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Appraising Incremental Oil with Numerical Quadrature: A Simpson Rule and Trapezoidal Rule Approach

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ABSTRACT

Technically, the performance of any successful enhanced oil recovery (EOR) project is given by the amount of incremental oil achieved from it. This paper adapted the Simpson quadrature in predicting the incremental oil from successful enhanced oil recovery processes. The error term of the quadrature was accounted for using the Finite difference approach, unlike in past works where it was neglected. This ensures that the differential equations component of the error formulae was adequately catered for numerically. This is important as the data considered were not defined by a model, typical of field data. Experimental data were used for the analyses. With the measured Incremental oil values from the laboratory used as comparative standards, results from the analyses showed that the Simpson quadrature gives a better incremental oil estimation among the numerical quadrature considered. This is lucid from the ‘‘error profile’’ plots for the two case studies where the Simpson and Trapezoidal recorded 4.567% and 5.41% for surfactant flooding and 5% and 18% for polymer flooding respectively. The superiority of the Simpson rule in incremental oil can be attributed to its adoption of higher order polynomial in modelling data points.

1.0 Introduction

Incremental oil is a critical factor in quantifying the success and profitability of any EOR process. Despite its unquestionable high importance, a careful perusal through past literature shows that attaining accuracy in its prediction have been a challenge in the past. The methods adopted have been questioned due to the relatively high error incurred in applying them as compared with observed field data. Typical examples of such predictive methods are the Decline curve and Trapezoidal rule. Mathematically, the concept of predicting the incremental oil from any successful EOR process involves integrating to estimate the area under the EOR rate-time curve. Research, however have shown that the Decline curve fails to properly estimate this due to the challenge of inaccuracy in shifting. [1] and its inability to capture unusual observation in rate data [2]. On the other hand, recent researches into the application of numerical methods in predicting incremental oil have shown some promising results. The Cubic spline can be adapted to rate-time data and when such are integrated

at each point and summed; predicted the incremental oil better, reducing some of the inherent problems faced in the past [3].
Some experimental works where data can be obtained for use includes the works of Onuoha and [4], [5], [6], [7], [8], [9], [10], [11]. Some other works of Onuoha et al (2017) [12], [13], [14], and Taiwo et al (2019) [15] were also of great contributions. To understand the different numerical methods used in estimating the incremental oil from EOR the works of [16],[17], and [18], [19],) [20], [21]) [22], [23], [24] and [25] were very useful.
In order to further reduce these problems and attained better accuracy in Incremental oil prediction, this paper propose to apply a relatively new numerical quadrature to the concept of incremental oil prediction; the Simpson quadrature. Furthermore, we also seek to improve the performances of the Trapezoidal rule by accounting for the error term in the formula.

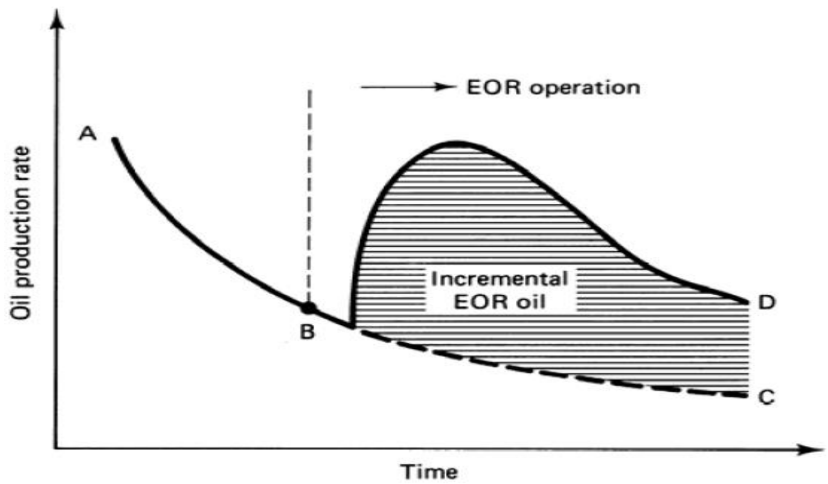

Fig 1. Incremental Oil Recovery from Typical EOR Response curve [26] and [27]

2.1 Simpson's Rule
In calculus, Simpson rule is a method for estimating value of integrals using quadratic and other higher polynomial expressions. Therefore, more complicated approximations formula an improve the accuracy for curves, these include using 2nd and 3rd order polynomial. Two versions of the rule are widely known and applied.

