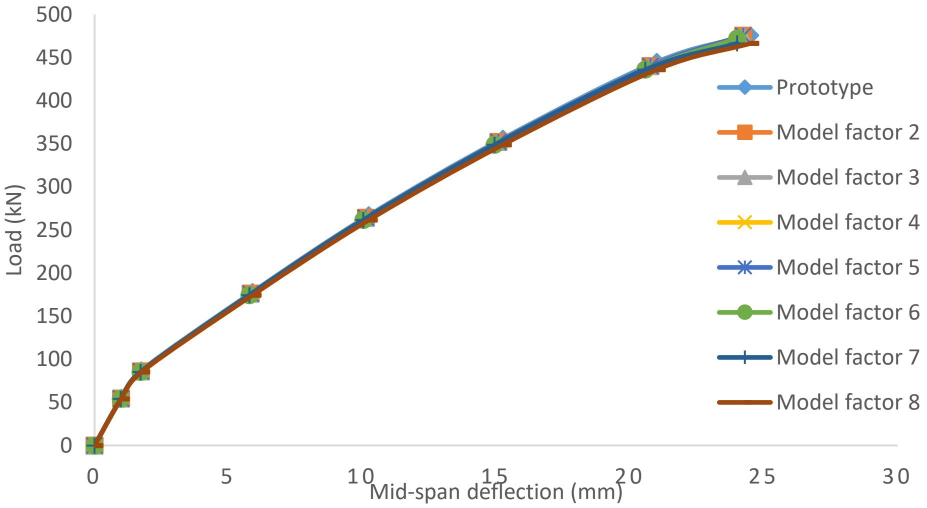


Figure 7 Load-deflection curve for the prototype and the predicted prototype beams

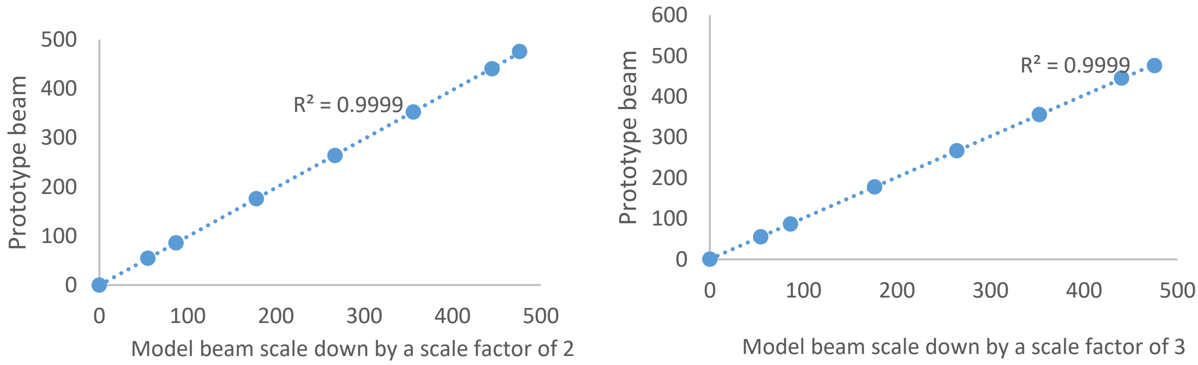
Figure 7 illustrates the load-deflection relationship of the beams, comparing both the prototype and the predicted prototype, beams based on the scaled-up results from the model beams. Overall, the predicted prototype results closely align with Kachlakev & McCurry's [21] data, demonstrating strong agreement in the load-deflection plot. Notably, the predicted load-deflection curve based on the 1:2 scale model beam (B) showed the closest agreement with the experimental results reported by Kachlakev et al. [22] and Hamid et al. [23] for the prototype beam (A). The ultimate load predicted from the 1:2 scale model was 472 kN, which is within 1% and 2% of the experimentally reported values of 476 kN [22] and 482 kN [23], respectively. The closeness of the 1:2 scale model predictions to the actual prototype behavior can be attributed to the relatively larger scale factor used, minimizing the size effect discrepancies. As the scale factor increased (i.e., smaller model sizes), the predictions deviated further from the prototype behavior, likely due to the amplified size effect and other scaling inaccuracies.

