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Modelling of UDP Downstream Throughput (UDPdownT) Dependence on the SNR in IEEE802.11b/g WLANs

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ABSTRACT

Network protocols are the world's most popular open-system protocol suite because they can be used to communicate across any set of interconnected networks and are equally well suited for WLAN communications. Packets transported by UDP applications have led to users experiencing disruption in seamless connections. In this work, an empirical model was used for the prediction of UDPdownT scenario dependence on the SNR in IEEE802.11b/g WLANs losses in WLAN network.

1. Introduction

There have been various means of communication in the past, ranging from the use of smoke signal from handheld lamps, to the use of town criers in villages [1, 2]. However, a more efficient form of communication was born, which replaced the old methods of communication. The use of smart devices such as android phones, tablets, computers, and various networks such as Wireless Local Area Network (WLAN) has made this method of communication possible [3].

Wireless LANs are most important access networks technologies in the internet, enabling mobility support for new mobile devices [4, 5]. It is a network device that can connect wirelessly without the need for cables unlike wired LAN where device communicates over Ethernet cables. WLAN regularly gives a network connection using an Access Point (AP) to the wider internet. It enables users in different areas like; offices, laboratories, universities or even libraries, to offer a network or have complete access to the internet without wiring the building with Ethernet. It is not limited to the numbers of ports on the router but can connect to multiple server and client. [6].

As more users are now heavily dependent on using WLAN, the goal is to ensure that wireless networks are optimized to meet users' expectations and there are various protocols that facilitate effective communication to enhance human and machine interaction [7]. This network protocols identify and make connections with each other. Protocols are set of rules that govern how packet is being sent over the internet. They operate in the Open System Interconnection (OSI) model, which is a layered model comprising of seven (7) layers, each representing a specific network function which has the best Protocols called TCP/IP [8].

In the transport layer of the Open System Interconnection (OSI) model, lies the Transmission Control Protocol (TCP) and User Datagram Protocol (UDP). In terms of initial connection, TCP has a reliable data connection while UDP has unreliable data connection [9, 10]. In TCP, packet sent get to its destination successfully, while in UDP, packet sent is either missing, duplicated or sent out of order. The UDP is faster and more efficient for applications that do not require acknowledgement [11] such as, in the areas of audio and video streaming, Domain Name System (DNS), Trivial file transfer protocol (TFTP), Dynamic Host Configuration Protocol (DHCP), Simple network management protocol (SNMP) among others.

Therefore, there is need for more predictive tools while carrying out network design and installation to achieve better customer satisfaction.

2. Methodology

The research work was carried out in a single user environment of the WLAN network. This environment was given attention due to the fact that these are the environment that the WLAN are likely going to be used in real-life. The different environments are; Open Space (Environment 1), Hallway (Environment 2), and Offices (Environment 3).

2.1. Experimental Setup

This research work introduces a new tool for better estimation of the actual UDP throughput being experienced by WLAN users. The tools used in the research are; 1 Access point (AP), 2 laptops (one represented the server device while the other laptop was use as the client device) with a wireless LAN card corresponding to the vendor of the access point.

The AP was setup as a bridge network to connect with the laptops. When connecting the device to the network, Power over Ethernet (POE), a direct Ethernet cable (RJ45) cable and a Power Adaptor was used. One cable was connected to the POE entering, another connected to the device, the other connected between the LAN entering, and the Ethernet computer access, then the power adaptor was connected to a Direct current (DC) source. The specification of the WLAN system used is shown in Table 1.

In monitoring the performance of the WLAN network, two laptops were used to measure for single users on the network, one representing the server while the other laptop represented the client and a continuous sending and receiving of different types of QoS traffic type for UDP through the network. The specifications of the device used for measurements are given in Table 2.