2.1.1 Simpson 1/3th Rule
In this rule, the function is approximated by a second-order degree polynomial between successive points. It corresponds to using second-order Lagrange polynomials in modelling data points.
Mathematically, it is given by the model:

$$\boldsymbol{I} = \frac{h}{3}[f(x_0) + 4[f(x_1) + f(x_3) \ldots + f(x_{2n-1})] + 2[f(x_2) + f(x_4) \ldots + f(x_{2n-2})] + f(x_n)] \qquad 1$$

The error associated with this rule is given by the model;

$$Error = -\frac{nh^5}{90} f^4(x) \qquad 2$$

2.1.2 Simpson's Three-Eight Rule
This rule corresponds to using third-order polynomial to fit four points. Integrating over the four points gives the model as shown in Eq (3).

$$\boldsymbol{I} = \frac{3h}{8}\lfloor f(x_0) + 3[f(x_1) + \cdots + f(x_{3n-1})] + 2\big(f(x_3) \ldots + f(x_{3n-3})\big) + f(x_{3n})\rfloor - \frac{nh^5}{80}f^4(x) \quad 3$$

The error associated with this rule is given by the model;

$$Error = -\frac{nh}{80}f^4(x) \quad 4$$

2.2 Trapezoidal Rule

The trapezoidal is the first of Newton-Cotes integration formulae. Trapezoidal Rule is an already established numerical integration formula which involves the use of the integral under a straight line to approximate the integral under a curve. The respective mathematical model for the rule is clearly outlined as follows.

Mathematically,

$$I = \int_a^b fx\,dx \quad 5$$

For a single strip (two ordinates)

$$I = \frac{h}{2}\lfloor f(x_0) + f(x_1)\rfloor - \frac{h^3}{12}f''(x) \quad 6$$

For multiple strips ($n > 2$ ordinates)

$$I = \frac{h}{2}\lfloor f(x_0) + 2\big(f(x_1) + f(x_2) + f(x_3) + \cdots + f(x_{n-1})\big) + f(x_n)\rfloor - \frac{h^3}{12}f''(x) \quad 7$$

$$Error = -\frac{h^3}{12}nf''(x) \quad 8$$

Where I = Area under the graph

2.3 Numerical Error Evaluation

It should be noted that, so far for the numerical quadrature include differentials to the second order and third order respectively in the error model for Trapezoidal rule and Simpson rule respectively. Interestingly, past works on these Quadrature have shown that they have the tendencies to over/under predict sometimes, depending on the data architecture. It therefore becomes pertinent that the differentials for the respective quadrature be accounted for in order to evaluate each error term and consequently get a more accurate result.

Since this work utilizes experimental data – which are carbon copy of field data- it becomes pertinent that we find a way to generate those differentials (as contained in respective error model) using the data points. In order to solve this challenge, this paper utilizes the Numerical differentiation technique, known as the Finite difference. The mathematical manipulation is shown below.

Recall,

$$\frac{dy}{dx} = \frac{\triangle f_i}{\triangle x} \quad 9$$

This can be extended to the second and fourth differentials in order to fully account for the error in the Trapezoidal and Simpson's rules respectively.

$$\frac{d^2y}{dx^2} = \frac{f_{i+2} - 2f_{i+1} + f_i}{\triangle x^2} \quad 10$$

$$\frac{d^4y}{dx^4} = \frac{\triangle^4 f_i}{\triangle x^4} = \frac{f_{i+5} - 4f_{i+3} + 6f_{i+2} - 4f_{i+1} + f_i}{\triangle x^4} \qquad 11$$

Therefore, the **Newton forward difference** table would be drawn for each data to account for their respective differentials in order for accurate error computations.