These findings support the applicability of the similitude theory and the proposed methodology for predicting the behavior of full-scale reinforced concrete beams using scaled-down models. However, it is evident that the scale factor plays a crucial role in the accuracy of the predictions, with larger scale factors (closer to the prototype size) yielding more reliable results.

Having obtained enough information on the model and prototype beams, we can evaluate the viability of using the failure load of the model beams to predict the prototype beam, using Equation (18). The ultimate capacity of the prototype beam, as reported in Table 2, was 476 kN and 482 kN, respectively, in the two previous studies. This aligns with the findings from the model beam with a scale factor of 2, 3, 4, 5, 6, 7, and 8, which had an ultimate capacity of 118.8, 57.27, 29.7, 19, 13. 17, 9.5, and 7.1 kN kN, respectively. Applying the scaling factor of 2, 3, 4, 5, 6, 7, and 8 (as per Equation 18), the expected ultimate capacity of the model beams corresponding to the prototype would be as presented in Table 3.

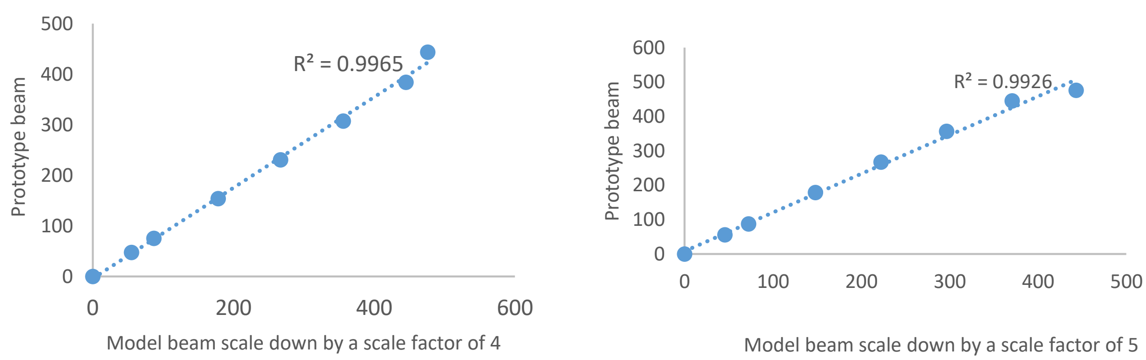
Table 3: Predicted values

Samples group	Scale Factor (S_i)	*Squaring the Scale factor* (s_i^2)	Failure load of the Model beams Q_m (kN)	Equation (18) Q_P(kN) $(Q_P = s_i^2 \times Q_m)$	Error %	Predicted deflection at failure Load (mm)
A	-	-	-	476.3	-	24.50
B	2	4	118.8	475.2	0.23	24.24
C	3	9	52.78	475.02	0.27	24.42
D	4	16	29.7	475.2	0.23	24.41
E	5	25	19.0	475.0	0.27	24.0
F	6	36	13.17	474.12	0.46	24.0
G	7	49	9.5	465.5	2.27	23.45
G	8	64	7.1	454.4	4.60	23.36



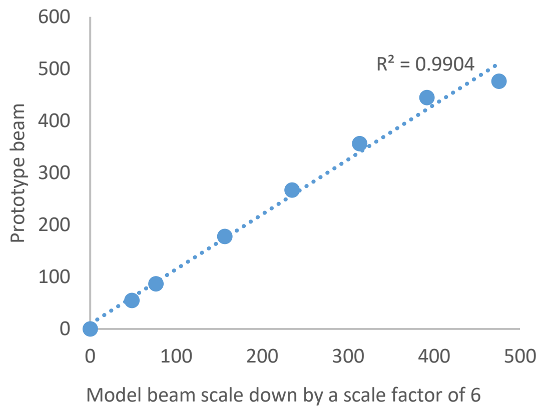
(a) (b)