Table 1: WLAN Systems Specification Information

System Information							
Processor Specs					Atheros AR2315 SOC, MIPS 4KC, 180MHz		
Memory Information					16MB SDRAM, 4MB Flash		
Networking interface					1 X 10/100 BASE-TX (Cat. 5, RJ-45) Ethernet Interface		
Radio Operating Frequency 2412-2462 MHz							
TX SPECIFICATIONS				RX SPECIFICATIONS			
	DataRate	TX Power	Tolerance		DataRate	Sensitivity	Tolerance
802.11b	1Mbps	26 dBm	+/-1dB	802.11b	1Mbps	-97 dBm	+/-1dB
	2Mbps	26 dBm	+/-1dB		2Mbps	-96 dBm	+/-1dB
	5.5Mbps	26 dBm	+/-1dB		5.5Mbps	-95 dBm	+/-1dB
	11Mbps	26 dBm	+/-1dB		11Mbps	-92 dBm	+/-1dB
802.11g OFDM	6Mbps	26 dBm	+/-1dB	802.11g OFDM	6Mbps	-94 dBm	+/-1dB
	9Mbps	26 dBm	+/-1dB		9Mbps	-93 dBm	+/-1dB
	12Mbps	26 dBm	+/-1dB		12Mbps	-91 dBm	+/-1dB
	18Mbps	26 dBm	+/-1dB		18Mbps	-90 dBm	+/-1dB
	24Mbps	26 dBm	+/-1dB		24Mbps	-86 dBm	+/-1dB
	36Mbps	24 dBm	+/-1dB		36Mbps	-83 dBm	+/-1dB
	48Mbps	23 dBm	+/-1dB		48Mbps	-77 dBm	+/-1dB
	54Mbps	22 dBm	+/-1dB		54Mbps	-74 dBm	+/-1dB
Other Important Parameters							
Wireless Approvals					FCC Part 15.247, IC RS210, CE		
RoHS Compliance					YES		
Max Power Consumption					4 Watt		
Power Supply					12V, 1A (12 Watts). Supply and injector included		
Power Method					Passive Power over Ethernet (pairs 4,5+; 7,8 return)		
Operating Temperature					-20C to +70C		

Table 2: Specifications of Device used for Measurement

Computer Name / Use	Processor	Operating System	Installed Measurement Software	Network Card	RAM size
Laptop1/running Single user Server	**AMD Turion Dual-core RM-75 2.20GH**	**32 bit operating system**	**Tamosoft throughput test and insider**	**Atheros AR5007 802.11b/gWiFi Adapter**	**3GB**
Laptop2/ Running single client	**Intel (R) Pentium (R) CPU B960 @ 2.20GHz 2.20GHz**	**64-bit operating system**	**Tamosoft throughput test and insider**	**Dell wireless** 1702 **802.11b/g/n**	**4.00GB**

3. Result and Discussion

The statistical parameters of UDPdownT field data for a received SNR in combined single user scenario are presented in Table 3. It gives a high standard deviation of 6.872, which shows that UDPdownT vary considerably in the single user environment on the network.

Table 3: Statistical Data for UDPdownT Single Users Combined Environments

Statistical Parameters	**$UDP_{down}T$ (Mbps)**
Sample size (N)	951
Mean (Mbps or ms)	20.30
Std. Deviation (Mbps or ms)	6.872
Median (Mbps or ms)	23.60
Grouped Median	23.60
Std. Error of Mean	0.223
Range	60
Variance	47.226

The UDPdownT data for all single user was put together; we took the average of data in all the environments that was determined. A plot of the graph against calculated SNR describing the behavior is shown in Fig. 1 For the graph of UDPdownT plotted against calculated SNR, it is observed that as the SNR increases the throughput performance increases.