3.0 Results and Discussion

3.1. Incremental Oil

In this research paper, the Simpson quadrature along with Trapezoidal rule were adapted to rate-time data for incremental oil prediction. Multiple data points were used for each case study to replicate what is obtainable in field data. The equations for each projected declines, assuming the EOR process had not been initiated were obtained using Excel trendline function for the case studies Therefore, Incremental oil as shown in Fig 1, is defined mathematically as given;

$$\text{Incremental oil} = \int_a^b (\text{Composite function})dt - \int_a^b f(t)dt$$

Where the composite function= Simpson rule/Trapezoidal rule.

$$\int_a^b f(t)dt = \text{Equation of projected decline}$$

The rate-time curves of the case studies considered in this analysis are shown below

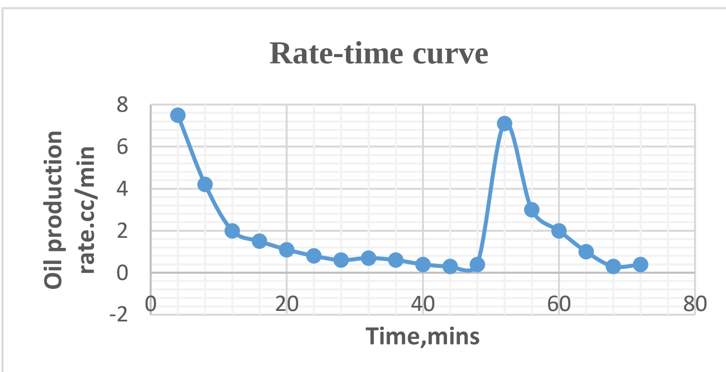

Figure 2. *Rate-time profile for the polymer flooding case study*

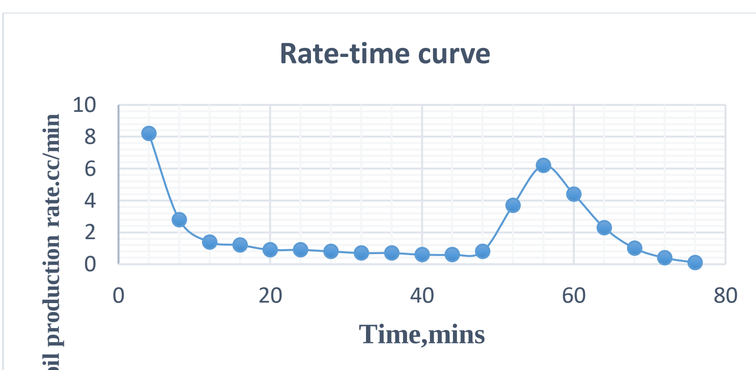

Figure 3. Rate-time profile for the polymer flooding case study

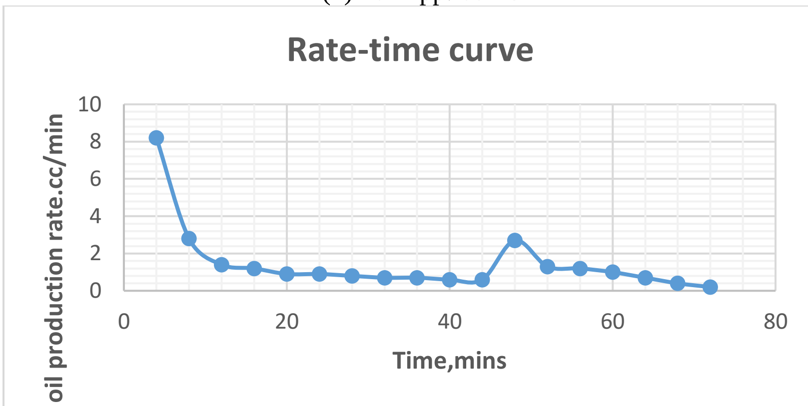

Figure 4 Rate-time profile for the alikaline surfactant polymer flooding case study

The incremental oil prediction from each case study using the respective models have been outlined below. Also, the error profile was also included to show clearly the accuracy of each method.