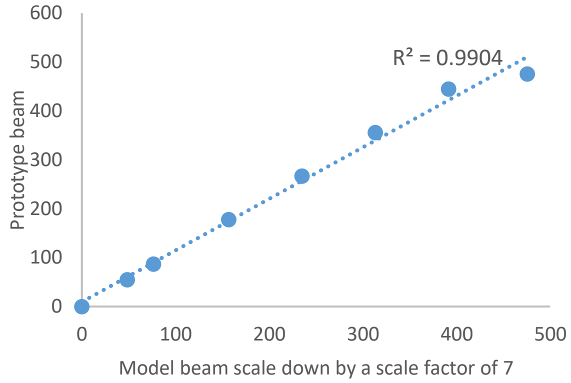
Figure 8: (a) Prototype vs predicted prototype beam by a scale factor of 2; (b) Prototype vs predicted prototype beam by a scale factor of 3



(a) (b)

Figure 9: (a) Prototype vs predicted prototype beam by a scale factor of 4; (b) Prototype vs predicted prototype beam by a scale factor of 5





(a) (b)

Figure 10: (a) Prototype vs predicted prototype beam by a scale factor of 6; (b) Prototype vs predicted prototype beam by a scale factor of 7

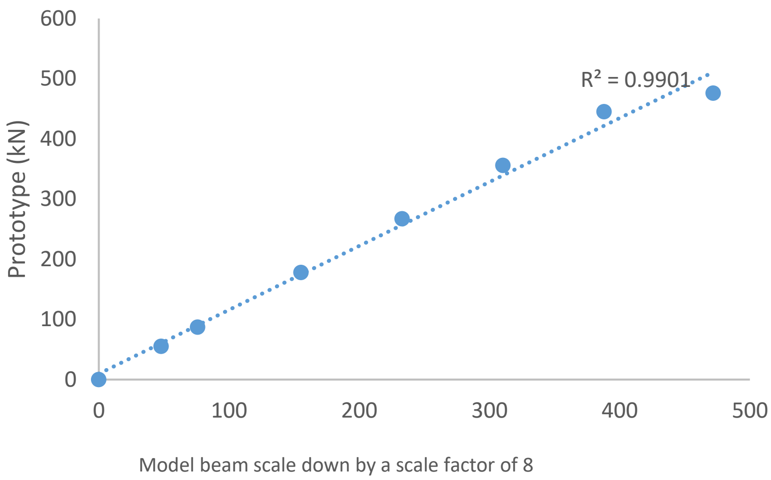


Figure 11: Prototype vs predicted prototype beam by a scale factor of 8

Figures 8, through 11 describe a comparison between the prototype beam outlined by Kachlakev & McCurry [21] and Hamid et al. [23] and those predicted through model tests. The figures presented show an acceptably agreement between the prototype beam and predicted beams in all models. This confirms the accuracy of the similitude method adopted for model testing. As shown in Figures 8, 9, 10, and 11, a significant number of the data points are located around the trendline, indicating comparable predicted model beams and the capacity of the prototype, which conform to the findings of Noor & Boswell [28]. This indicates a close relationship between the two results. However, a few data points show slight variances, presumably due to mild test errors. The coefficient of determination (R^2) was obtained to be 0.9999, 0.9999, 0.9965, 0.9926, 0.9904, 0.9902, and 0.9901, demonstrating around 99.99%, 99.99%, 99.65%, 99.26%, 99.04%, 99.02% and 99.01% predictive ability for model factors 2, 3, 4, 5, 6, 7 and 8 respectively, as clearly shown in the figures.

4. Conclusion

This article presents a model factor determination from a prototype RC beam. The scale factors 2, 3, 4, 5, 6, 7, and 8 were investigated and validated with existing experimental data in the literature. Similitude has proven to be a stimulating and valuable tool in engineering. The following conclusions are drawn from the results presented and discussed in section 3.0. They are:

i. scale factors 2 to 5 were adequately large and gave the best prediction concerning similarity with the full-scale beam called prototype, the models proved satisfactorily yielding best results in terms of similarity to the actual prototype;
ii. the study revealed that when performing model tests for RC structure, the effect of the Scale factors must at all times be taken into account;
iii. scale effect, the size effect has to be well thought out, and a clear difference must be reached between the two. As well, little variation in the material properties could alter the behaviour.
iv. load-deflection curve for the prototype beam, as reported by Kachlakev & McCurry (2020) [21] and model beams exhibit a similar trend, with an initial linear relationship.

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