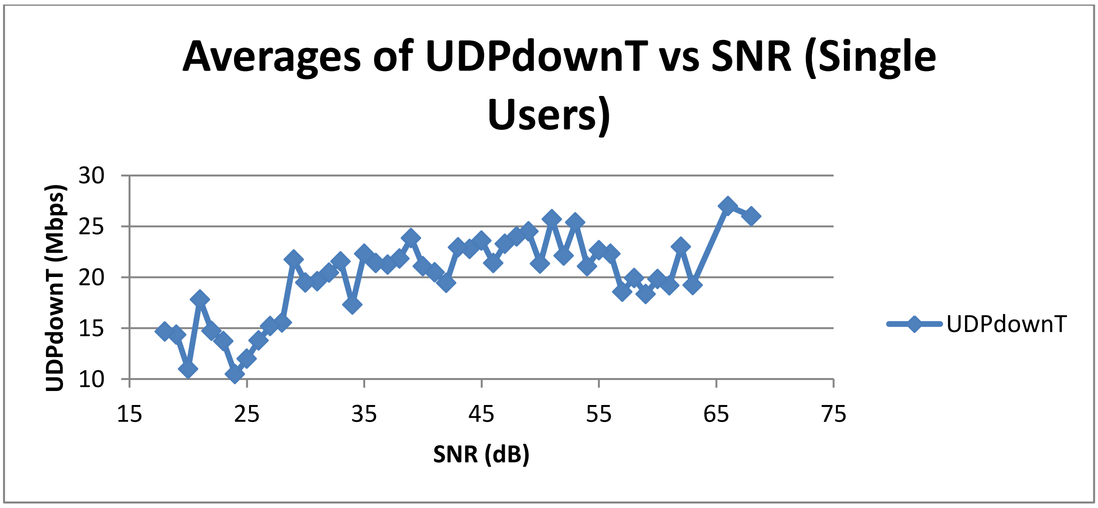

Fig 1. Average of $UDP_{down}T$ Single User Field Data vs. SNR

4. Empirical Model

The channel for UDPdownT was mathematically modelled, the model on Table 4 was developed by running regression on the data obtained from empirical method for the single user environment. Hence, the model equations that best fit the data in terms of description were selected. This was developed by selecting different model descriptions such as; compound, power, cubic, S, Growth, Exponential, and Logistic models to know the particular model that best emulate the behavior of the network. This is based on the highest R square value, lowest standard error of estimate and it being zero significance in nature, thus a model was chosen. For the dependent variable (UDPdownT), Power model best describes the network are evident in Table 4.

Table 4. Averages of $UDP_{down}T$ Single User Field Data vs. SNR

Summary of Model Parameters						
Model Summary	**R^2 Value**	**Standard Error of Estimate (Mbps)**	**F Test**	**T Test**	**Level of significance of the model (%)**	**Level of significance of the model coefficients (%)**
Power Model	**0.965**	**0.558**	**26016.233**	**161.295**	**0.000**	**0.000**

$$\text{SU UDPdownT Model}_2 = f(SNR) = \begin{cases} SNR^{b} & SNR \geq 41dB \\ SNR^{b} + e_1 & 29dB \leq SNR \leq 39dB \\ SNR^{b} + e_2 & 18dB \leq SNR \leq 28dB \end{cases} \quad (1)$$

The model performance indicates a high R^2 value of 0.965, signifying that the system is robust and accurate, the model also shows a low standard error of estimate of 0.558 and 0.000 significant in nature. Equation (1) was derived from the model data which yield a power equation and it describes where the throughput and SNR deviated, where e_1 and e_2 is a constant.

From the model table, we check for the F test from the F distribution table and the models were compared at 1% level of significant for their respective degree of freedom.

The comparison between the variances are given as;

$F\alpha, V_1, V_2$ (for the model) > $F\alpha, V_1, V_2$ (for tables in Appendix B) (The F value from the model must be greater than the F value from the F table).

Where α = Level of significance

The level of significant can be in 0.01 (1%), 0.025 (2.5%), 0.05 (5%), etc.

V_1 = The number of population parameters that will be estimated in the model equation.

$V_2 = N - V_1$

Where N is the sample sizes.

The 1% level of significance shows that there is 1% out of 100% chance that the Model will fail.

Carrying out the test, the Hypothesis are shown in Table 5. were defined.

T test from the T table value was done to see the percentage failure of the model and it was done for the model coefficient shown in Table 5. If the modulus of the T value from the generated model is greater than that obtained from the T Table, the model is accepted at that level of significance and degree of freedom, same implies for the F test. The F test and T test passed at stated level of significance and acceptable at 1% and 0.5% coefficient level. (The model will fail at 1% out of 100% and 0.5% out of 50%). For F test, the null hypothesis is rejected while the alternative is accepted, this means that the dependent and independent variable use in the model development are accepted to have a coherent variance at stated level of significance and degree of freedom and the model can be depended on within some stated boundaries.