Case one:

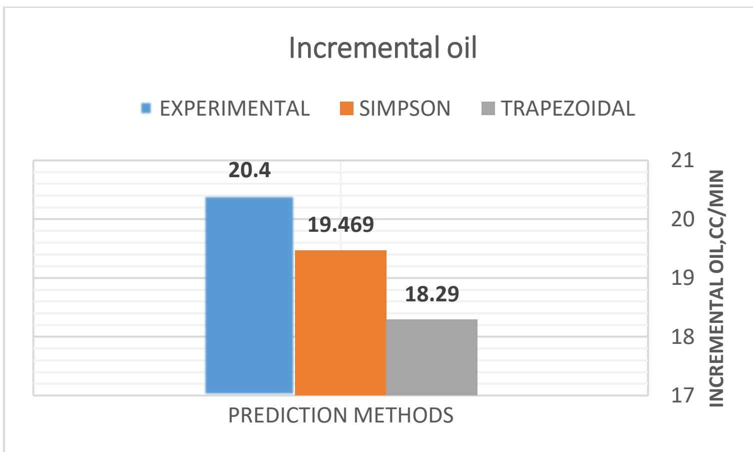

Figure 5a. Comparison of incremental oil predictions for the polymer flooding case study

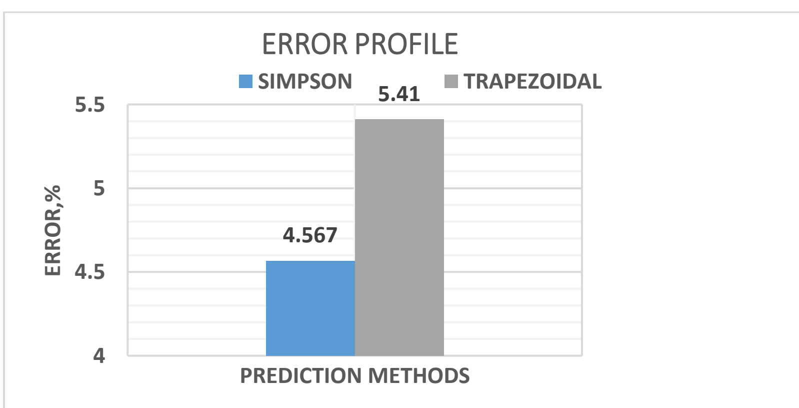

Figure 5b. Error incurred by the prediction methods for polymer flooding case study

Case two:

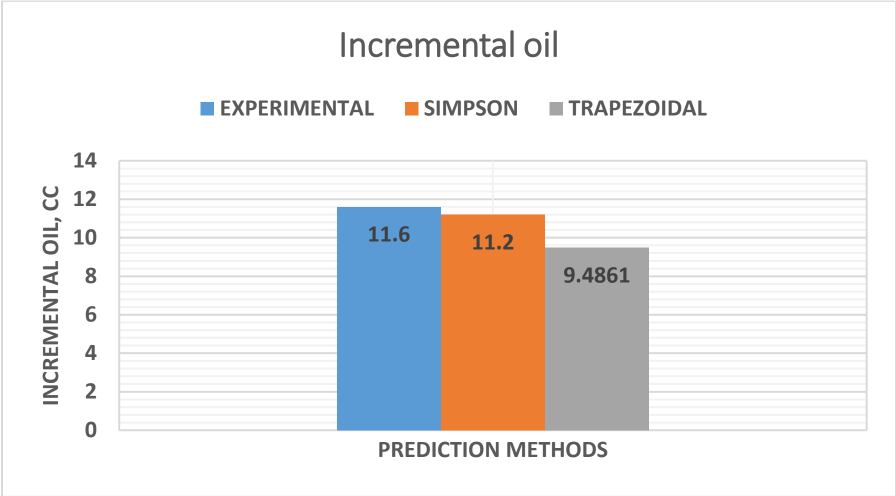

Figure 6a. Comparison of incremental oil prediction for the polymer flooding case study

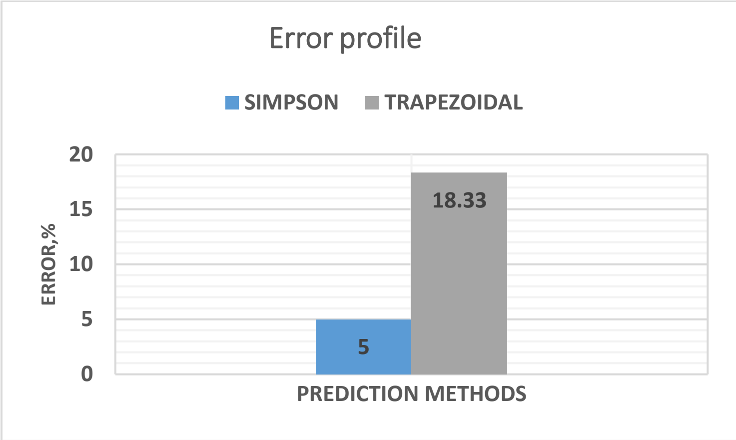

Figure 6b. Error incurred by the prediction methods for polymer flooding case study

Case three:

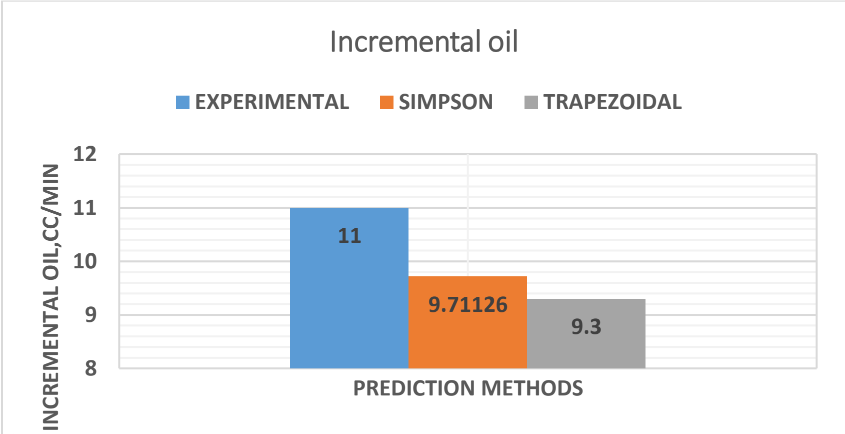

Figure 8a. Comparison of incremental oil prediction by the for the alikaline surfactant polymer flooding case study

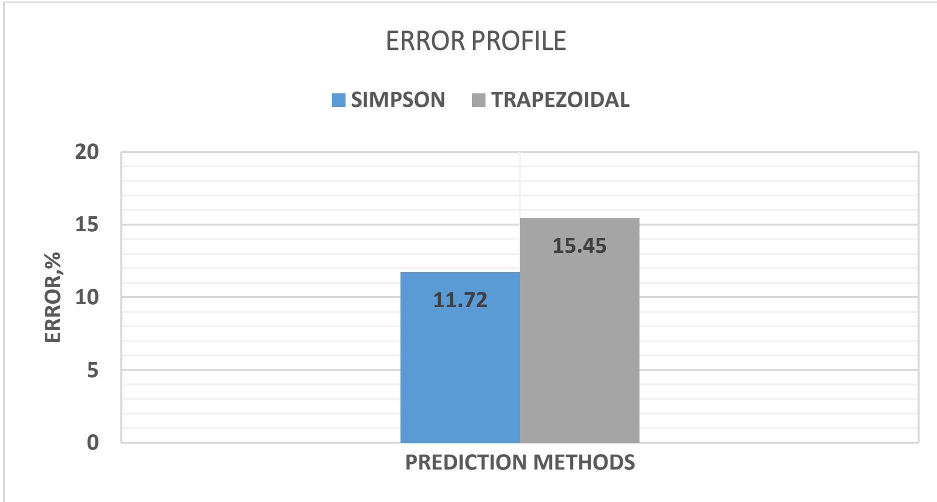

Figure 8b. Error incurred by the prediction methods amount for alikaline surfactanrt polymer flooding case study

The results from the analyses as presented in figures 4.1a to 4.3b shows the superiority of the Simpson rule in predicting incremental oil from successful EOR process compared to the Trapezoidal rule, which has previously been one of the most popular tools used in the past. This is shown clearly in the "error profiles" plots for each EOR process considered as the Simpson rule return closer incremental oil values to those measured in the laboratory

One of the reasons for this impressive performance by the Simpson rule can be attributed to its adoption of higher polynomials in modelling data points as compared to the straight line used in Trapezoidal rule. This ensures that rate data were well modeled and defined with appropriate functions.

It is worthy to note that our introduction of Finite difference in this research work has helped effectively in accounting for the error term in both the Simpson's rule and trapezoidal rule. In fact, it has further refined the performance of the Trapezoidal when compare the past observation when this was not taken in cognizance. This, which have been left unaccounted for in past works, shows truly that these rules sometimes can over/under estimate depending on the data we faced with and therefore should not be neglected or with contempt.