Table 5. F test and T test result

Combined Environment for Single User Models							
Variable	**F Value for $Model_1$**	**F Value from the F Table**	**Remark**	**T Value from $Model_1$**	**T Value from the T Table**	**Coefficient**	**Remark**
UDPdownT	$F_{0.01, 1, 950}$= 26016.233	6.63	H_{01} is rejected and model was accepted at 1% level of Significance.	161.295	$T_{0.005, 950}$= 2.58	b_1	H_{01} is rejected and model was accepted at 0.5% level of Significance.

The models developed were validated with the field data shown in Fig. 2. From Fig. 2, the graph, the average field data for UDPdownT and model data for UDPdownT are plotted against SNR. It was observed that the Model data followed the field data value closely. This means that the developed model data can be a good representation of the field data for UDPdownT, single user.

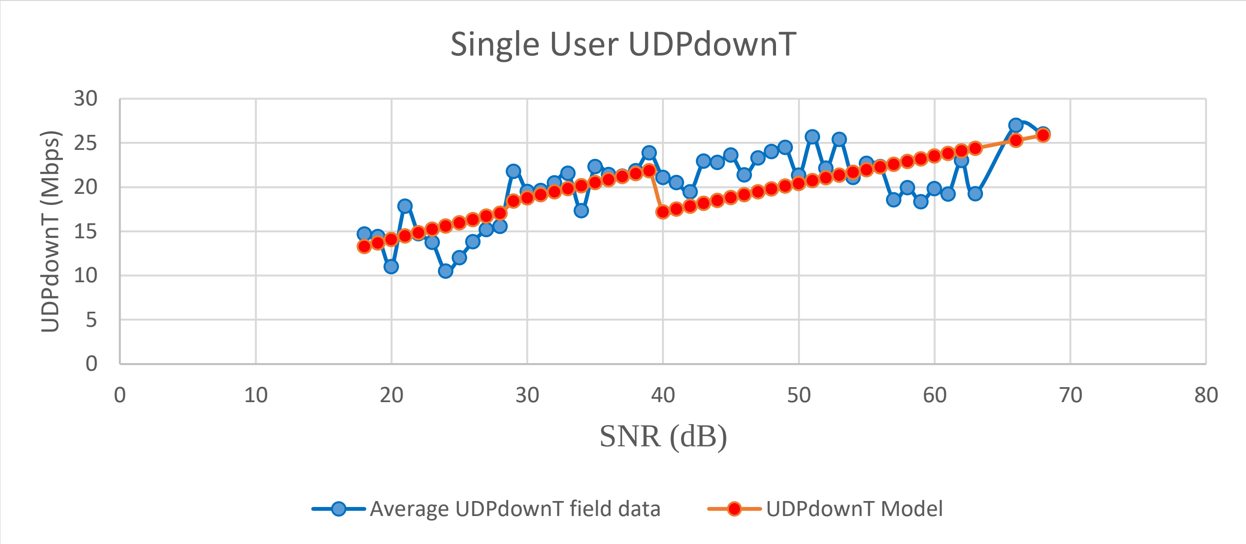

Fig. 2. UDPdownT Validation data plotted against SNR for Single User

5. Conclusion

This work has introduced a new prediction model of UDPdownT model. This was implemented based on the calculated SNR observed in IEEE 802.11b/g WLAN network. The result shows that the developed model demonstrates a strong dependence on UDPdownT model. The model would enable the Network Engineer design the network based on the users' experience.

Nomenclature

DHCP	Dynamic Host Configuration Protocol
DNS	Domain Name System
F	F-test
IP	Internet Protocol
Mbps	*Megabits per second*
OSI	Open System Interconnection Model
QoS	Quality of Service
R^2	Coefficient of determination or goodness of fit
UDP	User Datagram Protocol
UDP_{down}	User Datagram Protocol Downstream throughput
T	T-test
WLANs	Wireless local area networks

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My sincere gratitude to Almighty God for his guidance, protection and for giving me success in this program. I dearly appreciate my supervisor: Engr. Prof. F.O Edeko, my co-supervisor Engr. Prof. J.O Emagbeterefor their knowledge in making this work a success.