4.0. Conclusion and Recommendation

We have convincingly developed a more accurate alternative methodology for estimating the incremental oil**.** The incremental oil predicted by the Simpson rule was more accurate compare to the Trapezoidal rule and close to that measured from the laboratory. The improved accuracy of Simpson quadrature can be attributed to the adoption of higher polynomial in modelling data points; that is rate points. This gives a robust definition of each data point, giving a more accurate result for the respective integral evaluation**.** The concept of finite difference introduced was helpful in the error calculation and thus ensuring an accurate incremental oil prediction by the quadrature The difference between the integral under the incremental oil recovery curve and the integral under the corresponding curve of the projected decline represents the incremental oil**.** The performance of the quadrature will contimue to yield impressive results regardless of the EOR method that is under consideration and the number of data point being used.The Incremental prediction by the quadratures provides a "unique" solution. However, we do like to recommend the use of more numerical methods for oil recovery analysis.

Nomenclature

EOR	Enhanced oil recovery
x	Independent variable
x_i	Node
$f'_{i,i+1}$	First derivatives
$f''_{i,i+1}$	Second derivatives
Δf_i	First forward difference
N_p	Cumulative production(cc)
q	Recovery rate(cc/min)
y_i	Knot
h	Height (mins)
n	Number of panels
I	Area under graph
$f(x_n)$	Independent variable (Rate)
$\frac{d^2y}{dx^2}$	Second order differentials
$\frac{d^4y}{dx^4}$	Fourth order differentials

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References

[1] John M., C (1960). Oil property Evaluation. Englewood Cliffs: Prentice - Hall, Inc., 1960, pp.200

[2] Sutawanir D., Budi N., Asep K.,(2009). ''Robust Decline Curve Analysis''.J.Indones. Math. Soc (MIHMI) Vol.15, No. 2, 2009

[3] Mammud A and Olafuyi O. A. (2015) "Application of cubic spline numerical modelling on displacement mechanism." M.S thesis, University of Benin, Benin-City, Nigeria, 2015.

[4] Onuoha S.O and Olafuyi O.A (2013). Alkali/Surfactant/Polymer flooding using Gum Arabic; A comparative analysis. This paper was presented at the Nigeria Annual International Conference and Exhibition held in Lagos, Nigeria, 30 July-1 August 2013.

[5] Orivri, D.U, Taiwo, O.A. and Olafuyi, O.A. (2014). "Characterizing wettability Effect on Recovery from Surfactant Flooding in a light Oil Porous Media". The Journal of Nig. Institution of Prod. Engineers, Vol. 17, Page 143-152, June, 2014

[6] Atsenuwa J.B. Taiwo O.A., Mohammed I.U., Dala, J.A and Olafuyi, O.A, (2014). "Effect of Viscosity of Heavy Oil (Class-A) on Oil Recovery in SP Flooding Using Lauryl Sulphate and Gum Arabic". SPE 172401, Presented at the Nigeria Annual International Conference and Exhibition (SPE-NAICE), Lagos, August 5-7, 2014. doi: https://doi.org/10.2118/172401-MS.

[7] Yebu W. Y. F, Taiwo O. A and Olafuyi O. A, (2015). "Oil Recovery Performance of Surfactant/Polymer solutions in Flooding (A Comparative study)". This paper was presented at the 1st International Conference on Oilfield Chemistry & Flow Assurance (Oilflow 2015) at CPRT UNIPORT, Port Harcourt, Nigeria 22 – 24 June, 2015

[8] Uwari F, Taiwo O. A, Olafuyi O.A, (2015). Nigeria. "Surfactant-Polymer Flooding Schemes (A Comparative Analysis)". Presented at the Nigeria Annual International Conference and Exhibition (SPE-NAICE) in Lagos, Nigeria, 4 – 6 August 2015. doi: https://doi.org/10.2118/178367-MS

[9] Kharma R, Taiwo O. A, Olafuyi O. A, (2015) "Flooding For Light Oil Recovery In Strongly Water Wet Cores Using Sodium Hydroxide, Lauryl Sulphate, And Gum Arabic". Presented at the Nigeria Annual International Conference and Exhibition (SPE-NAICE) in Lagos, Nigeria, 4 – 6 August 2015. doi: https://doi.org/10.2118/178363-MS

[10] Taiwo O. A, Bello K, Mohammed I, Olafuyi O. (2016). "Characterization of Surfactant Flooding for Light Oil Using Gum Arabic", International Journal of Engineering Research in Africa, Vol. 21, pp. 136-147, 2016

[11] Mamudu A, Taiwo O. A and Olafuyi O. A (2016). "Cubic Spline and Graphical Techniques For Determining Unsteady State Relative Permeabilities In Field Cores", a paper presented at the SPE Nigeria Annual International Conference and Exhibition (SPE-NAICE), Lagos, Nigeria, August. (2016).

[12] Onuoha O. S, Olafuyi O. A and Taiwo O. A (2017). "Grain Size Effect on Permeability and Pressure Drop in Alkali-Surfactant-Polymer Flooding" a paper presented at the SPE Nigeria Annual International Conference and Exhibition (SPE-NAICE), Lagos, Nigeria, August. 2017.

[13] Ojukwu I. N, Taiwo O. A, Gimba S. B. A, Dala J, and Olafuyi O. A (2018). "Visualization of Heavy Oil Recovery Processes Using Hele-Shaw Cell", a paper presented at the SPE Nigeria Annual International Conference and Exhibition (SPE-NAICE), Lagos, Nigeria, August. 2018. DOI: https://doi.org/10.2118/193502-MS

[14] Madugu A, Tukur H, Gimba A,, Taiwo O, A, Bello K, Ismail M and Olafuyi O. A, (2018). "Comparative Analysis on Rate Dependent Polymer Flooding Using Bio and Synthetic Polymers" a paper presented at the SPE Nigeria Annual International Conference and Exhibition (SPE-NAICE), Lagos, Nigeria, August. 2018. doi: https://doi.org/10.2118/193529-MS

[15] Taiwo O. A, Bello K and Olafuyi O. A, (2019). Polymer Injection Performance in Multi Porous Medium *International Journal of Petroleum and Petrochemical Engineering* 2019, 5(3) : 10-16

[16] Russell J., Larry W., Garry P., Bill R. (2014) "Defining Enhanced Oil Recovery" in Fundamental of Enhanced Oil Recovery. United States: Society of Petroleum Engineers, 2014, pp.1-15.

[17] Gao, S., Zhang, Z., & Cao, C. (2011). Differentiation and Numerical Integral of the Cubic Spline Interpolation. Journal of Computers Vol. 6, No. 10, 2011. pp 2037-2044.

[18] Steven C., Chapra Raymond, P. C., (2010) Numerical Methods for Engineers. New York: McGraw Hil, 2010, pp.631-51.

[19] Alan J, (2022) Advanced Engineering Mathematics. San Diego: Harcourt/Academic Press, 2002, pp.1045

[20] Peressini, A. F, (2010) "Numerical analysis and its (Invisible?) Role in Mathematical Application" in Philisophical Perspectives on Mathematical Practice. South Africa: College Publishers, pp.331-349.

[21] Don W.G., Paul W G.,(1998). Enhanced Oil Recovery. Texas: Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers, 1998, pp.1-11

[22] Jeffrey R.C., (2012). Introduction to Numerical Methods. Hong Kong: The Hong Kong University of Science and Technology, 2012, pp.35-39.

[23] Jerome, F., Trevor, H., Robert, T.(2009). "Basic Expressions and Regularization'' in The Element of Statistical Learning, 2nd ed. Standford: Springer, 2009, pp.115-164.

[24] Laura R.Z, (2012)/ Advances in enhance oil recovery processes. Canada: Intech Open, 2012, pp.1-44

[25] Boyun G., William L., Ali G., (2007) ''Petroleum Decline Analysis in Petroleum Production Engineering. Louisiana: Elsevier Science and Technology.

[26] Prats M, (1982). Thermal recovery. New York: SPE of AIME; 1982.Google Scholar.

[27] Oloyede Olaoye, Oluwaseun Taiwo, Olalekan Olafuyi (2019). "Application of Numerical Methods for the Analysis of Incremental Oil", International Journal of Petroleum and Petrochemical Engineering (IJPPE), 5(4), pp.14-22, DOI: http://dx.doi.org/10.20431/2454-7980.0504003