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Appendix

Table A1: Upper Percentage Points of the F-Distribution α=0.01

α = 0.01

v_1 →	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30
1	4052.29	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5	6055.8	6106.3	6157.3	6208.7	6234.6	6260.6
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.50
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.41	4.25	4.10	4.02	3.94
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.26	4.01	3.86	3.78	3.70
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.07	3.17	2.98	2.84	2.70	2.55	2.47	2.39
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70

Source:Chart Field (1978)

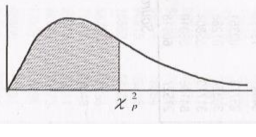


Table A2: Upper Percentage Points of the F-Distribution α=0.025

α = 0.025

ν_1 → / ν ↓	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63	976.71	984.87	993.10	997.25	1001.4
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46
3	17.44	16.04	15.44	15.10	14.83	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.41	4.36
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.56
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57

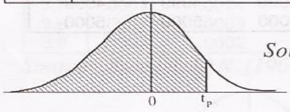
Table A3: Upper Percentage Points of the F-Distribution α=0.05

α = 0.05

ν_1 / ν_2 ↓	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	28.01	249.05	250.10
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.84	2.77	2.74	2.70
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46

Table A4: Percentile values, t_p for student's t distribution with v degree of freedom (shaded area equals p)

v	$t_{.995}$	$t_{.99}$	$t_{.975}$	$t_{.95}$	$t_{.90}$	$t_{.80}$	$t_{.75}$	$t_{.70}$	$t_{.60}$	$t_{.55}$
1	63.66	31.82	12.71	6.31	3.08	1.376	1.000	.727	.325	.158
2	9.92	6.96	4.30	2.92	1.89	1.061	.816	.617	.289	.142
3	5.84	4.54	3.18	2.35	1.64	.978	.765	.584	.277	.137
4	4.60	3.75	2.78	2.13	1.53	.941	.741	.569	.271	.134
5	4.03	3.36	2.57	2.02	1.48	.920	.727	.559	.267	.132
6	3.71	3.14	2.45	1.94	1.44	.906	.718	.553	.265	.131
7	3.50	3.00	2.36	1.90	1.42	.896	.711	.549	.263	.130
8	3.36	2.90	2.31	1.86	1.40	.889	.706	.546	.262	.130
9	3.25	2.82	2.26	1.83	1.38	.883	.703	.543	.261	.129
10	3.17	2.76	2.23	1.81	1.37	.879	.700	.542	.260	.129
11	3.11	2.72	2.20	1.80	1.36	.876	.697	.540	.260	.129
12	3.06	2.68	2.18	1.78	1.36	.873	.695	.539	.259	.128
13	3.01	2.65	2.16	1.77	1.35	.870	.694	.538	.259	.128
14	2.98	2.62	2.14	1.76	1.34	.868	.692	.537	.258	.128
15	2.95	2.60	2.13	1.75	1.34	.866	.691	.536	.258	.128
16	2.92	2.58	2.12	1.75	1.34	.865	.690	.535	.258	.128
17	2.90	2.57	2.11	1.74	1.33	.863	.689	.534	.257	.128
18	2.88	2.55	2.10	1.73	1.33	.862	.688	.534	.257	.127
19	2.86	.254	2.09	.173	1.33	.861	.688	.533	.257	.127
20	2.84	2.53	2.09	1.72	1.32	.860	.687	.533	.257	.127
21	2.83	2.52	2.08	1.72	1.32	.859	.686	.532	.257	.127
22	2.82	2.51	2.07	1.72	1.32	.858	.686	.532	.256	.127
23	2.81	2.50	2.07	1.71	1.32	.858	.685	.532	.256	.127
24	2.80	2.49	2.06	1.71	1.32	.857	.685	.531	.256	.127
25	2.79	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
26	2.78	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
27	2.77	2.47	2.05	1.70	1.31	.855	.684	.531	.256	.127
28	2.76	2.47	2.05	1.70	1.31	.855	.683	.530	.256	.127
29	2.76	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
30	2.75	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
40	2.70	2.42	2.02	1.68	1.30	.851	.681	.529	.255	.126
60	2.66	2.39	2.00	1.67	1.30	.848	.679	.527	.254	.126
120	2.62	2.36	1.98	1.66	1.29	.845	.677	.526	.254	.126
∞	2.58	2.33	1.96	1.645	1.28	.842	.674	.524	.253	.126



Source: Spiegel, M.R. (1961)